GENERAL COURSE OBJECTIVE (ENCE 4318) : To familiarize the students with the application of the basic principles of fluid mechanics to problems associated with pipe and open channel flows. The applications in this course will involve pipelines, pipe networks, flow measurement, flow in rivers, design of canals, and hydraulic structures.

SPECIFIC COURSE OBJECTIVES (ENCE 4318) :
On the successful completion of this course the students should have the ability to:
a) understand and applied the continuity equation to steady open-channel flows and steady incompressible pipe flows;
b) understand and applied the energy principle to steady open-channel flows and steady incompressible pipe flows;
c) understand and applied the momentum principle to steady open-channel flows and steady incompressible pipe flows;
d) apply the principles of hydrostatic to hydraulic structures.
e) apply the Manning Equation for uniform open channel flow;
f) design simple lined and unlined open channels;
g) compute water surface profiles for gradually varied steady flow;
h) compute the steady state operating flow and head for a pipeline with a pumping station;
i) design a culvert with inlet or outlet control.
j) design a simple spillway and stilling basin.

ADDITIONAL COURSE WORK FOR ENCE 4318G Credit:
Graduate students will be required to do additional work for credit in this course as outlined below.

PREREQUISITE: UNO course ENCE 3318 or an equivalent undergraduate course in fluid mechanics.
CO-REQUISITE: UNO course ENCE 4319.

TEXT: ***Elementary Hydraulics*** by James F. Cruise; Mohsen M. Sherif; Vijay P. Singh, Thomson, 2006.

REFERENCES: Terry W. Sturm, (2001) "Open Channel Hydraulics", McGraw-Hill Book Co., New York, NY.
Ven te Chow, (1959) "Open Channel Hydraulics", McGraw-Hill Book Co., New York, NY.
US Army Corps of Engineers. *Hydraulic Design Manuals*
Giles, J.B. Evett, and C Liu . "Fluid Mechanics and Hydraulics,"
Schaum's Outlines, McGraw Hill, New York, Third Edition, 1995.

INSTRUCTOR: Alex McCorquodale, Ph.D. P.Eng., P.E.
Room EN 817 or CERM 315
Telephone 280 6074 (same telephone for both offices)
jmccorqu@uno.edu

OFFICE HOURS: Thursday 9 am-12:00 pm or by appointment. Please email or arrange for an appointment at the time of the lecture.

GRADING SCHEME:

1. Assignments & Quizzes	**15%.**
2. Two 80 minute Mid-term tests (open book)	**45%.**
3. Final examination (open book)	**40%.**

4. Grades
   - A 89.5-100
   - B 79.5-89.5
   - C 69.5-79.5
   - D 59.5-69.5
   - F Less than 59.5.

TENTATIVELY TESTS WILL BE:

Last week in September/First week in October
First or second week in November

ENCE 4318 COURSE GUIDE

*CLASS MEETINGS
Bulletin

STUDENT LEARNING PROCESS
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The tutorial and lab assignments are design to strengthen theoretical concepts that are covered in the lectures. In addition weekly assignments will be given to give the students the opportunity to test their understanding of the course material. All assignments will be graded and feed-back provided to the students.

*EXAMS
There will be one mid-term test and a comprehensive Final examination at the end of the semester. Check the Blackboard calendar for the exam schedule. All tests and the final examination will be of the "open-book" type. The term tests (1 or 2) and the final examination will be 2 hours in duration.

COURSE FOLDER
You are encouraged to maintain a course folder that contains class handouts, your complete solutions to all of the assigned homework problems.

*HOMEWORK, TUTORIALS, LABS AND QUIZZES
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TEXT BOOK
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Additional materials will be handed out in class or posted on Bb.

REQUIRED SUPPLIES
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Tuesday and Thursday 9:00 am to 11:00 noon or by appointment on Wednesday 9-11 am. Please call, email or arrange for an appointment at the time of the lecture.

ENCE 4318 COURSE GUIDE

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***GRADING SCHEME / SCALE**

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3. Final examination (open book) — 40%.
4. Grades
   - A 89.5-100
   - B 79.5-89.5
   - C 69.5-79.5
   - D 59.5-69.5
   - F Less than 59.5.

***ATTENDANCE POLICY**

Attendance at lectures is REQUIRED and attendance will be recorded. The tutorials (ENCE 4319) are MANDATORY. A zero grade will be assigned for the report or design if the student does not attend the respective lab or tutorial session. Handouts will only be available for those attending the respective class.

***ACADEMIC DISHONESTY POLICY** click
http://alt.uno.edu/stud_handbook.html#five

COMMUNICATIONS POLICY

As a matter of policy at UNO, all Blackboard accounts are created using only UNO email addresses. If you wish to use a different email address other than your UNO address, it is up to you to set up forwarding from your UNO email account to your desired email address. This can be done in one of two ways: by sending a request to or going in person to the UCC Help Desk, or by going to http://mail-service.ucc.uno.edu:7633/popstore_user/, logging in, and using UNO email forwarding options. It is the student's responsibility to obtain access information (username and password) for your UNO email account. To obtain UNO email account information, click here. To simplify matters in communication, I will only use your UNO email addresses (e.g., student@uno.edu). I will post important course information on blackboard.

***ACCOMMODATIONS FOR STUDENTS WITH DISABILITIES**

Students who qualify for services will receive the academic modifications for which they are legally entitled. It is the responsibility of the student to register with the Office of Disability Services (UC-260) each semester and follow their procedures for obtaining assistance.

QUIZ 1
NAME Donald Scrolleman 4/4

At a site on the Mississippi River, it is proposed to install run-of-the-river-turbines.

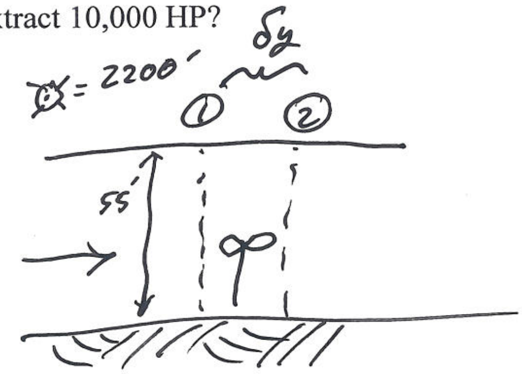
Assume that the River current at the site is 5 ft/sec, the D~ R = 55 ft and the P = 2200 ft. $\cong b = B$ b/c very wide vs. shallow

1. What is the flow in the River? 605,000 cfs

2. What will be the impact on the water depth if the turbines extract 10,000 HP?

Select the best answer:

a) Upstream depth will increase by about 0.15ft
b) Upstream depth will decrease by about 0.15ft
c) Downstream depth will increase by about 0.15ft
d) Downstream depth will decrease by about 0.15ft
e) None of the above.



NOTE 1HP =550 ft-lbs/sec. $\times 10{,}000^{HP} \approx 5{,}500{,}000 \frac{ft\,lb}{s}$

Show proof!

Assume: upstream velocity $\approx$ downstream velocity

$Q = AV$
$= 5\frac{ft}{s}(55 ft)(2200 ft)$
$= 605{,}000 \frac{ft^3}{s}$

$\alpha_1 = \alpha_2 = 1$
$h_{z_1} = h_{z_2}$
$h_L = \varnothing$ b/c elevation = b/t points

$N_{F_1} = \frac{5}{(32.2(55))^{0.5}} = << 1$
∴ depth is prob. more important than velocity

* Assume pt. ① & ② = same elevation

$V_1 \sim V_2 \sim 5 \frac{ft}{s}$

$y_1 = y_2 + (h_{z_2} - h_{z_1}) + \left(\frac{\alpha_2 V_2^2}{2g} - \frac{\alpha_1 V_1^2}{2g}\right) + \frac{h_{L_{1\to2}} + 5500000}{62.4(605000)}$

$y_1 - y_2 = 0.146 \sim 0.15$ ft (upstream change)

Head extracted by turbine

Energy Equ.

mechanical energy head

Rate of Power out

$H_{T_1} = H_{T_2} + h_{L_{1-2}} + H_{Tb}$

$y_1 + h_{z_1} + \alpha_1 \frac{V_1^2}{2g} = y_2 + h_{z_2} + \alpha_2 \frac{V_2^2}{2g} + h_{L_{1\to2}} + \frac{P'_{out}}{\gamma Q}$

Correct for Velocity

If $V_2 = 5$ ft/s ; $V_1 = \frac{V_2 A_2}{A_1} = \frac{605{,}000}{(55+0.15)(2200)}$
$= 4.99 \frac{ft}{s}$

$\frac{5^2}{64.4} - \frac{4.99^2}{64.4} = 0.002$ (very little effect)

Term Test No. 1- ENCE 4318

Duration 1 hour and 25 minutes

This is an open-book test; you may use text books, class notes and assignments/tutorials.

Laptop computers are not permitted.

Please attempt all four questions (4) questions.

Enter your answers in the space provided on the question sheet.

State any assumptions that you make in solving the problems.

Your Name Donald Jerolleman

Student No. 2330000

Marks

Question 1:	15	15
Question 2:	15	13
Question 3:	15	13
Question 4:	10	10
Total	55	51

good paper!

1. Determine the force on the support for the 90° elbow shown in Figure 1. The elbow weighs 400 lbs.

    a. the pressure head at section (2) is closest to:{Answer 36, 38, 40, 44} ft

    b. the force on the support is (express as magnitude and direction):

Answer F_x = 23.815 kips → or ←: F_y = 21.83 kips ↓ or ↑ :

Assume:
$\alpha = \beta = 1$
Neglect energy loss and friction.
Given:

Upstream pressure head (section 1) =	50	ft
Diameter =	3	ft
Q =	80	cfs
Elbow radius =	6	ft

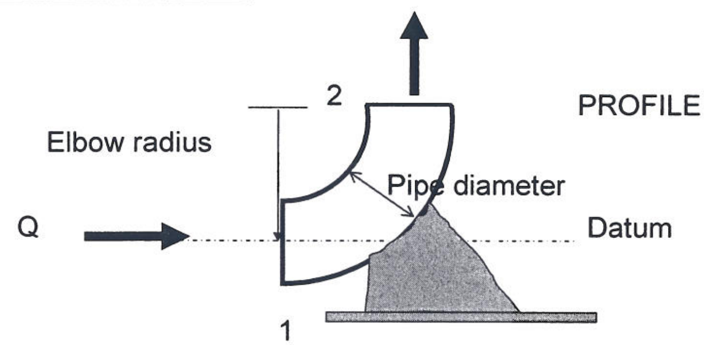
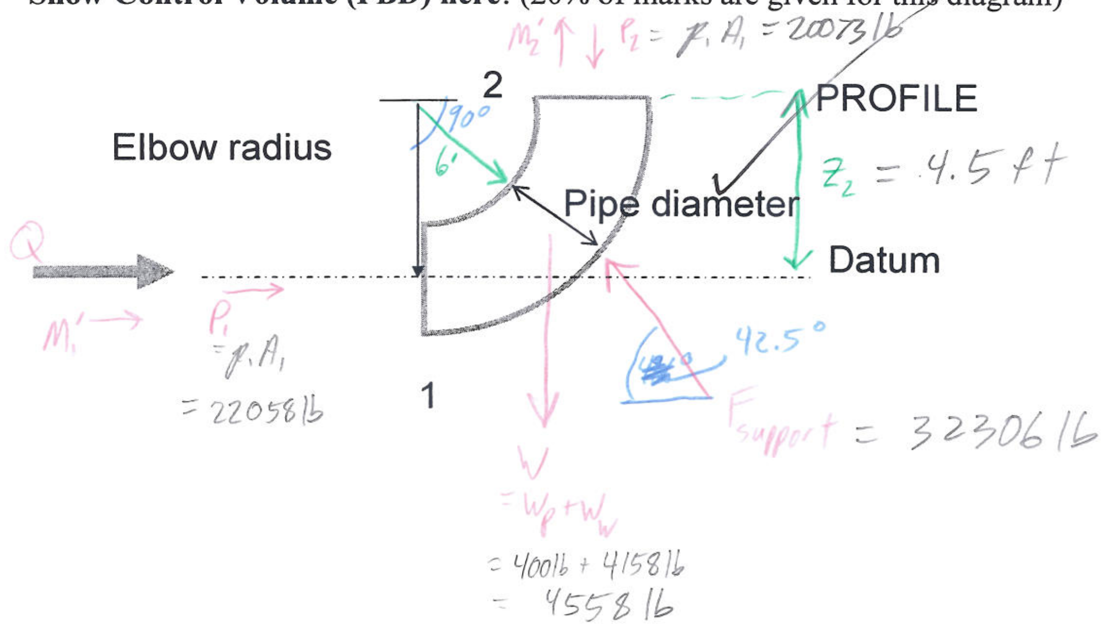


Figure 1.

Solution:
Show Control Volume (FBD) here! (20% of marks are given for this diagram)



$A_p = \frac{\pi}{4}(3ft)^2 = \underline{7.07\ ft^2}$

$\forall_{pipe} = A_p(r)(\theta^{rad}) = 7.07(6)\left(\frac{\pi}{2}\right) = \underline{66.6\ ft^3}$

$w_p = \underline{400\ lb}$

$W_w = 66.6\ ft^3(62.4\ lb/ft^3) = \underline{4158\ lb}$

$h_{L_{1-2}} \sim \varnothing$

$V_1 = V_2 = \frac{Q}{A} = \frac{80\ ft^3/s}{7.07\ ft^2} = \underline{11.32\ ft/s}$

$\frac{p_1}{\gamma} = 50'$

$P_1 = p_1 A_1 = 50(62.4)(7.07) = \underline{22058\ lb}$

$Q = 80\ cfs$

radius = 6'

Impulse - Mom

x-direction

$\Sigma F_x = \rho Q V_{2_x} - \rho Q V_{1_x}$ ($\rho Q V_{2_x} \to \varnothing$)

$-N_x + p_1 A_1 = -\rho Q V_1$

$50(62.4)(7.07) + (1.94)(80)(11.32) = N_x$

$N_x = \underline{23815\ lb} \leftarrow$

Energy Bal

$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_{L_{1-2}}$ ($z_1, \frac{V_1^2}{2g}, \frac{V_2^2}{2g}, h_{L_{1-2}} \to \varnothing$)

$50' = \frac{p_2}{62.4} + 4.5' \to p_2 = 2839\ psf$

$P_2 = p_2 A_2 = 2745(7.07) = \underline{20073\ lb}$

Y-direction

$\Sigma F_y = \rho Q V_{2_y} - \rho Q V_{1_y}$ ($\rho Q V_{1_y} \to \varnothing$)

$N_y - P_2 = \rho Q V_2$

$N_y = (1.94)(80)(11.32) + 20073$

$N_y = \underline{21830\ lb} \uparrow$

$N = \sqrt{N_x^2 + N_y^2} = \underline{32306\ lb}$

$N_\theta = \tan^{-1}\left(\frac{21168}{23815}\right) = \underline{42.5^\circ}$

Force of water on support

$F_{w \to sup.} = \underline{31863\ lb}$

$F_{w \to sup\ \theta} = \underline{42.5^\circ}$

2. For the concrete dam in Figure 2 with a seepage barrier of length C:
a) Determine the Factors of Safety against sliding and overturning.
b) Will there be any tension in the foundation?

Assume: the specific weight of concrete = 150 lbs/ft^3
Neglect: ice, silt and earthquake forces.

H	60	ft
Friction factor at the base =	μ = 0.7	
C	30	ft

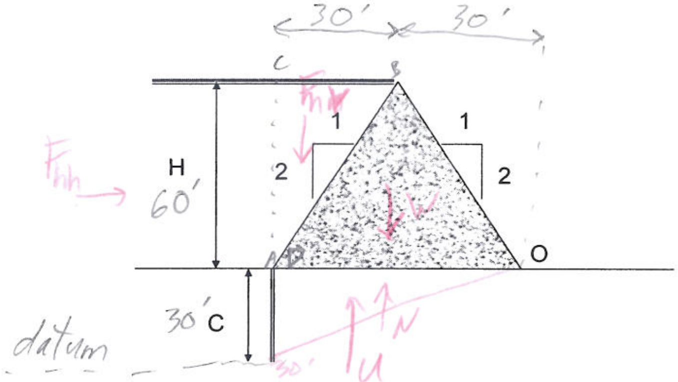


Figure 2.
Circle the closest answer here!

Factor of safety against sliding = _{ <1, 1 – 1.5, 1.5- 2, >2 } (Safe), (Unsafe)

Factor of safety against overturning = {<1, 1 – 1.25, 1.25 – 1.5, >1.5} (Safe), (Unsafe)

Tension in base: (Yes) (No)

Solution:

Force ID	**F$_x$** →+	**F$_y$** ↑+	**Moment arm**	**+Moment uprighting**	**-Moment overturning**
Hydro Horizontal	112.32 K	0	60/3 = 20'	0	2246.4 k-ft
" Vertical	0	(-56.16 K)	30 + 2(30)/3 = 50'	2808 k-ft	0
weight Dam	0	(-270 K)	30'	8100 k-ft	0
U	0	56.16 K	2(60)/3 = 40'	0	2246.4 k-ft
SUM			Σ	10908 k-ft	4493 k-ft

N= 213.8 K

F$_{fmax}$ = 150 K

x$_N$ = 30.0

w/in middle 1/3

$N = W - U = 213.8^K$

$\mu N = 149.66^K \sim 150^K$

$X_N = \frac{\Sigma+ - \Sigma-}{N} = 30.0$

$F_{OS\,OT} = \frac{\Sigma+}{\Sigma-} = 2.428$

$F_{OS\,slid} = \frac{\mu N}{\Sigma F_x} = \frac{150}{112.32} = 1.34$

3. a) Determine the operating point and efficiency for two pumps operating in parallel as shown in Figure 3 for the pump curve in Figure 4.
b) Will the pump cavitate?
The pump curve is attached.
Given: Kps = 1;
Kpd= 4.
Wet Well Level = 100 ft
Reservoir Level = 135 ft

Circle the closest answer below:

Qo = { 1.0, 1.1, 1.4, 1.6, 2.0 } cfs

Ho = {35, 39, 40, 45, 47 } ft

Efficiency in %=

40 48 75 80

Cavitation: (Yes) (No)

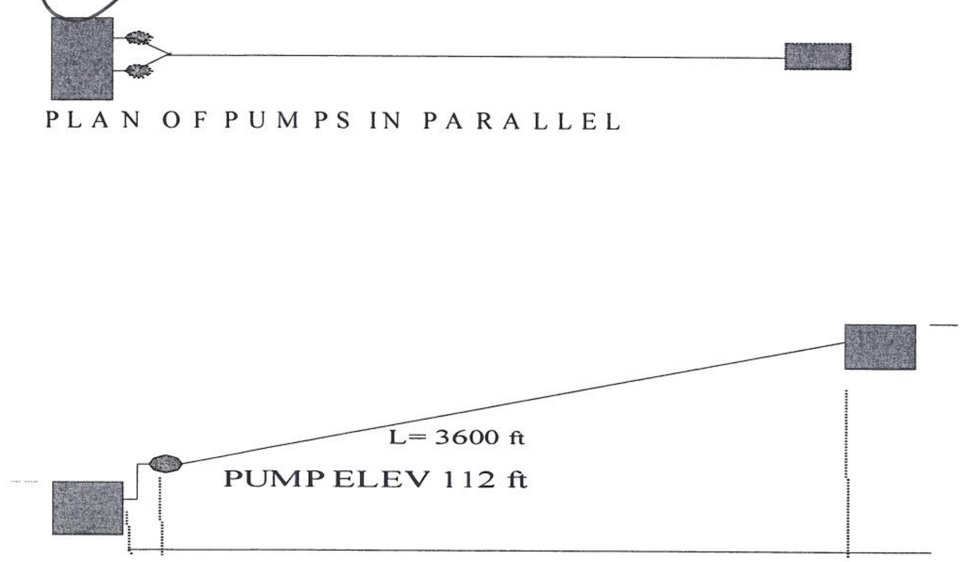


Figure 3.

$= Q_s^2$ $= 4(Q_d)^2$ $H_{sys} = H_{st} +$

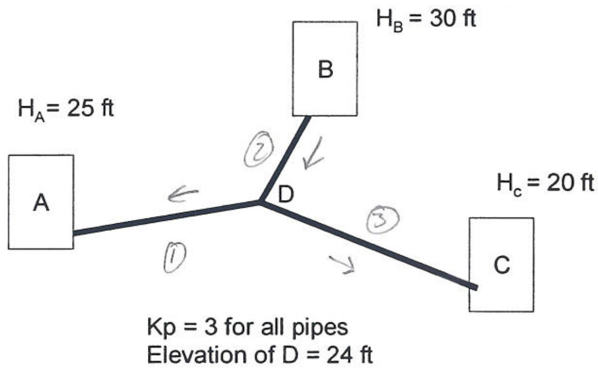
Qd	Qs	Hst	hLs	hLd	Hsys
1	0.5	35'	0.25	4	39.25
1.1	0.55	35	0.303	4.84	40.14
1.2	0.6	35	0.36	5.76	41.12
1.3	0.65	35	0.423	6.76	42.18
1.4	0.7	35	0.49	7.84	43.33
1.5	0.75	35	0.563	9	44.56
1.6	0.8	35	0.64	10.24	45.88

Qd	Qs	hLs	hLd	Hsys
1.8	0.9	0.81	12.96	48.77
1.7	0.85	0.723		47.3

4. In the three reservoir problem illustrated below, the value of H_D is closest to:

{ 28, 26, 25, 24 } ft

10



Assume $H_D = \frac{30+25+20}{3} = 25$

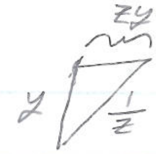
$\Delta H = H_{res} - H_D$ $\quad Q = \sqrt{\frac{\Delta H}{Kp}}$

Figure 5.

Pipe	D(ft)	Kp	ΔH(ft)	Q(cfs)	Res	H(ft)
1	—	3	∅	∅	A	25
2	—	3	5	1.291	B	30
3	—	3	(-5)	-1.291	C	20

$\Sigma = \varnothing$

$R = \frac{A}{P}$ $R_n = \frac{RV}{\nu}$



Hydro Test 2 Review Nov. 8 (1)

(1) Given b, Q, E

* Find Alt Depths

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2} = y + \frac{Q^2}{2g(A)^2}$$

(put in terms of y)

Solve polynomial for y_u & y_e

* Find y_c $E_c = y_c + \frac{D_c}{2}$; $D_c = \frac{A}{B}$

	Triangle	Trapazoid	Rectangle
A	zy^2	$y(zy+b)$	$y(b)$
B	$2zy$	$b+zy(2)$	b
P	$2y\sqrt{z^2+1}$	$b+2y\sqrt{z^2+1}$	$2y+b$

* Find y_c using Q_c

$$Q_c = Q_{max} = V_c A_c = \sqrt{g D_c}\, A_c \; ; \; D_c = \frac{A}{B}$$

* Given S_o, n Find y_n

$$\frac{n\,Q}{c' S_o^{1/2}} = AR^{2/3} = A\,\frac{A^{2/3}}{P^{2/3}}$$

$\{ c' = 1 \,(S.I.) = 1.486 \,(U.S.)$

Hydro Review test 2 (3)

(3) Step : (a) Find N_{F_1} (b) y_2 for H.J. if NO $\mathcal{S}$ "step"
(c) y_2 for H.J. w/ $\mathcal{S}$ (d) E loss $1 \to 2$ (e) Hydro pressur @ 1 & 2

(a) $N_F = \frac{V_1}{\sqrt{g y_1}}$ (b) $y_2 = \frac{1}{2} y_1 \left(\sqrt{1 + 8 (N_{F_1})^2} - 1 \right)$

(c) $P_1 + \rho Q V_1 = P_2 + \rho Q V_2$ whole term squared !! Remember

$$\frac{1}{2} \gamma y_1^2 W + \rho \frac{Q^2}{A_1} = \frac{1}{2} \gamma (y_2 + \mathcal{S})^2 W + \rho \frac{Q^2}{A_2} \; ; \; A_2 = y_2 (W)$$

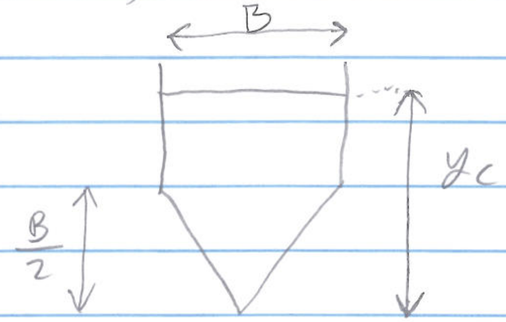
$\rho = 1.94$ slugs

(d) $y_1 + \frac{V_1^2}{2g} = y_2 + \mathcal{S} + \frac{V_2^2}{2g} + h_L$ solve for h_L

(e) $P_1 = y_1 \gamma \; ; \; P_2 = y_2 \gamma$

2

Hydro Test 2 Review (2)



(a) Find y_c

(b) Find Q_{max}

∴ Slope = 1:1

$Q_c = Q_{max}$

(a) $E_c = y_c + \left(\frac{D_c}{2}\right)$

where $D_c = \frac{A_\Delta}{B_\square} + \frac{A_\square}{B_\square}$ (taken from bottom of rectangle) [y ↓ from]

(b) using y_Δ, y_c (above) plug into $Q_c = A_c\sqrt{D_c(g)}$

Hydro Test 2 Review

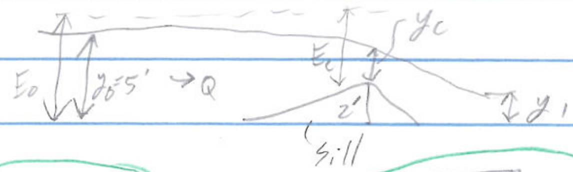
(4) given Q, z, n, S_o, b Find y_n, y_c

Manings $\frac{nQ}{c' S_o^{1/2}} = AR^{2/3} = \frac{A^{5/3}}{P^{2/3}}$ solve for y_n

$Q_c = A_c V_c = \sqrt{g}\sqrt{D_c}\, A_c$ solve for y_c

(5)

Hydro Test 2 Review



W = 3'

$$E_c = \frac{3}{2} y_c \qquad y_c = \sqrt[3]{\frac{\left(\frac{Q}{W}\right)^2}{g}}$$

$$E_0 = E_c + S = \frac{3}{2} y_c + S = \frac{3}{2} \sqrt[3]{\frac{\left(\frac{Q}{W}\right)^2}{g}} + S$$

$$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8N_F^2} - 1\right)$$

UNKNOWN

$$E_0 = y_0 + \frac{Q^2}{2g(y_0 W)^2} = E_c + S$$

$$E_1 = y_1 + \frac{Q^2}{2g(y_1 W)^2} = E_c + S$$

~~Find $N_f = \frac{V_1}{\sqrt{g y_1}}$~~ ~~$E_c + S = y_1 + \frac{Q^2}{2g(W y_1)^2}$~~

~~solve for y_1~~

~~$Q = V_1 (y_1 W)$~~

$$y_0 + \frac{Q^2}{2g(y_0 W)^2} = E_c + S \quad \text{sub } y_c \text{ into } E_c$$

$$= \frac{3}{2}\sqrt[3]{\frac{\left(\frac{Q}{W}\right)^2}{g}} + S$$

* solve for Q = [52.9 cfs]

* $y_1 + \frac{Q^2}{2g(y_1 W)^2} = E_c + S$ solve y_1

* $E_1 - y_1 = \frac{V_1^2}{2g}$ solve for V_1

* $N_{F_1} = \frac{V_1}{\sqrt{g y_1}}$ solve N_{F_1}

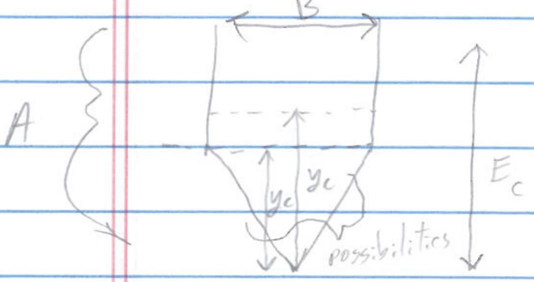
* $y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8N_{F_1}^2} - 1\right)$ solve y_2

Hydro Test 2 review

Know all: - calc ult., critical, alternate depths, normal depth
- specific energy
- find critical flow

$Q_{max} = Q_c$

max flow = critical flow



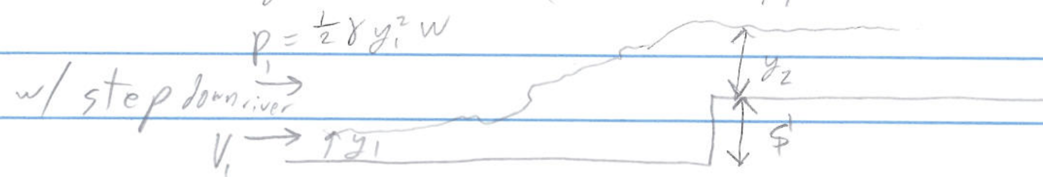
① Find y_c

$E_c = \left(y_c + \frac{D_c}{2}\right)$

$D_c = \frac{A}{B} = (A_\triangle + A_\square)/B = \left(z\left(\frac{B}{2}\right)^2 + \left(y_c - \frac{B}{2}\right)B\right)/B$

If multi-choose test Try answers for y_c in & see if D_c is ✓

$y_2 = ?$ $y_2 = \frac{1}{2} y_1 \left(\sqrt{1 + 8 N_{F_1}^2} - 1\right)$ *No step



* a good guess is $y_2 - \$ = y_1$

spec. Force $F_1 = F_2 \rightarrow P_1 + \rho Q V_1 = \frac{1}{2}\gamma(y_2 + \$)^2 W + \rho Q V_2$ (with M_1 and P_2 marked over the terms)

Could use $P_1 + \frac{\rho Q^2}{A_1} = \frac{1}{2}\gamma(y_2 + \$)^2 W + \rho \cdot \frac{Q^2}{A_2}$

where $A_2 = y_2(W)$

A

Term Test No. 2- ENCE 4318

11:00am to 12:30pm

November 9, 2010

- This is an open-book test; you may use text books, class notes and assignments/tutorials. Laptop computers are not permitted.
- Please attempt all four (4) questions.
- Enter your answers in the space provided on the question sheet. Marks **will not** be deducted for wrong answers in the multiple choice questions.
- State any assumptions that you make in solving the problems.

Your Name Donald Serolleman

Student No. 2330000

Marks

Question 1:	20	18
Question 2:	12	12
Question 3:	18	18
Question 4:	12	12
Total	62	62

Excellent!

1. Circle the nearest answer in the following multiple choice questions:

Given a triangular channel that has a 1H:1V side slope and a specific energy of 20 ft:

~~$E_o = E_c = \frac{2}{3} y_c$~~

~~$y_c = 30$~~

$E_c = y_c + \frac{D_c}{2}$ $D_c = \frac{zy^2}{2zy} = \frac{1}{2} y$

$20 = y_c + \frac{1}{4} y_c = y_c(1 + \frac{1}{4}) \rightarrow y_c = 16$ ft

a) The critical depth is closest to:
{Answer **[6.7], [13.3] [16] [20]** units ft

b) If the critical flow is closest to:
{Answer **170; 1400; 2000, 2200, 4000)** units cfs

$Q_c = \sqrt{g D_c}\, z y_c^2 = \sqrt{g \frac{1}{2} y}\, y^2 = 4108$ cfs

$c' = 1.486$

c) If the critical slope is closest to (given n = 0.015):
{Answer **0.012; 0.0012; 0.003; 0.004)** units N/A

$S_o = \left(\frac{n Q_c P^{2/3}}{c' A^{5/3}}\right)^2 \rightarrow = 40.38 \frac{[4 y_c^2 (z+1)]^{1/3}}{z^{5/3}\, y_c^{11/3}} = 0.0197$

d) If Q = 200 cfs and E = 20 ft the alternate depths are closest to:
{Answer
[20, 2.5] [19.98, 2.62], [19.99, 2.63] [19.99. 2.44]
units__

$20 = y + \frac{Q^2}{2g z^2 y^4}$

$20 = y + 621.1 \frac{1}{y^4} \rightarrow \phi = y^5 - 20y^4 + 621.1$ $\{19.99, 2.44\}$

e) If the slope 0.009 and n = 0.015 the normal depth is closest to:
{Answer **1.2; 4.0; 5.1, 5.8, 6.3)** units ft

$\frac{n Q}{c' S_o^{1/2}} = \frac{A^{5/3}}{P^{2/3}} \rightarrow C_Q = 21.28 = \frac{y^{11/3}}{(4y^2(2))^{1/3}} = \frac{y^{11/3}}{2 y^{2/3}} = \frac{1}{2} y^{9/3}$

($Q = 200$, C_Q)

$(42.56)^{3/9} = y_n =$ ~~9.48~~ 3.49 ft

2. The specific energy for the open channel in Figure 1 is 12 ft. Given B= 10 ft

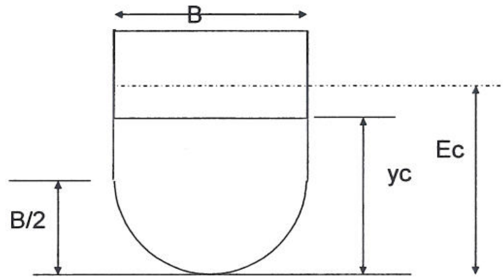


Figure 1.

a) The critical depth is closest to:
{Answer: **5.0, 6.5; 7.1; 8.4)** units__

$E_c = y_c + \frac{D_c}{2}$

$D_c = \frac{A}{B} = \frac{A_o}{B_{\square}} + \frac{A_{\square}}{B_{\square}} = \frac{(0.125)\pi b^2}{b} + \frac{yb - \frac{b^2}{2}}{b} = 0.3927b + y - 0.5b$

$= y - 0.1073b = y - 1.073$

$b = 10'$

$A_o = \frac{1}{2}\pi\frac{b^2}{4}$

$A_{\square} = (y - \frac{b}{2})b = yb - \frac{b^2}{2}$

$B_{\square} = b$

$12 = y_c + 0.5y_c - 0.05365$

$12.05 = 1.5y_c \Rightarrow y_c = 8.03$

b) The maximum flow is closest to:
{Answer **440; 620; 830; 1125; 2060)** units ft³/s

$Q_c = Q_{max} = \sqrt{gD_c}\,A_c = \sqrt{g}\sqrt{y - 1.073}\left(\frac{1}{8}\pi b^2 + yb - 0.5b^2\right)$

~~$= 5.675\sqrt{y - 1.073}\,(10y - 10.73)$~~

$= \sqrt{g}\sqrt{8.03 - 1.073}\left(\frac{\pi(10)^2}{8} + 80.3 - 50\right)$

~~$Q^2 = 32.2y - 34.56(10y - 10.73)$~~

~~$322y^2 - 345.5y - 345.6y + 370.8$~~

Forgot I knew y_c...

$= 14.967(69.57)$

$= 1041$

3. For the stilling basin shown in Figure 2, determine:
   a. the Froude number at section 1
      {Answer **4.2,** **5.2,** **6.2,** **8.2,** **8.9**)

   b. the depth y2 for a hydraulic jump to form if there wasn't a step

      {Answer **15.5,** **18.3,** **25.3,** **30.3**) Units

$y_2 = \frac{1}{2} y_1 \left(\sqrt{1+8N_{F_1}^2} - 1\right) = 14.56$

   c. the depth y2 for a hydraulic jump to form with the step

      {Answer **15.,** **18.0,** **25.2,** **30.3,** **30.8.**) Units

   d. the energy loss from section 1 to 2.
      {Answer **0,** **45,** **65,** **75,** **105.**) Units

Given: q = 100 cfs/ft $= \frac{Q}{W} = Q$

y_1 =2.5 ft (Initial depth)

s = 3.0 ft (Step height)

$A_1 = y_1(W) = 2.5\ ft^2$

$V_1 = \frac{Q}{A_1} = \frac{100}{2.5} = 40\ ft/s$

$N_{F_1} = \frac{V_1}{\sqrt{g y_1}} = \frac{40}{\sqrt{g(2.5)}} = 4.46$

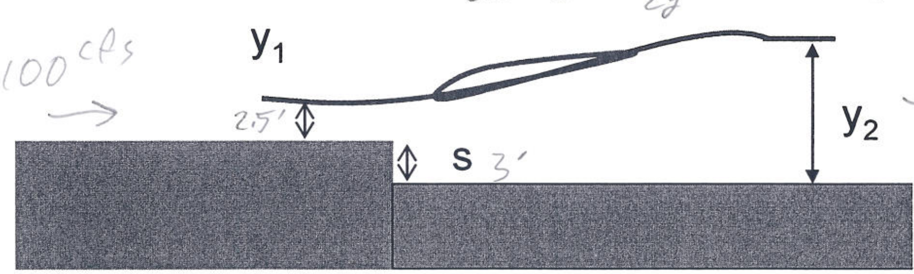
Assume: α = β = 1
Hydrostatic pressure at sections 1 and 2

$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L$

$V_2 = \frac{Q}{A_2} = \frac{Q}{15.45(1)} = 6.47\ ft/s$

W=1.0 ft

$2.5 + 3 + \frac{40^2}{2g} = 15.45 + \frac{6.47^2}{2g} + h_L$



$\rightarrow h_L = 14.24\ ft$

$P_1 + \rho Q V_1 = P_2 + \rho Q V_2$

Figure 3.

$\frac{1}{2}\gamma (y_1+s)^2 w + \rho Q V_1 = \frac{1}{2}\gamma (y_2)^2 w + \rho \frac{Q^2}{y_2(w)}$

Solution:

$8703.8 = 31.2 y^2 + 19400 \frac{1}{y_2}$

~~$\phi = 31.2 y_2^2 + 187.2 y_2 + 280.8 + 19400 \frac{1}{y_2} - 7955$~~

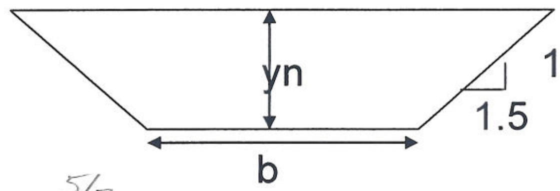
~~$\phi = 31.2 y_2^3 + 187.2 y_2^2 - 7674.2 y_2 + 19400$~~

$\phi = 31.2 y^3 - 8703.8 y + 19400$ $\{15.45, 2.27\}$

~~$y_2 = \{2.811, 11.18\}$~~

4. Given: Q= 5000 cfs; z = 1.5; n = 0.015; S_o = 0.0009 and y_n = 8 ft.

The value of **b** is closest to {**110**, **135**, **155**, **200**} units _____



$A = y(zy+b) = 12+8b$

$P = b + 2y\sqrt{z^2+1} = b + 28.84$

C_Q

$$\frac{nQ}{c' S_o^{1/2}} = \frac{A^{5/3}}{P^{2/3}}$$

$$C_Q = 1682.4 = \frac{(12+8b)^{5/3}}{(b+28.84)^{2/3}}$$

Try b = 155 → 4498

try 110 → 3082

Final Exam Review

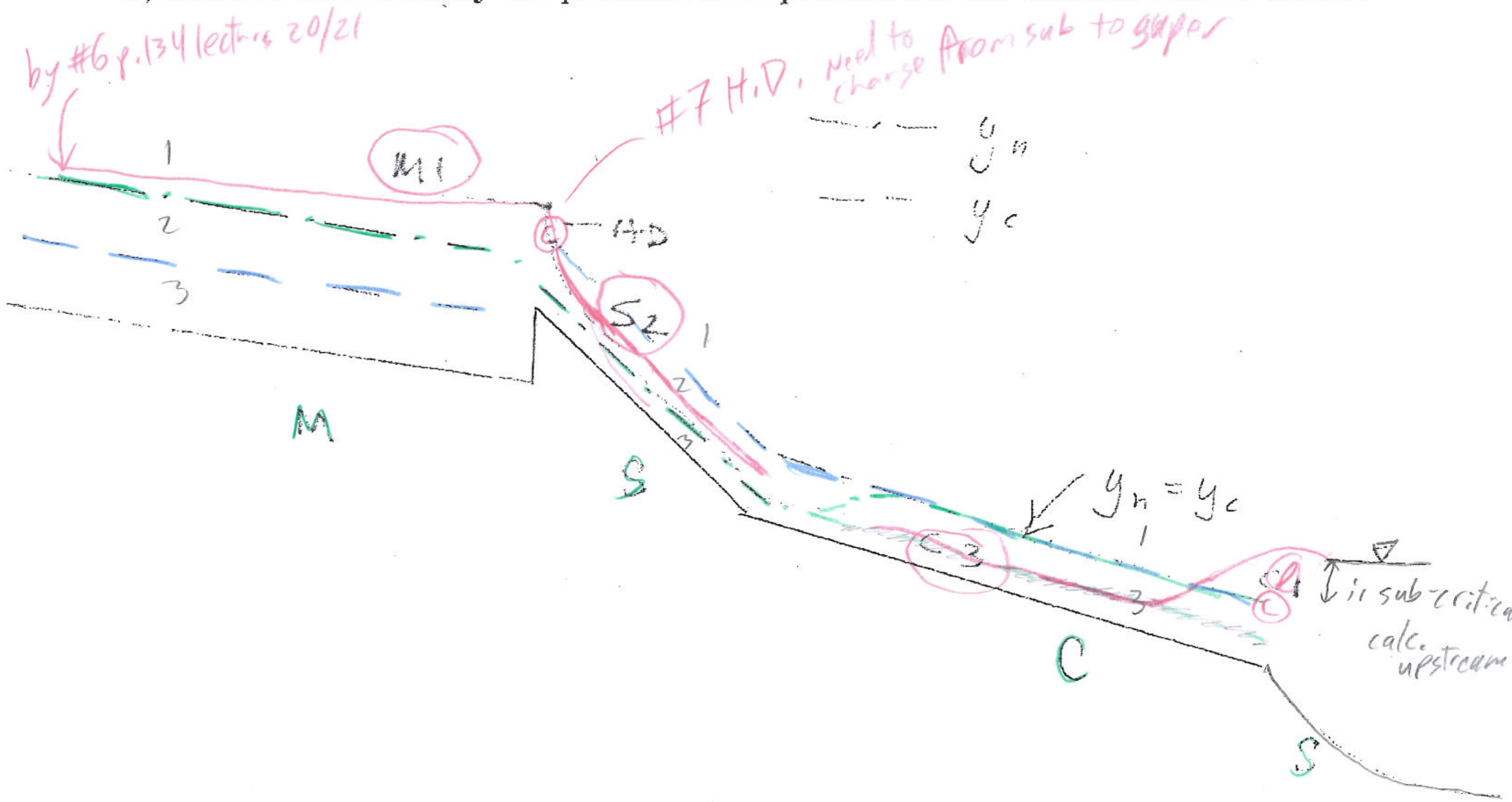
1. Given: Q= 3000 cfs; z = 1.5; n = 0.015; So = 0.0007 and b = 18 ft.
   Find: yn , yc and dy/dx if y = 1 ft.

2. Design a concrete channel to carry 1000 cfs on a So of 0.0007.
   Assume Stiff clay.

3. Design an unlined channel to carry 3500 cfs with So= 0.0003.
   Bed material D50 = 2.2 mm, D75 = 3 mm
   Soil friction angle = 32°
   Use Strickler Eq for n and add n1 = 0.012 for bed forms .
   CHECK by maximum velocity method.

4. a) Given: Q = 1500 cfs; b=26 ft; n = 0.03; So = 0.0008; z=2.5 and y1 = 1.2yn
   Find: x where y2 = 1.1yn

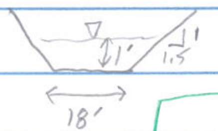
   b) Sketch and **Classify** the possible flow profiles for the channel shown below.



Hydro Final Review

Find y_n, y_c, $\left(\frac{dy}{dx} \text{ if } y = 1\text{ft}\right)$ ①

1) given $Q = 3000\text{cfs}$; $z = 1.5$; $n = 0.015$; $S_o = 0.0007$; $b = 18'$



$$C_Q = \frac{nQ}{c' S_o^{1/2}} = \frac{0.015\,(3000\text{ cfs})}{(1.486)(0.0007)^{1/2}} = 1144.58$$

$$C_Q = \frac{A^{5/3}}{P^{2/3}} = \frac{[y_n(b + z y_n)]^{5/3}}{(b + 2 y_n \sqrt{1+z^2})^{2/3}}$$

Solve for y_n so left = right (C_Q) (answer)

(use multiple choice answers for y_n)

$$\frac{Q}{\sqrt{g}} = (b + z y_c)\, y_c \sqrt{\frac{(b + z y_c)\, y_c}{b + 2 z y_c}}$$

Solve for y_c same as ↑

Extra Info.

	Rect.	TRi	TRAP
A	by	zy^2	$y(b+zy)$
B	b	$2zy$	$2zy + b$
P	$2y + b$	$2y\sqrt{z^2+1}$	$b + 2y\sqrt{z^2+1}$

$E_c = y_c + \frac{D_c}{2}$ $D_c = \frac{A}{B}$

$Q_c = V_c A_c = \sqrt{g D_c}\, A_c$

$R = \frac{A}{P}$

$$\frac{dy}{dx} = \frac{S_o\left(1 - \left(\frac{C_Q}{A R^{2/3}}\right)^2\right)}{1 - \left(\frac{Q}{A\sqrt{D}\sqrt{g}}\right)^2}$$

solve for $\frac{dy}{dx}$

$A = y(b + zy)$

$R = \frac{y(b+zy)}{b + 2y\sqrt{z^2+1}}$

$D = \frac{A}{B} = \frac{y(b+zy)}{2zy + b}$

Set $y = 1\text{ft}$ as asked for & plug in

Hydro Final Review ②

2) Design Lined Channel given $Q = 1000$ cfs ; $S_o = 0.0007$

Assume Stiff Clay , lined w/ concrete

Lecture 17 $z = 0.5 \to 1$ choose 1 concrete $\to n = 0.013$

$$C_Q = \frac{nQ}{c' S_o^{1/2}} \quad \{c' = 1.486\} = \underline{calc} \; ; \; \frac{b}{y} = 2\left(\sqrt{1+z^2} - z\right) = \underline{calc}$$

$$y = \frac{C_Q^{3/8}\left(\frac{b}{y} + 2\sqrt{1+z^2}\right)^{1/4}}{\left(\frac{b}{y} + z\right)^{5/8}} = \underline{calc.} \qquad b = \frac{b}{y} \cdot y = \underline{calc.}$$

$FB_1 = 0.44 \ln Q - 1.5 = \underline{calc.}$

$FB_2 = 0.475 \ln Q - 0.2 = \underline{calc.}$

$A = y\left(\frac{b}{y} \cdot y + z y_n\right) = \underline{calc.}$

$B = b + 2zy = \underline{calc.}$

$D = A/B = \underline{calc}$

Draw it

$V = Q/A = \underline{calc}$ V_{min} (2–2.5 ft/s) ; V_{max} (15–20 ft/s concrete) — lecture 17

$N_F = \frac{V}{\sqrt{gD}} = \underline{calc.}$ < 1 OKAY

3) Design Unlined Channel   given $Q = 3500$ cfs, $S_0 = 0.003$

Bed Material $D50 = 2.2$ mm, $D75 = 3$ mm

Soil friction angle $= 32°$ Use Stricker eq for n & add $n1 = 0.012$ for bed forms

→ $n = 0.034\,(D50_{[ft]})^{1/6}$

Lecture 15

solve for n then add $n1$ to it to account for bed forms.

– ~~Check max velocity method~~

– max shear method

$\tau_b = 0.4\ D75[in]$

$C_s = 0.75$
$C_b = 0.97$
lecture 19

Set $\tau_b = \tau_o = C_b\,\gamma\,y_n\,S_o$ (γ = 62.4) ⇒ solve for y_n

$$\phi = \sin^{-1}\left[\left(\sin\theta_f\left[1-\left(\frac{C_s}{C_b}\right)^2\right]\right)^{1/2}\right] = \text{calc}$$

← soil friction angle given

$y_n = 2.6'$
$\phi = 19.6°$
$z = 2.87$
$\frac{b}{y} = 285.4$

$$z = \frac{1}{\tan\theta} = \text{calc}$$

$$Q = \left(\frac{c'}{n}\right) \frac{(y(b+zy))^{5/3}}{(b+2y\sqrt{z^2+1})^{2/3}} S_o^{1/2} \rightarrow \text{solve for } b$$ (744 ft in example)

check $\frac{b}{y}$ $C_Q = \frac{nQ}{c' S_0^{1/2}}$

$$f_v = C_Q - AR^{2/3} = \emptyset$$

$$V = \frac{Q}{A} = 1.8 < 3 \text{ okay}$$

$$N_f = \frac{V}{\sqrt{gD}} = \frac{V}{\sqrt{g\,\frac{A}{B}}} = \frac{V}{\sqrt{g\,\frac{y(b+zy)}{2zy+b}}} < 1 \text{ OKAY}$$

Regime

$$y = \frac{P \pm (Disc)^{1/2}}{2\left[2\sqrt{1+z^2} - z\right]}$$

$$Disc = P^2 + 4a\left(z - 2\sqrt{1+z^2}\right)$$

$$b = P - 2y\sqrt{1+z^2}$$

4) Water Profile

Given: $Q = 1500$ cfs $b = 26'$ $n = 0.03$ $S_o = 0.0008$ $z = 2.5$

$y_1 = 1.2\,y_n$ $y_2 = 1.1\,y_n$ Find ΔX $c' = 1.468$

$$C_Q = \frac{nQ}{c'} S_o^{1/2} \qquad C_Q = \frac{A^{5/3}}{P^{2/3}} = \frac{\left[y_n\,(b + z y_n)\right]^{5/3}}{\left(b + 2y_n\sqrt{1+z^2}\right)^{2/3}}$$ solve for y_n

$$Q = D_c^{3/2}\sqrt{\frac{(b + z D_c]^3}{b + 2z D_c}(g)} \qquad \left\{ D_c = \frac{A_c}{B_c} \right\}$$ solve for y_c

	y	A	P	R	D	dx/dy	Avg dx/dy	D_y	D_x	X (ft)
y_1	$1.2y_n$	$y+(b+zy)$	$b+2y\sqrt{1+z^2}$	$\frac{A}{P}$	$\frac{A}{b+2yz}$	○ →	$\frac{+}{2}$	∅	∅	∅
y_2	$1.1y_n$	"	"	"		○ →		□	□	□

$AR^{2/3}$ $AD^{1/2}$ — use for $\frac{dx}{dy}$

$X = X_1 + D_x$

$$\frac{dx}{dy} = \frac{1 - \left(\frac{Q}{\sqrt{g}\,A\sqrt{D}}\right)^2}{S_o\left(1 - \left(\frac{C_Q}{AR^{2/3}}\right)^2\right)}$$

$D_y = y_2 - y_1$

$D_x = \left(dx/dy\right)\left(D_y\right)$

9-7. Sketch the possible flow profiles in the channels shown in Fig. 9-13.

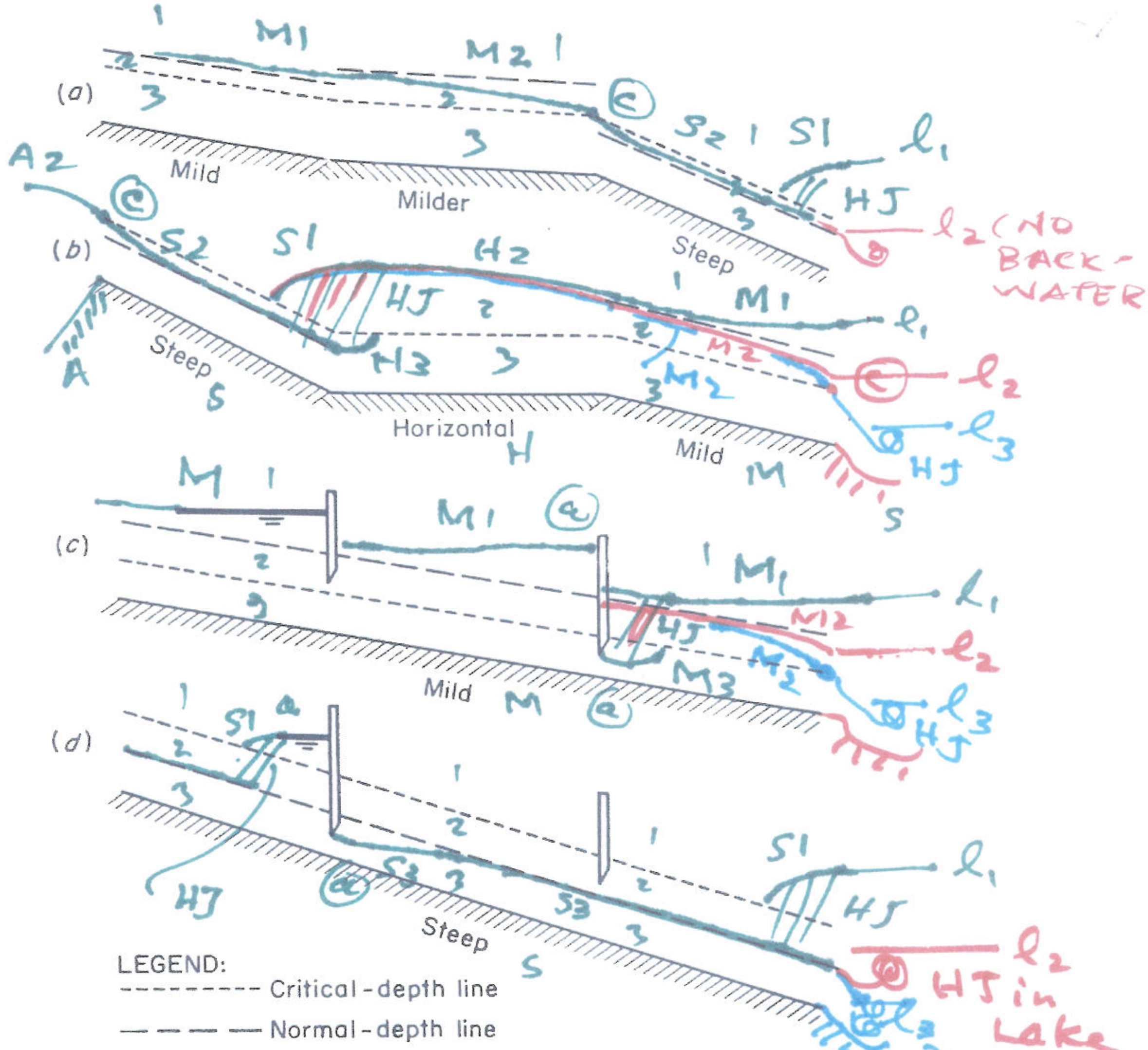


FIG. 9-13. Channels for Prob. 9-7. The vertical scale is exaggerated.

Donald Jerolleman

Assume $\gamma = 62.4\ lb/ft^3$

9/10

10/10

Assumptions
- Neglect weight of water
$W = \gamma (width)(depth) L$
$= 62.4(6)(1)(2) = 748\ lb$

$P_1 = \gamma w d_1^2/2$

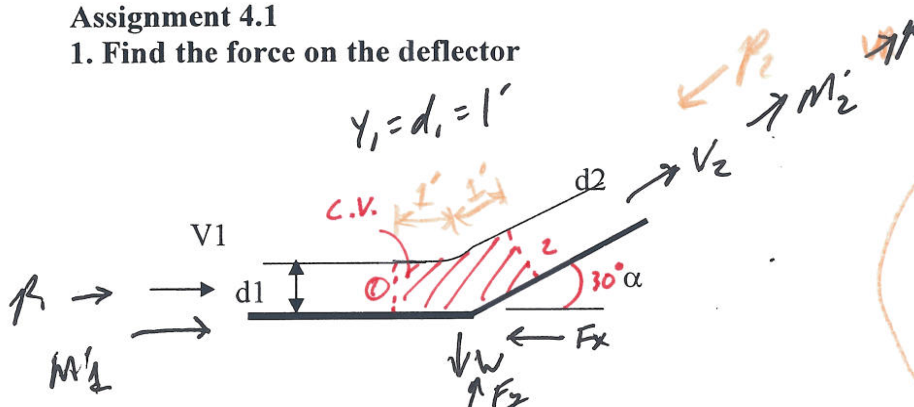
$P_2 = \frac{1}{2}\gamma w d_2^2 (\cos\alpha)$

$P_{2y} = P_2(\cos\alpha)$

Applications of Linear Momentum

Assignment 4.1

1. Find the force on the deflector



W = 6 ft
d1= 1 ft
V1=40 ft/sec
α = 30°

Assume d1=d2 and neglect the weight of the water.

$Q = V_1 d_1 w = 40(1)(6) = 240\ cfs$

$Q = V_2 d_2 w \ ; \ V_2 = \frac{Q}{d_2 w} = \frac{240}{1(6)} = 40\ ft/s$

$\Sigma F_x = P_1 - F_x - F_{f_x} - P_2 \cos 30° = M_2' \cos 30° - M_1'$ (small b/c very close; $P_1 \approx P_2$)

$\beta = 1$

$-F_x = \rho Q V_2 \cos 30 - \rho Q V_1 \rightarrow F_x = \rho Q V(1 - \cos 30°) = 2495\ lbs$ *

* neglecting pressure & friction

$\Sigma F_y = F_y - W - P_2 \sin 30° - F_{f_y} = M_2' \sin 30° - M_1' \sin\phi$ (b/c small; $\to 0$)

$F_y = M_2' \sin 30 = \rho Q V_2 \sin 30 = 9312\ lbs$
(1.94 slug/ft³)(240)(40)(0.5)

Forces of Deflector on water

with weight of water

$P_1 =$

correction for w etc?

$\phi = \tan^{-1}\left(\frac{F_x}{F_y}\right) = 15°$

$R = \sqrt{F_x^2 + F_y^2} = 9640\ lbs$

For force on Deflector = & opposite of

Assignment 4.2.

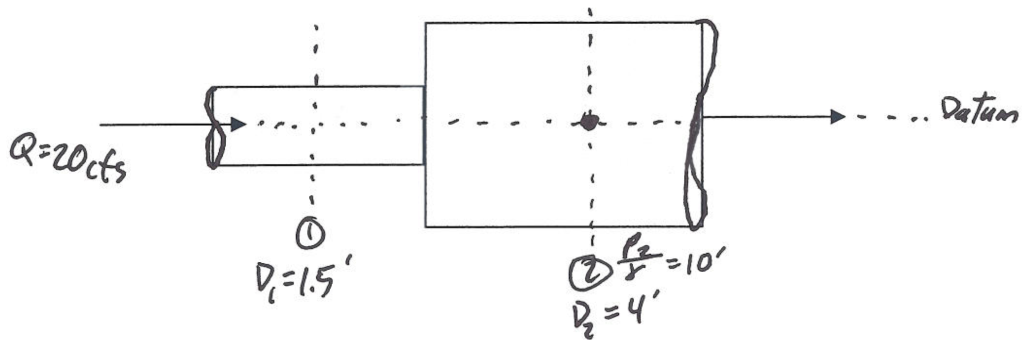
Determine the head loss at the Abrupt Expansion in the pipe shown below.

Find the pressure head at section 1.

Given: D_1=1.5 ft; D_2=4 ft; Q = 20 cfs; p_2/γ = 10 ft.

Assume: $\alpha_1 = \alpha_2 = 1$; $\beta_1 = \beta_2 = 1$., $h_f = \emptyset$

10



Section 1 Section 2

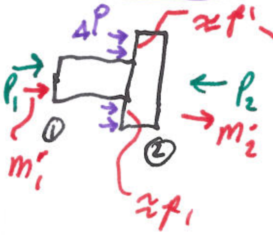
Apply Continuity

$Q = V_1 A_1 = V_2 A_2$

$V_1 = \frac{Q}{A_1}$; $V_2 = \frac{Q}{A_2}$

$V_1 = \frac{20\ ft^3/s}{\pi \frac{(1.5)^2}{4}} = 11.32\ ft^2$ $V_2 = \frac{20}{\pi \frac{4^2}{4}} = 1.592$

Apply Momentum



$\Sigma F_x = p_1 + \Delta P - p_2 = M_2' - M_1' = \rho Q V_2 - \rho Q V_1$

$p_1 A_1 + p_1 (A_2 - A_1) - p_2 A_2 = \rho Q (V_2 - V_1)$

$\rightarrow p_1 A_2 - p_2 A_2 = \rho Q (V_2 - V_1)$

(A) - A_2

$\rightarrow p_1 = p_2 + \frac{\rho Q}{A_2}(V_2 - V_1) = 10'(62.4\ lb/ft^3) + \frac{\left(\frac{62.4\ lb/ft^3}{32.2\ ft/s^2}\right)(20\ ft^3/s)}{12.57\ ft^2}(1.592\ ft/s - 11.32\ ft/s)$

$624\ lb/ft^2 + (-30)\ lb/ft^2 = 594\ lb/ft^2$

$h_{press} = \frac{p_1}{\gamma} = 9.519\ ft$

Energy

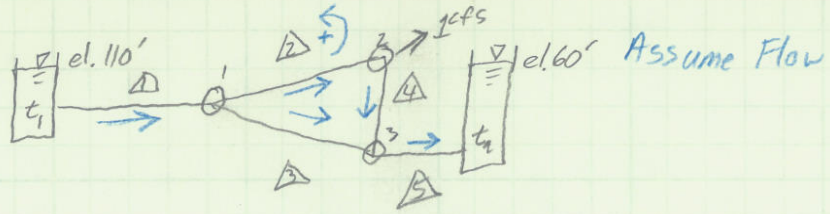
$H_{T_1} = H_{T_2} + h_L \rightarrow \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_{z_1} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_{z_2} + h_L = \frac{594}{62.4} + \frac{(11.32)^2}{64.4} + \emptyset = 10' + \frac{(1.592)^2}{64.4} + h_L$

$h_L = 1.46\ ft$

compare above w/

$h_L = \frac{(V_1 - V_2)^2}{2g} = 1.47\ ft$ ✓

(For sudden expansion of flow)



5/5

$t = 2$
$l = 1$
$e = t - 1 = 1$
$j = 3$ el. of all $j = 10'$
$N_p = 5$

contistancy equ. $N_p = l + e + j$
$5 = 1 + 1 + 3 = 5$ OKAY

Continuity equ: $Q_{in} - Q_{out} = \emptyset$

Energy equ. (loop): $h_{L_3} - h_{L_4} - h_{L_2} = \emptyset \{\Sigma h_L = \emptyset\}$ Based on +

Energy equ. (path): $\Sigma h_L = E_{drop} = W.L._1 - W.L._2$ (t_1) (t_2)

energy drop, water level

Put Both Energy equ. into terms of Q

$h_{L_p} = k_p Q_p^2 = k_p Q_p |Q_p|$

Loop: $\Sigma k_p Q_p |Q_p| = \emptyset$
Path: $\Sigma k_p Q_p |Q_p| = E_d$

① ASSUME a Q_{p_i} $\Sigma Q_{p_i} = \emptyset$ @ $j = 1, 2, 3$

Assume $Q_1 = 3$, (j_1: $Q_2 = 2$, $Q_3 = 1$), (j_2: $Q_4 = 1$), (j_3: $Q_5 = 2$)

② Q_{p_i} loop $\Sigma h_{L_p} = 0 \rightarrow \Sigma k_p Q_p |Q_p| = \emptyset$

Accounts for error

$\rightarrow \Sigma(h_{L_{p_i}} + \Delta h_{L_{p_i}}) = \emptyset = \Sigma(k_p Q_{p_i} |Q_{p_i}| + \Delta h_{L_{p_i}}) = \emptyset$

$h_L = k_p Q_p^2$
$d(h_L) = 2 k_p |Q_p| \Delta Q$
$\Delta h_{L_{p_i}} = 2 k_p |Q_{p_i}| \Delta Q$

sub. in: $\Sigma(k_p Q_{p_i} |Q_{p_i}| + 2 k_p |Q_{p_i}| \Delta Q) = \emptyset$

$$\rightarrow \Delta Q = \frac{-\Sigma k_p Q_{p_i} |Q_{p_i}|}{\Sigma 2 k_p |Q_{p_i}|}$$

Given: $k_{p_1} = k_{p_5} = 2$, $k_{p_2} = k_{p_3} = k_{p_4} = 1$

$E_{drop} = W_{L_1} - W_{L_2} = 110' - 60' = 50'$

Path $\Delta Q_{PATH} = \dfrac{-(\Sigma K_p Q_{pi}|Q_{pi}|) + E_{drop}}{\Sigma\, 2 K_p |Q_{pi}|}$

APPROX. CORRECTION

$$\Delta Q_{PATH} = \frac{-((2\cdot3\cdot3)+(1\cdot1\cdot1)+(2\cdot2\cdot2))+50}{(2\cdot2\cdot3)+(2\cdot1\cdot1)+(2\cdot2\cdot2)} = \frac{23}{22} \sim 1$$

(numerator: K_{p_3} Q_3 $|Q_3|$, K_{p_5} Q_5 $|Q_5|$; denominator: 2 K_{p_1} $|Q_1|$, 2 K_{p_3} $|Q_3|$, 2 K_{p_5} $|Q_5|$)

ADD ΔQ_{PATH} to Assumed Path Flows

$Q_1 = 3+1 = \underline{\underline{4}}$; $Q_3 = 1+1 = \underline{\underline{2}}$; $Q_5 = 2+1 = \underline{\underline{3}}$

Loop ③-④-② (+) $\Delta Q_{LOOP} = \dfrac{-(\Sigma K_p Q_{pi}|Q_{pi}|)}{\Sigma\, 2 K_p |Q_{pi}|}$

Use new values from path if applicable

$$\Delta Q_{loop} = \frac{-((1\cdot2\cdot2)-(1\cdot1\cdot1)-(1\cdot2\cdot2))}{(2\cdot1\cdot2)+(2\cdot1\cdot1)+(2\cdot1\cdot2)} = \frac{1}{10} = 0.1$$

(numerator: K_{p_3} new Q_3 Q_3, K_{p_4} Q_4, K_{p_2} Q_2; denominator: 2 K_{p_3} Q_3, 2 K_{p_4} Q_4, 2 K_{p_2} Q_2)

ADD ΔQ_{loop} to Assumed Loop Flows

$Q_3 = 2 + 0.1 = 2.1$; $Q_4 = 1 - 0.1 = 0.9$ (subtracted b/c of assumed (+))

$Q_2 = 2 - 0.1 = 1.9$ (ditto)

MAKE Tollerence = 1% of given Q_{out} (0.01)

Need to reiterate ∴ ΔQ approaches ∅ USE NEW VALUES

$$\Delta Q_{path} = \frac{-((2\cdot4\cdot4)+(1\cdot2.1\cdot2.1)+(2\cdot3\cdot3))+50}{(2\cdot2\cdot4)+(2\cdot1\cdot2.1)+(2\cdot2\cdot3)} = (-0.137)$$

New: $Q_1 = 3.86$; $Q_3 = 1.963$; $Q_5 = 2.863$

$$\Delta Q_{loop} = \frac{-((1\cdot1.963\cdot1.963)-(1\cdot0.9\cdot0.9)-(1\cdot1.9\cdot1.9))}{(2\cdot1\cdot1.963)+(2\cdot1\cdot0.9)+(2\cdot1\cdot1.9)} = 0.06$$

New: $Q_3 = 2.02$; $Q_4 = 0.84$; $Q_2 = 1.84$

$Q_1 = 3.86$, $Q_2 = 1.84$, $Q_3 = 2.02$, $Q_4 = 0.84$, $Q_5 = 2.863$

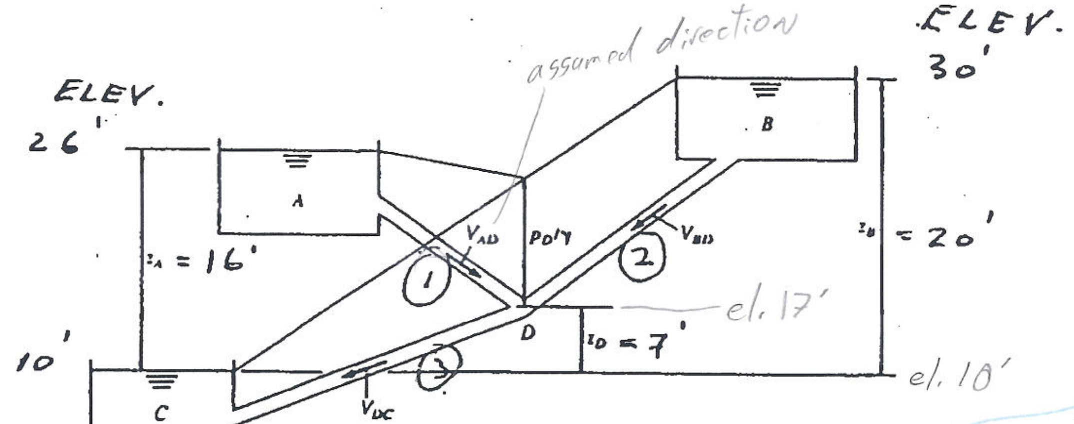
Donald Jerolleman

Three Reservoir Problem

Assignment 11.1
Three reservoirs (A, B, C) connected at a common point (D) as shown on the following page.
Pipes are numbered as # 1 A-D

2 B-D

3 C-D



$\frac{P_D}{\gamma} = H_D - el_D$

$H_{D_{avg}} = 22'$ assumed

$K_p = \frac{8fL}{g\pi^2 D^5}$

Solution:

$\Delta H = H_{res} - H_D$ — sign of ΔH taken out

Assumed H_D =

Pipe #	D ft (diameter)	F (friction)	L ft (length)	g	Kp	ΔH ft (Head loss)	Q cfs $= \pm\sqrt{\frac{\Delta H}{K_p}}$	RES	H ft (height)
1	1	0.025	2000	32.2	1.26	26-22=4	1.78	A	26
2	1	0.025	2000	32.2	1.26	30-22=8	2.52	B	30
3	1.5	0.022	3000	32.2	0.219	10-22=(-12)	(-7.4)	C	10
							Σ (-3.1)		

(should be ∅) Try H_D = 17' (pressure = ∅ @ D)

Assumed H_D =

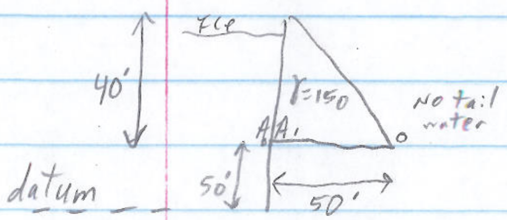
Pipe #	D ft	F	L ft	g	Kp	ΔH ft	Q cfs	RES	H ft
1	1	0.025	2000	32.2	1.26	9	2.67	A	26
2	1	0.025	2000	32.2	1.26	13	3.21	B	30
3	1.5	0.022	3000	32.2	0.219	(-7)	(-5.65)	C	10
							Σ +0.23 ≠ ∅		

try H_D = 17.4

Assumed H_D =

Pipe #	D ft	F	L ft	g	Kp	ΔH ft	Q cfs	RES	H ft
1	1	0.025	2000	32.2	1.26	8.6	2.613	A	26
2	1	0.025	2000	32.2	1.26	12.6	3.162	B	30
3	1.5	0.022	3000	32.2	0.219	(-7.4)	(-5.813)	C	10
							Σ (-0.038)		

Hydro test 1 practice



$W_{dam} = \frac{1}{2}(50)(40)(150)(1) = 150^k$

$\bar{x}_o = 33.3$ ft

~~Hydro = 40 (62.4) = 2.5^k~~

$U: \phi A_0 = 40' + \text{datum} = 90' \qquad \phi_0 = 50'$

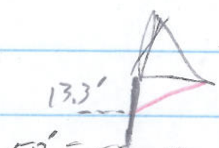
$\phi A_1 = \phi A_0 + \Delta\phi\left(\frac{S_{A_1}}{S_e}\right) = 90 + (-40)\left(\frac{100}{150}\right) = 63.3' \qquad \therefore \Delta\phi = \phi_0 - \phi A_0 = -40$

$S_e = 50 + 50 + 50 = 150'$

F	F_x	F_y	Arm_o	↺+ M	↻− M
W	θ	$(-150)^k$	33.3 ft		θ
Ice	5^k	θ	40 ft	θ	
Hydro	2.5^k	θ	$\frac{40}{3} = 13.3$ ft	θ	
U	θ	20.75^k	33.3 ft	θ	

$\frac{1}{2}\gamma h^2$

datum h_z	S	φ	$\frac{P}{\gamma}$
A_0 50	θ	90	40
A_1 50	100	63.3	13.3
O 50	150	50	θ



$U: 0.5(13.3)(50)(1)(62.4) = 20.75^k$

$N = W - U = 150 - 20.75 = 129.3$

μN = max friction

$FOS_{OT} = \frac{↺+}{↻-} \qquad FOS_{slide} = \frac{\mu N}{\Sigma F_x}$

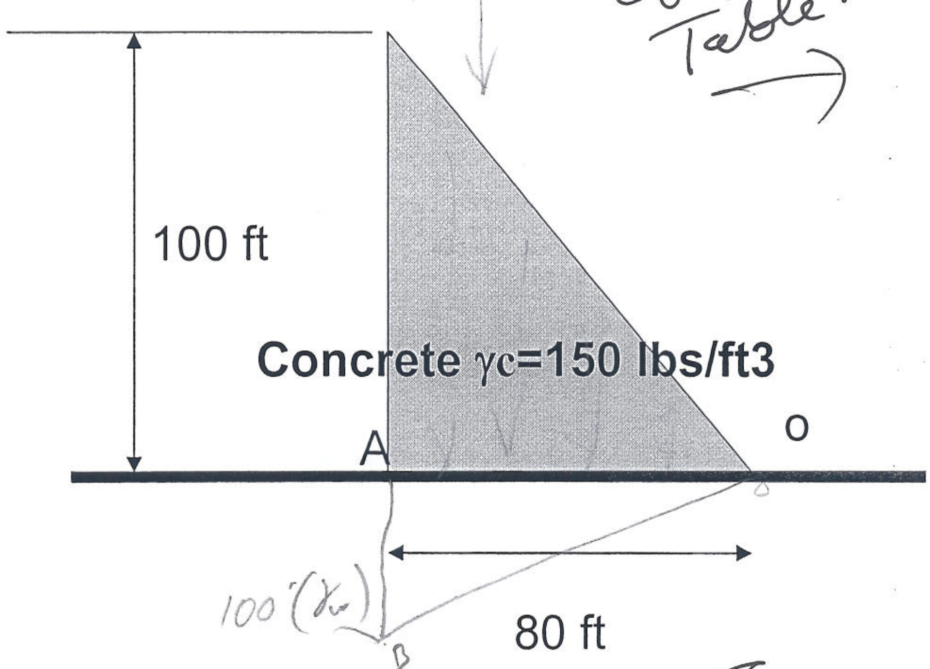
$X_N = \frac{(↺+) - (↻-)}{N}$

put in 1/3

Assume no shear stress

1. Check the stability of this dam under normal loads as shown.
Assume friction factor = 0.7

Donald Jerolleman



1. Fos ot = 1.35
2. Fos sl = 0.79
3. xN = 23.65 ft and tension (exists) (does not exist)

9.5/10

1) $F_h = A_v \gamma_w \bar{y}_{A_v} = 100ft(1ft)(62.4\ lb/ft^3)(\frac{100ft}{2}) = 312^K$

$W = A_c w \gamma_c = (\frac{1}{2})80'(100')(1')(150\ lb/ft^3) = 600^K$

$U = A_{ABO}\ w = \frac{1}{2}(\gamma_w)(100')(80')(1') = 249.6^K$ @ X_u from O $= 53.3'$

	$\overrightarrow{+}$ F_x	$\uparrow +$ F_y	M_{arm} ab/o	W/R ↺	O/T ↻
F_h	312^K	∅	$\frac{100}{3} = 33.3'$		10390 ft-K
U	∅	249.6^K	$\frac{80(2)}{3} = 53.3'$	~~13304~~	13304 ft-K
W	∅	-600^K	$= U = 53.3'$	31980 ft-K	
Σ	312 →	350.4 ↓		31980 ft-K ↺	23694 ft-K ↻

$F_{OS_{OT}} = \frac{↺+}{↻_{OT}} = \frac{31980}{23694} = 1.35$

$F_{OS_{SL}} = \frac{\mu(N)}{\Sigma F_x} = \frac{0.7(350.4^K)}{312^K} = 0.79$

$X_N = \frac{31980 - 23694}{350.4} = 23.65\ ft$ middle 1/3 = 26.7 → 53.33

∴ Less than middle 1/3

---

2) $F_h = 312^K$, $W = 600^K$, $U = \gamma_w 2000' = 124.8^K$

	F_x →+ (K)	F_y ↑+ (K)	M_{arm} ab/o	W/R ↺+	O/T ↻
F_h	312		33.3		10390 ft-K
U		124.8	53.3		6652 ft-K
W		-600	53.3	31980 ft-K	
Σ	312 →	475.2 ↓		31980 ↺	17042

$F_{OS_{OT}} = 1.88$

$F_{OS_{SL}} = \frac{0.7(475.2)}{312} = 1.07$

$X_N = 31.4\ ft$ middle 1/3 = 26.7 → 53.33 OKAY

90/100

Fluid Lab

ENCE 4319

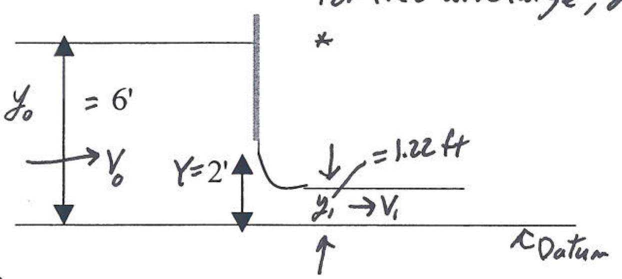
Donald Serolleman

Tutorial Assignment 1.

Due 9/7/10

1. Using energy and continuity principles estimate the flow under the sluice gate shown in the sketch below. Assume: no head loss, $\alpha = \beta = 1$, W = 6 ft, Gate opening = 2 ft; y_o = 6 ft and Cc = 0.61.

For free discharge, 'y' contracts to a depth $y_1 = C_c Y = 0.61(2) = \underline{1.22\ ft}$



Assumptions:

Friction Loss = Ø ; α(K.E. correction factor) = β(momentum correction factor) = 1

Equations:

Continuity: $A_oV_o = A_1V_1$; $A = W \cdot y_i$ ∴ $W \cdot y_o \cdot V_o = W \cdot y_1 \cdot V_1 \rightarrow y_o V_o = y_1 V_1$

Energy: $H_o = H_1 + h_{L_{0-1}}$; $y_o +$ [redacted] $\frac{V_o^2}{2g} = y_1 +$ [redacted] $+ \frac{V_1^2}{2g} + h_L$ (Ø)

OPEN CHANNEL: $H = y + \frac{V^2}{2g} + z$

Solution: $V = \frac{Q}{A}$; $V_o = \frac{Q}{6'(6')}$ ($A_{b/f\ gate}$) ; $V_1 = \frac{Q}{6(1.22)}$ ($A_{a/f\ gate}$) ; $6'(V_o) = 1.22'(V_1) \rightarrow V_o = \frac{1.22(V_1)}{6}$

$6' + \frac{V_o^2}{2g} = 1.22' + \frac{V_1^2}{2g}$

$\rightarrow (2g)6' + \left(\frac{1.22V_1}{6}\right)^2 - 1.22(2g) = V_1^2$

$\rightarrow 307.8 = V_1^2 - 0.0413V_1^2$

$\rightarrow V_1^2 = 321.1 \rightarrow$ $V_1 = 17.91$ ft/s

$\Rightarrow 6'(V_o) = 1.22'(17.91\ ft/s)$

$\rightarrow$ $V_o = 3.64'$

$\Rightarrow$ $Q = 3.64(36) = 131.2$ ft³/s

$Q = 17.91(7.32) = 131.1$ ft³/s } ✓

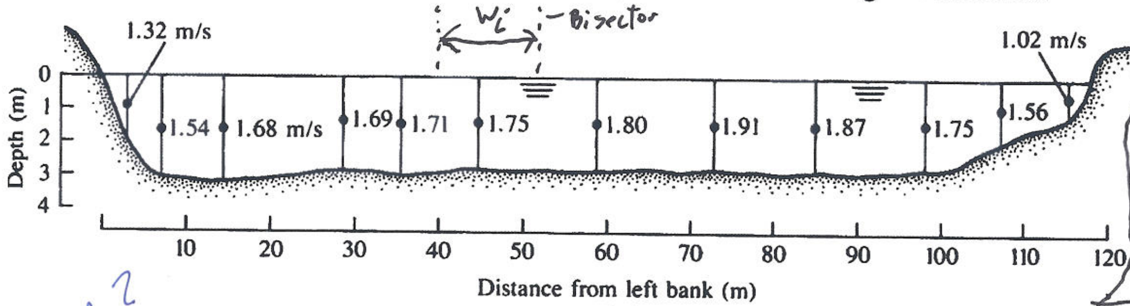
Q = 131.15 ft³/s ✓

30/30

Donald Jerolleman

2. Find the discharge, energy and momentum correction factors for the channel shown in below.

Theory and experimental verification indicate that the mean velocity along a vertical line in a wide stream is closely approximated by the velocity at 0.6 depth. If the indicated velocities at 0.6 depth in a river cross section are measured, what is the discharge in the river?



$A_i = w_i(y_i)$

$A = \sum_{i=1}^{N} A_i$

$Q = \sum_{i=1}^{N} (A_i(V_i))$

$V = \frac{Q}{A}$

momentum correction factor

$\beta = \frac{\sum A_i V_i^2}{AV^2}$

K.E. correc. factor

$\alpha = \frac{\sum A_i V_i^3}{AV^3}$

What's the value of α and β? -10

$Q = 543.8\ m^3/s$ ✓

25/35

3. Estimate the velocities and pressures in the venturi device shown below.
Assume: a) no loss in contracting flow, b) for expanding flow $hL = 0.1(V_{max}-V_{min})^2/2g$, c) neglect friction loss, d) α = Kinetic energy correction factor = 1, e) vapor pressure 0.3 m, and f) pressure head at entrance is 250 m.

① $\frac{V_1^2}{2g} + \frac{p_1}{\gamma} = \frac{V_2^2}{2g} + \frac{p_2}{\gamma}$ ($\frac{p_1}{\gamma}$ = 250 m)

② $\frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + h_{L_{2-3}}$

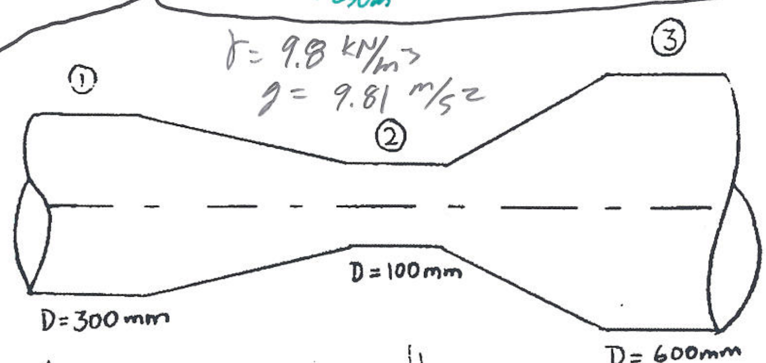
Continuity equ.: $A_1V_1 = A_2V_2 = A_3V_3 = Q = 0.2\ \frac{m^3}{s}$

Energy Balance Equ.: ① $H_{T_1} = H_{T_2} + h_{L_{1-2}}$ ($h_{L_{1-2}} \to 0$)

② $H_{T_2} = H_{T_3} + h_{L_{2-3}}$

$\gamma = 9.8\ kN/m^3$

$g = 9.81\ m/s^2$



$A_1 = 0.07069\ m^2$

$A_2 = 0.007854\ m^2$

$A_3 = 0.2827\ m^2$

$V_1 = \frac{Q}{A_1} = 2.82925\ m/s$ ✓

$V_2 = \frac{Q}{A_2} = 25.4647\ m/s$ ✓

$V_3 = \frac{Q}{A_3} = 0.70746\ m/s$ ✓

$p_1 = 250m(\gamma) = 2450\ kN/m^2$ ✓

① $\frac{(2.82925\ m/s)^2}{2g\ [m/s^2]} + 250m = \frac{(25.4647\ m/s)^2}{2g} + \frac{p_2}{9.8\ [kN/m^3]}$

$\rightarrow p_2 = 2130\ kN/m^2$ ✓

② $\frac{2130\ kN/m^2}{9.8\ kN/m^3} + \frac{(25.4647)^2}{2(9.81)} = \frac{p_3}{9.8\ kN/m^3} + \frac{(0.70746)^2}{2(9.81)} + \frac{0.1(V_2-V_3)^2}{2(9.81)}$

$\rightarrow p_3 = 2423\ kN/m^2$ ✓

35/35

Fluid Lab

Donald Jerolleman

ENCE 4319 Tutorial Assignment 1, Problem 2

z	y_i	$V_{0.6}$ (@ that depth (V_i))	W_i	A_i ($= W_i(y_i)$)	$A_i V_i$	$A_i V_i^3$
0	0	0		Ø	Ø	Ø
3	1.9	1.32	4 ($= \frac{3-0}{2} + \frac{8-3}{2}$)	7.6	10.03	12.48
8	3	1.54	5.5	16.5	25.41	60.26
14	3.1	1.68	10.5	32.55	54.68	154.34
29	2.85	1.69	11	31.35	52.98	151.32
36	2.9	1.71	8	23.20	39.67	116
45	2.8	1.75	11.5	32.2	56.35	172.57
59	2.8	1.8	13.5	37.8	68.04	220.45
72	2.8	1.91	13	36.4	69.52	253.63
85	2.8	1.87	13.5	37.8	70.69	247.18
99	2.8	1.75	11.5	32.2	56.35	172.57
108	2.75	1.56	8	22	34.32	83.52
115	1.25	1.02	4.5	5.63	5.74	5.97
117	0	0		Ø	Ø	Ø
Σ				315.23	543.79	1655.3

$$V = \frac{Q}{A} = \frac{543.79}{315.23} = 1.725$$

$$\beta = \frac{\Sigma A_i V_i^2}{AV^2} \quad ?$$

$$\alpha = \frac{\Sigma A_i V_i^3}{AV^3} \quad ?$$

Late

Donald Jerolleman

Find the operating point (Qo,Ho).

Given: Kps = 0.2; Kpd = 3.0

WL1 10ft; WL2 = 60ft

Pump El = 20ft.

Vapor pressure head = 1 ft.

Atmospheric pressure = 34 ft.

Will the pump cavitate? (Very close, but No)

Assume One Pump. Yes

$\frac{P_{atm}}{\gamma} - \frac{P_{vap}}{\gamma} - \Delta z - \Sigma$

$$H_{sys} = H_{st} + \Sigma h_L = H_{st} + f(Q^2) = H_{st} + K_{ps} Q_s^2 + K_{pd} Q_d^2$$

where H_{st} = Static Lift = WL1 – WL2

h_L = minor losses + friction losses

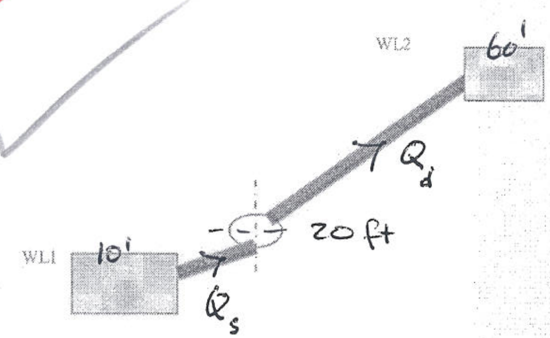
12/15

Qo =

Ho =

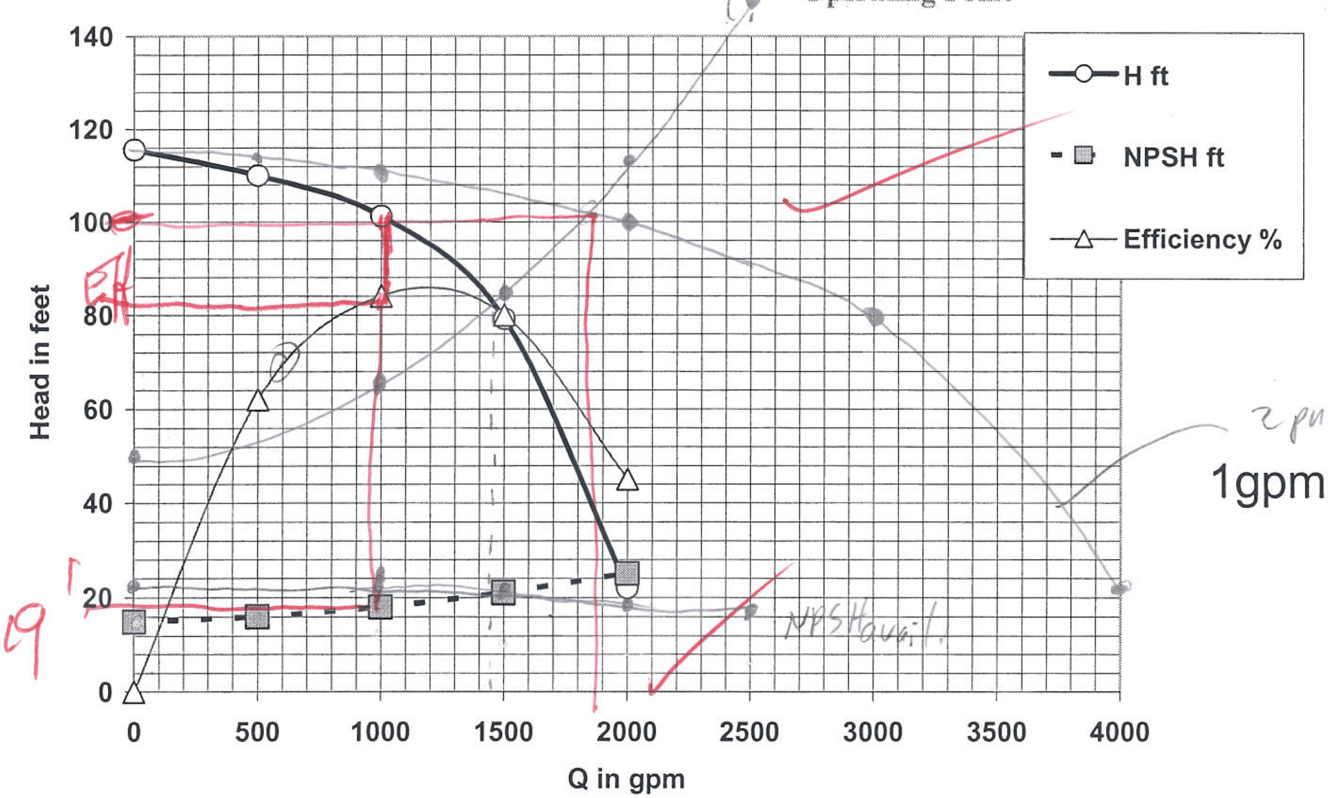
4/5

8/10



PUMP CURV

Operating Point



1gpm=0.00223 cfs

1gpm = 0.002228 ft³/s

Q	cfs (Q_d or Q_s)	Hst	hLs	hLd	Hsys	
			Kps 0.2	Kpd 3.0		
0	0	50	0	0	50	23
1000	2.228	50	0.993	14.89	65.9	22.0
1500	3.342	50	2.234	33.51	85.74	20.77
2000	4.456	50	3.971	59.57	113.54	19.03
2500	5.57	50	6.205	93.07	149.3	16.8

0	0	50	0	0	50	23
500	1.114	50	0.0248	0.37		
750	1.671	50	0.558	8.38		
1000	2.228	50	0.99	14.9		
1250	2.79	50	1.55	2.33		

sam

$\frac{P_{atm}}{\gamma} - \frac{P_{vap}}{\gamma} - \Delta z - K_p Q_s^2$

34' − 1' − 10ft − $K_p Q_s$

(23 − $K_p Q_s$)

Problem #2

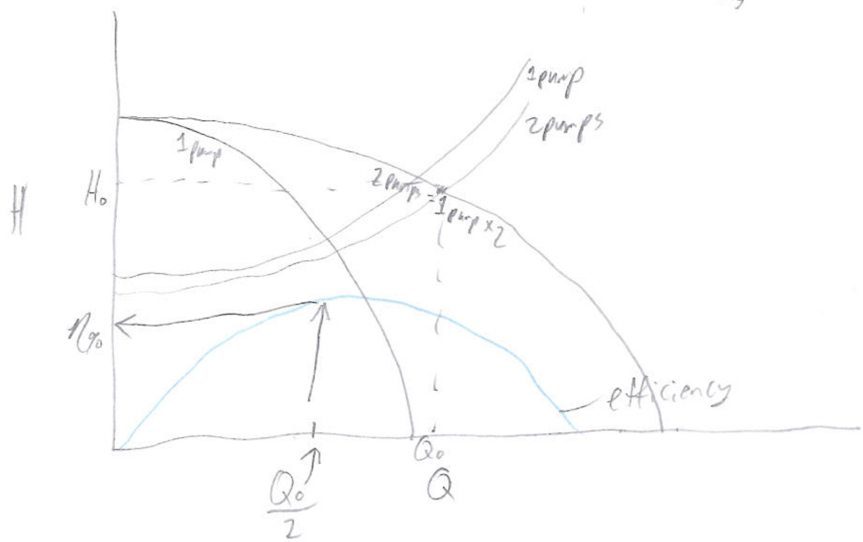
8/10

Hsys

Review

$H_{sys} = H_{st} + K_{ps}(Q_s)^2 + K_{pd} Q_d^2$ ($\frac{Q_d}{2}$ discharge) (Q_s = ?)

H_{st}: static lift



$H_{L_s} = K_{ps}(Q_s)^2$

$H_{Ld} = K_{pd}(Q_d)^2$

2 pumps: ~~Hsys~~ $Q_s = \frac{Q_d}{2}$

Donald Jerolleman

$L = 1000\ ft$

$a = 4000\ ft/s$

$V_o = 3\ ft/s$

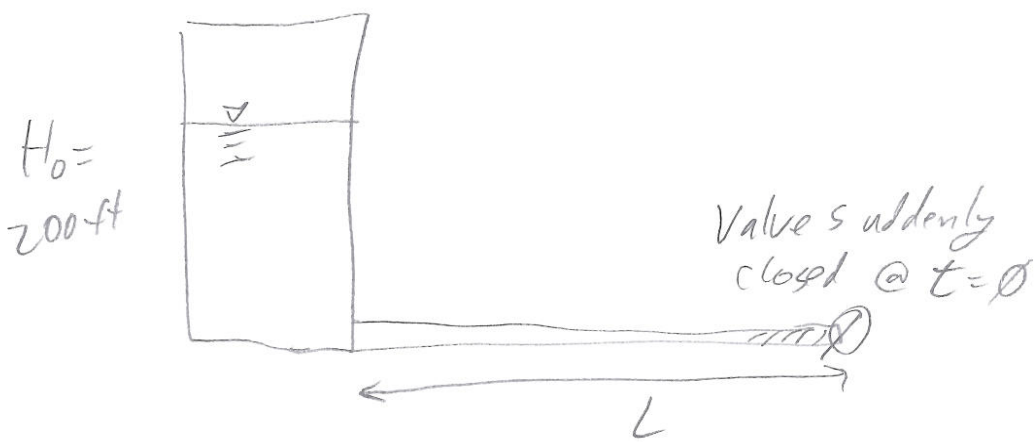
$P_o/g = 200\ ft$

Find

$$H_{max} = H_o + \Delta H = H_o + \frac{aV_o}{g} = 200\ ft + \frac{4000\ ft/s\,(3\ ft/s)}{32.2\ ft/s} = \boxed{573\ ft}$$

$$H_{min} = \frac{P_o}{g} - \Delta H = 200\ ft - 372.67\ ft = (-172.7\ ft) = (-33)$$ (b/c cannot go below vapor pressure)

$$t_c = \frac{2L}{a} = \frac{2(1000\ ft)}{4000\ ft/s} = \boxed{0.5\ s}$$



10

$K_p = \frac{8(f)(L)}{g\pi^2 D^5}$

late 10 − 1 = 9

Assignment Problem 11.3 Name Donald Jerolleman

a) Find the flow in all of the pipes in the network shown below.
b) Find the pressure at junction c.
Assume: f = 0.025 and L = 2000 ft in all pipes & K_m = 0.
All junctions at 0 elevation.

Kp	
1.259	**D1 = 12 inches**
3.132	**D2 = 10 inches**
9.558	**D3 = 8 inches**
40.28	**D4 = 6 inches**

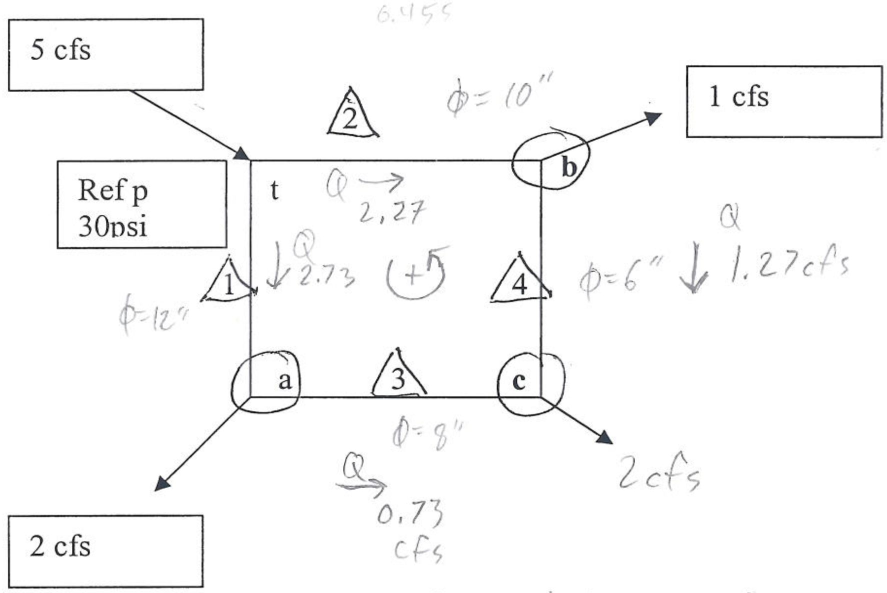
$n_p = l + e + j$

$t = 1,\ l = 1,\ e = t - 1 = \emptyset,\ j = 3,\ n_p = 4$

$4 = 1 + \emptyset + 3 = 4$

$\frac{10}{10+12} = 0.455$

$\frac{12}{10+12} = 0.545$



$$\Delta Q = \frac{-\left(K_{p_1}(Q_1)|Q_1| + K_{p_3}Q_3|Q_3| - K_{p_4}Q_4|Q_4| - K_{p_2}Q_2|Q_2|\right)}{2K_{p_1}|Q_1| + 2K_{p_3}|Q_3| + 2K_{p_4}|Q_4| + 2K_{p_5}|Q_5|}$$

ANSWERS:

Q1= 3.289 **cfs**

Q2= 1.711 **cfs**

Q3= 1.289 **cfs**

Q4= 0.711 **cfs**

Pressure at c = 17.2 **psi**

Attach calculation sheet.

8/8

Pipe	kp	8/gpi^2	f	l	d (in)	d (ft)	d^5
1	1.259	0.025174	0.025	2000	12	1	1
2	3.132	0.025174	0.025	2000	10	0.833333	0.401878
3	9.558	0.025174	0.025	2000	8	0.666667	0.131687
4	40.279	0.025174	0.025	2000	6	0.5	0.03125

$$K_p = \frac{8 f L}{g \pi^2 D^5}$$

All el. = Ø

$$\frac{P_t}{\gamma} = 30 \frac{lb}{in^2} \left(62.4 \frac{lb}{ft^3}\right)^{-1} \left(\frac{144 in^2}{1 ft^2}\right) = 69.2 ft$$

$$H_t = \frac{P_t}{\gamma} + El_t = 69.2 ft$$

$$H_a = \frac{P_a}{\gamma} + El_a^{\nearrow \emptyset} = H_t - h_{t-a} = 69.2 ft - \underbrace{1.259(3.289)^2}_{Kp_1 (Q_1)^2} = 55.58 ft$$

$$H_c = \frac{P_c}{\gamma} + el_c^{\nearrow \emptyset} = H_a - h_{a-c} = 55.58 ft - 9.558(1.289)^2 = 39.7 ft$$

$$P_c = 39.7 ft \left(62.4 \, lb/ft^3\right)\left(\frac{1 ft^2}{144 in^2}\right) = 17.2 \, lb/in^2$$

pipe	Kp	Q	|Q|
1	1.259	2.73	3.73
2	3.132	-2.27	2.27
3	9.558	0.73	1.73
4	40.28	-1.27	1.27

Delta Q 0.353571

pipe	Kp	Q	|Q|
1	1.259	3.084	3.084
2	3.132	-1.916	1.916
3	9.558	1.084	1.084
4	40.28	-0.916	0.916

Delta Q 0.1936694

pipe	Kp	Q	|Q|
1	1.259	3.277	3.277
2	3.132	-1.723	1.723
3	9.558	1.277	1.277
4	40.28	-0.723	0.723

Delta Q 0.0120231

pipe	Kp	Q	|Q|
1	1.259	3.289	3.289
2	3.132	-1.711	1.711
3	9.558	1.289	1.289
4	40.28	-0.711	0.711

Delta Q 4.67E-05

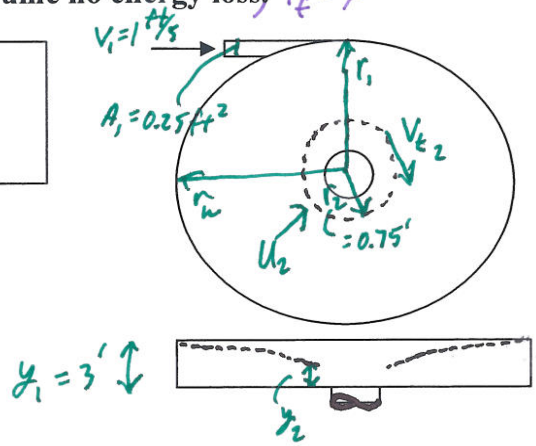
pipe	Kp	Q	|Q|
1	1.259	3.289	3.289
2	3.132	-1.711	1.711
3	9.558	1.289	1.289
4	40.28	-0.711	0.711
	Sum:	1 & 2	5.000

Example:
Consider a 10 ft diameter tank with a drain in the center of the floor with an outside wall depth of 3 ft. The tangential velocity at the perimeter is 1 ft/sec. What will the tangential velocity be at r = 0.75 ft? Plot the radial, tangential and depth as a function of radius. Neglect friction. Assume no energy loss, $F_f = \emptyset$

$h_{z1} = h_{z2}$

$A_{inlet} = 0.25$ ft^2

$A_{outlet} = 1.25$ ft^2



Continuity

$Q = V_1 A_1 = 1(0.25) = 0.25$ cfs

Angular Momentum

$V_t = \frac{C}{r}$ (free vortex b/c friction $= \emptyset$)

$C = r_1 V_1$; $r_w = \frac{10}{2} = 5'$; $r_1 = r_w - \frac{w}{2}$; $w = \frac{A_1}{y_1} = \frac{0.25}{3}$

$\therefore r_1 = 5 - \frac{\frac{0.25}{3}}{2} = \boxed{4.96 \text{ ft}}$ $\therefore C = 4.96(1) = \boxed{4.96 \frac{ft^2}{s}}$

$V_{t_2} = \frac{C}{r_2} = \frac{4.96}{0.75} = \boxed{6.61 \text{ ft/s}}$

Energy

$H_{T_1} = H_{T_2} + h_L^{\nearrow \emptyset} \rightarrow y_1 + \frac{V_1^2}{2g} + \cancel{h_{z_1}} = y_2 + \frac{V_{t_2}^2}{2g} + \frac{U_2^2}{2g} + \cancel{h_{z_2}}$

$\rightarrow y_2 = \left(y_1 + \frac{V_1^2}{2g}\right) - \frac{V_{t_2}^2}{2g} - \frac{U_2^2}{2g}$

$U_2 (\underbrace{2\pi r_2 y_2}_{\text{area of cylinder}}) = Q \rightarrow U_2 = \frac{Q}{2\pi r_2 y_2}$

– Assume U_2 is small but will have to check it after, or plug into energy equ & solve.

$\frac{U_2^2}{2g} = \frac{0.03}{64.4} \simeq \emptyset$ *small $\therefore$ Assumption OKAY

$y_2 \simeq 3 + \frac{1^2}{64.4} - \frac{6.61^2}{64.4} - \text{small} = 2.34$ ft

use approx y_2 to calc. U_2 to see if it is small

$U_2 = \frac{0.25}{2\pi(0.75)2.34} \cong 0.03$ *plug into correction

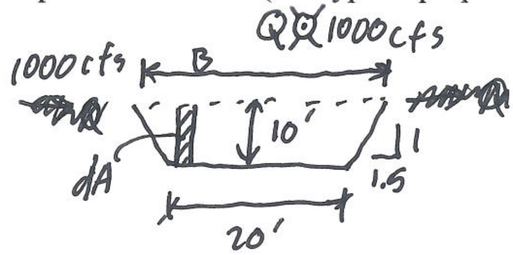
Donald Jerolleman

10/10

Assignment No. 1.1 Due Date : Next lecture.

1. Classify the flow regime in the following trapezoidal channel (use typical properties):

Q = 1000 cfs
b = 20 ft
z = 1.5
y = 10 ft
Kinematic viscosity = 10^{-5} ft ^{2}/sec



Area A = 350 ft²

$A = y(b + zy) = 10'(20' + 1.5(10')) = 350 ft^2$

Surface width B = 50 ft

$B = b + (2zy) = 20' + (2(1.5)(10')) = 50 ft$

Wetted perimeter P_w = 56.1 ft

$P_w = b + (2y\sqrt{1+z^2}) = 20' + (2(10')\sqrt{1+1.5^2}) = 56.1 ft$

Hydraulic radius R = 6.24 ft

$R = \frac{A}{P_w} = \frac{350 ft^2}{56.1 ft} = 6.24 ft$

Mean depth D = 7 ft

$D = \frac{A}{B} = \frac{350 ft^2}{50 ft} = 7 ft$

Velocity V = 2.86 ft/s

$V = \frac{Q}{A} = \frac{1000 ft^3/s}{350 ft^2} = 2.86 ft/s$

(N_F) Froude Number = 0.1905
Subcritical

$N_F = \frac{V}{\sqrt{gD}} = \frac{2.86 ft/s}{\sqrt{32.2 ft/s^2 (7 ft)}} = 0.1905$

(N_R) Reynolds Number = 1,784,640
Turbulent

$N_R = \frac{V(R)}{\nu} = \frac{2.86 ft/s (6.24 ft)}{10^{-5} ft^2/s} = 1784640$

2. Estimate the velocity head and momentum flow (M) for the channel in problem 1.
Assume that the kinetic energy and momentum correction factors are: α = Kinetic energy correction factor = 1.05; β = Momentum correction factor = 1.02.

Velocity head $\alpha V^2/2g$ = 0.1334 ft

$1.05(2.86^2 \ ft^2/s^2)/2(32.2 ft/s^2) = 0.1334 ft$

Momentum flow M' = 5665 slug·ft/s² (lbs)

$= \beta \rho V^2 A = 1.02(1.94 \frac{slug}{ft^3})(2.86 \frac{ft}{s})^2(350 ft^2)$

$= 5665 \frac{slug \cdot ft}{s^2}$

Lecture 19
Unit Tractive Force Method (Allowable Shear Stress Method)
Application of Critical Shear Stress

Critical shear stress *is the hydrodynamic boundary shear stress that initiates movement of bed sediment. Critical shear stress functions can be used to design channels for non-silting and non-scouring conditions. In actual channels the bed can be stable even if there is movement of the bed material; this can occur if there is a sediment load being transported in the channel and there is an equilibrium between erosion and deposition. Thus the allowable design shear stress in channels is a function of the sediment load and is often higher than the critical shear stress.*

Stable Channel Cross-section

If the bed slope is fixed (e.g. by topography), then the critical shear stress can be used to determine the channel depth and width for the case of negligible sediment transport and non-silting non-scouring conditions. The design procedure commonly is called the ***Maximum Permissible Unit Tractive Force Method***. The concept is that

Local Applied Shear Stress = Local Resistance of the Channel Boundary
= Allowable Design Shear Stress

$$\tau_o = \tau_b \geq \tau_c \qquad 19.1$$

The applied shear stress is the hydrodynamic shear on the boundary. The attached Figure 5.29 & 5.30 from the COE manual shows a typical stress distribution for a trapezoidal channel. The maximum bed stress is τ_o and is given by

$$\tau_o = C_b \gamma \; y S_o \qquad 19.2$$

and the maximum side stress is

$$\tau_{so} = C_s \gamma \, y \, So \qquad 19.3$$

where C_b and C_s are functions of b/y and z as shown in Figures 5.29 and 5.30 (COE Manual). For typical wide channels $C_b \sim 0.97$ and $C_s \sim 0.75 - 0.76$.

For other τ_b values see ven te Chow Table 7.3, Figures 7.10 and 7.11.

- e) Solve (c) for y, -> $y = \tau_b /(C_b \gamma S_o)$
- f) Solve (d) for φ and z,
  $\sin \varphi = \sin \theta_f [1- (C_s /C_b)^2]^{1/2}$
  $z = [1- (\sin \varphi)^2]^{1/2} /\sin \varphi$
- g) Solve for b from use Manning's $Q = (c'/n) AR^{2/3} S_o^{1/2}$.

- h) Check b/y and correct C_b and C_s if necessary; check Froude Number, Is n OK, e.g. are there bed forms? and
- Add Freeboard (FB2).

Example Problem: Design a channel with a slope of 0.0009 with bed material D50 = 1 inch and
D75 = 1.3 inches (well rounded gravel). The dominant flow is 20,000 cfs. Assume the Strickler equation for Manning's n.

<u>Cohesive Soils</u>
Figure 7.11 shows a graph that can be used to estimate the permissible shear stress for clay soils. The z for cohesive soils is approximated by the stable slope criteria discussed in Lecture 17.

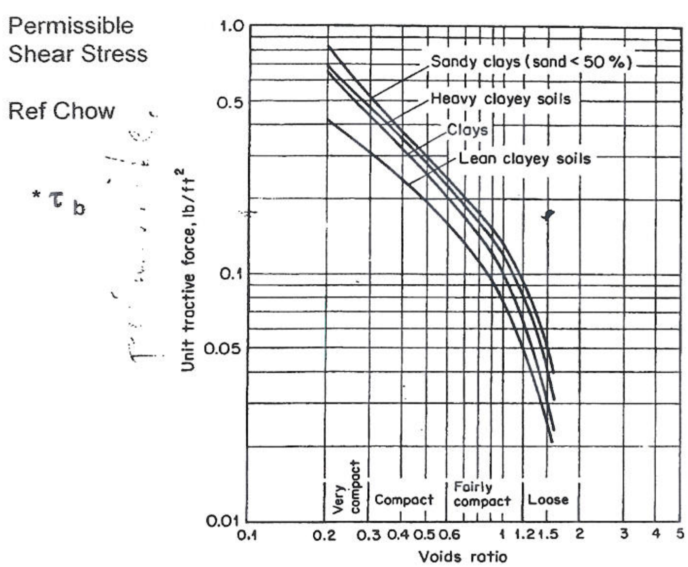
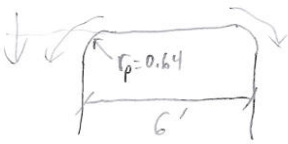


FIG. 7-11. Permissible unit tractive forces for canals in cohesive material as converted from the U.S.S.R. data on permissible velocities.

$r_p = 0.133\,H_d = 0.638 \sim 0.64$

$W_p = 2(r_p) = 1.28'$ but we want $6'$ so,



$\Sigma W_b = L_a =$ clear length

p.178 equ. 27-7, 27-8

calc C_d p.177 equ 27-3

$C_{d_0} = 3.97$

$C_d = 4.39$

L_e effective length = equ 27-7

N_b # of bays $= N_p + 1 = 101$

$W_{bay} = \frac{L_a}{N_b} = \frac{2348}{101} = 23.2'$

$L_t = L_a + N_p(W_p) = 2948'$

$Y = H_d\,K'\left(\frac{X}{H_d}\right)^n$ $\quad n = 1.85$, $K' = 0.5$

$\frac{dY}{dX} = \frac{1}{m} = 1\left(n\,H_d\,K'\left(\frac{X^{n-1}}{H_d^{\,n}}\right)\right)$ solve for X

This is X @ Tangent point (X_{TP})

then go back and get Y.

Bucket Radius

R_b p.179

+ V_1 equ. 27-10

$y_1 = \frac{Q}{W V_1} = \frac{385000}{2948(33.6)} = 3.89'$

$N_{F1} = \frac{V_1}{\sqrt{g y_1}} = 3$

y_2 equ 28.1 $= 14.67'$

Problem

y_2 avail = TWL − SBFL = 6 ft

So hydraulic jump will be swept out & tear up channel

next step guess @ (−9) ft for apron (aka SBFL = (−11) not Ø)

this changes Z ....

→ V_1 → Y_1 → N_{F1} → Y_2 req'd

now check against avail.

$V_1 = 42.8$

$y_1 = 3.05$

$N_{F_1} = 4.32$

$y_2 = 17.2$ req'd

$y_2 = 6 - (-11) = 17$ ft

try again w/ −11.3

$V_1 = 43.1$ ft/s ; $y_1 = 3.03'$

$N_{F_1} = 4.36$

$y_2 = 17.24$ ft

y_2 aval $= 17.3$ ∴ OKAY

Name Donald Jerolleman From Lecture 27

DESIGN SUMMARY

Maximum Flow Q =	385000	cfs	**Given**	
Maximum Pond Level =	23	ft	**Given**	
Crest Level =	12	ft	**Given**	
Maximum Head = H =		ft		
Approach Invert		ft	**Given**	
V_a	3	ft/sec	$V_a = Q/\{L_t y_a\}$; $y_a = [Max\ Pond\ Level - Approach\ Invert\ ]$	
H_e **max =**	11.14	ft		
h_p **=**	(-20)	ft	**Given**	**-20 ft**
Design Head H_d **=**	4.8	ft	**Use** $H_d = \frac{H_{e,max}}{\left[1 - \frac{h_p}{C_p H_{e,max}}\right]}$	**Assume Cp=1.35**
C_{do} **=**	3.97		**WES** Given	**3.97**
C_d **=**	4.39			
m =	1		**Assume** Given	**1.0**
X_{tp} **=**		ft		
Y_{tp} **=**		ft		
K_p **=**	-0.01	ft	**WES** Given	**-0.01**
L_e **=**	2358	ft	$L_e = \frac{Q_{max}}{C_d H_{e,mac}^{3/2}}$	
L_a **=** $L_e + N K_p H_e = L_e - 100(-0.01)(11.14)$	2348	ft		
Number of piers	100		**Given**	**100**
W_{bay} **=**	23.2	ft		
W_p	6	ft	**Min** Given	**6 ft**
L_t **=**	2948	ft		
TWL =		ft		
SB FL Elev. =		ft		
W_1 **=**		ft		
Y_1 **=**		ft		
V_1 **=** $\sqrt{2g(Z - H/2)}$	43.1	ft/sec		
N_{f1} **=**	4.36			
y_2 **=**	17.24	ft	$y_2 = \frac{y_1}{2}\left\{\sqrt{1 + 8N_{f1}^2} - 1\right\}$ equ 28.1	
[TWL-(SB FL Elev)] =		ft		

Lecture 27
Rapidly Varied Steady Flow - Spillway and Stilling Basin Design

Assignment Due Date : In class assignment.
Reference Corps of Engineers Manuals and Handouts.

Design Case Study
See separate handout.

Function of a Spillway
The function of a spillway is to safely pass the excess flood waters around, through of over a dam.

Types of Spillways
The following are examples of commonly used spillways:

1) Crest, e.g. Ogee, WES, weir,
2) Side Channel,
3) Drop Inlet, e.g. Morning Glory
4) Sluice,
5) Over-and-Under,
6) Fuse-plug,
7) Syphon,
8) Stepped,
9) In-built.

The spillway may be gated of free flow. In any case the gates are assumed to be fully open at the PMF.

Design Considerations
1. The most important design criterion for a spillway is the design flood. The selection of this flood must consider the consequences of exceeding the spillway capacity. Generally it is assumed that if the dam is overtopped it will fail. If this would cause any risk to human life then the probable maximum flood (PMF) must be used. This flood is determined by hydrologic studies of the existing floods, regional flood analysis, regional rainfall analysis, probable maximum rainfall analysis (maximum moisture content in air column and maximum efficiency of conversion to precipitation) and rainfall runoff models and flood routing models.
In rivers with very long and reliable flow records, the 1:10,000 year flood is sometimes used as the design flood. In this case, the extrapolation of the flood frequency curve is based on the probability function that best fits the available annual series of peak flows. The most common probability functions are the log-normal Pearson III and the Gumbel distribution.
2. Another criterion in designing a spillway is the maximum allowable reservoir level during the passage of the probable maximum flood. This is established by the overall project cost-benefit analysis. The cost side includes: the cost of building a higher dam, the cost of land and flood rights, environmental and transportation costs and present value of future costs such as operation s and maintenance. The benefits include: increased storage, increased hydroelectric

power, increased attenuation of flood peaks. Based on acceptable interests rates and inflation rates the annual benefits (income) must exceed the amortized capital costs plus the operating and maintenance costs. In fact the owners would like to maximize the return on their investment; in this case the height of dam that maximizes the return on the capital investment would be the design height that is selected. In other cases the dam height that give the maximum benefit to cost ratio is selected.

3. It is also necessary to know the tailwater level (river stage downstream from the dam) for the entire range of floods from the low flows to PMF. This is usually presented as a Stage versus Q curve which is also called a rating curve. This curve may change with time after the construction of the reservoir. For example, the river morphology will change due to removal of sediment load in the reservoir; this may cause degradation of the channel and lowering of the tailwater level. This information is needed to design the stilling basin and other outlet works for the dam. It is also need to estimate uplift on the structure and back pressure on turbines. Fish migration structures are designed for a specified tailwater range.

A tailwater rating curve can be established using existing flow and stage records; however, if these are not available it will be necessary to use models like HEC-RAS to estimate the rating curve - in this case calibration with actual stage-flow data is essential. The tailwater rating curve may exhibit hysteresis, i.e. on the rising limb of the flood the stage may be lower than normal and on the falling limb it may be higher than normal where normal refers to the stage that would exist for the same steady flow.

4. The normal pond elevation is often used to establish the sill of the spillway. It may also correspond to the ice loading elevation.

Crest Spillway Design

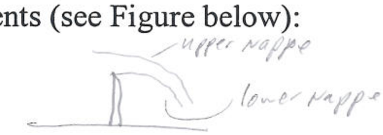
A complete spillway typically consists of the following elements (see Figure below):

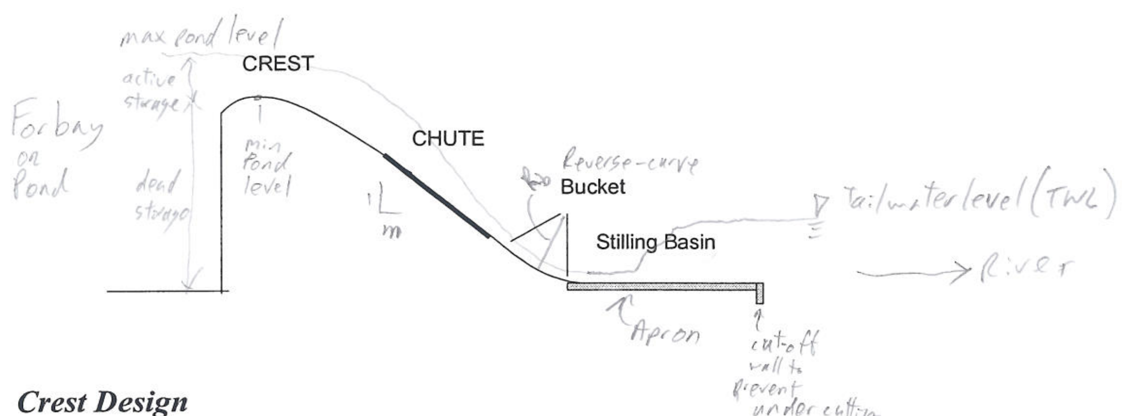
a) the crest section, m ~ 0.8 → 1

b) the chute,

c) the bucket,

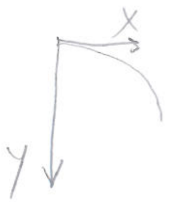
d) the stilling basin.





Crest Design

We need to select a spillway form that has low cost and high capacity. An early attempt to obtain an efficient shape was to take the lower nappe of the flow over an aerated sharp-crested weir as the shape of the concrete crest (see figure below). Of course the weirs were scale models of the actual spillway. The idea was to have nearly zero normal force between the water and the concrete and therefore have almost no frictional resistance for the selected head on the weir. This gave a parabolic spillway shape. $n = 1.85$, $K' = 0.5$, H_d = head that gave pressure ~ zero @ bottom "Design Head" safe H_d = Δ nappe not best answer though

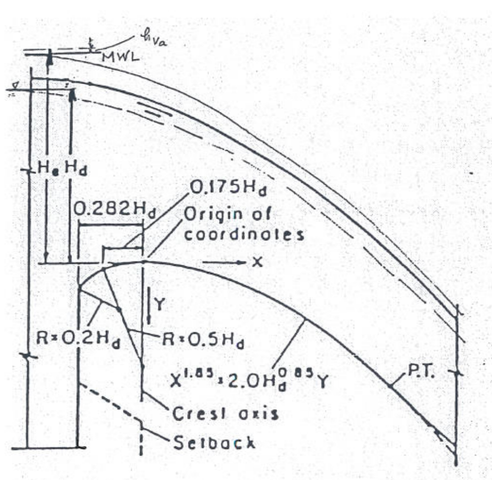


$Y = H_d K' \left(\frac{X}{H_d}\right)^n$

To generalize these results and make the crest easier to construct, the Waterways Experimental Station (WES) proposed the following dimensionless equation for the downstream portion of the crest (see Figure below):

$$\mathbf{Y/H_d = K' (X/H_d)^n} \qquad (27\text{-}1)$$

where H_d is the design head (not necessarily the maximum head); X, Y are Cartesian coordinates of the crest as shown below; K_d and n are constants that depend on the upstream batter and the relative height of the spillway (see attached table). For, typical vertical face spillway **K' =1/2 and n = 1.85.**



WES suggests a compound curve for the upstream portion of the spillway. The radii and offsets are proportional to H_d.

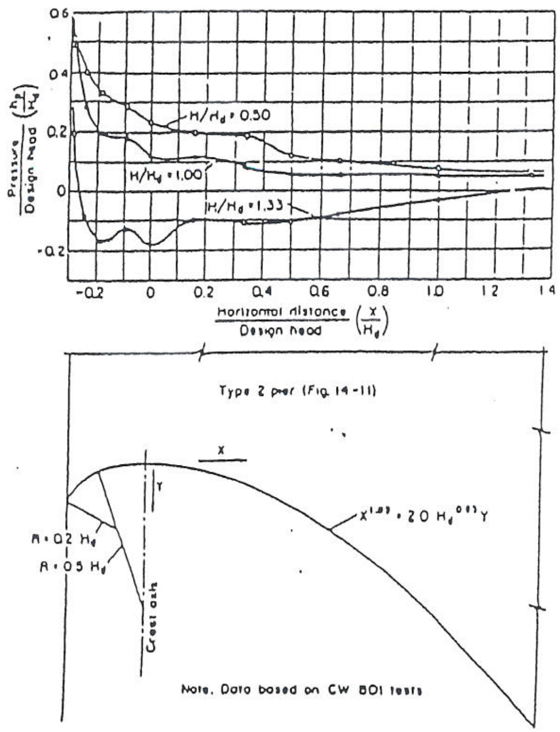


FIG. 14-13. Crest pressures on WES high overflow spillways (*continued*). (*b*) Along center line of pier bay. (*U.S. Army Engineers Waterways Experiment Station* [20], *Hydraulic Design Chart* 111-16/1, *WES* 3-55.)

w/ piers ; pressure in-between piers

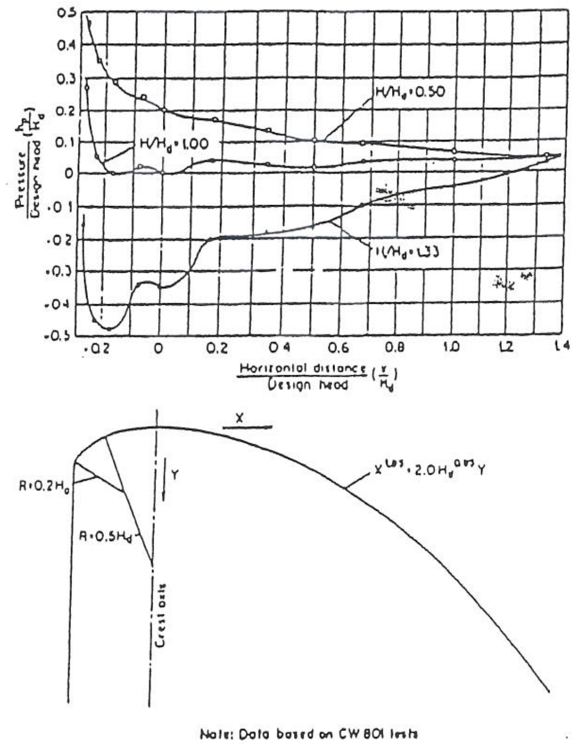


FIG. 14-13. Crest pressures on WES high overflow spillways. (*a*) No piers. (*U.S. Army Engineers Waterways Experiment Station* [20], *Hydraulic Design Chart* 111-16, *WES* 9-54)

w/out piers

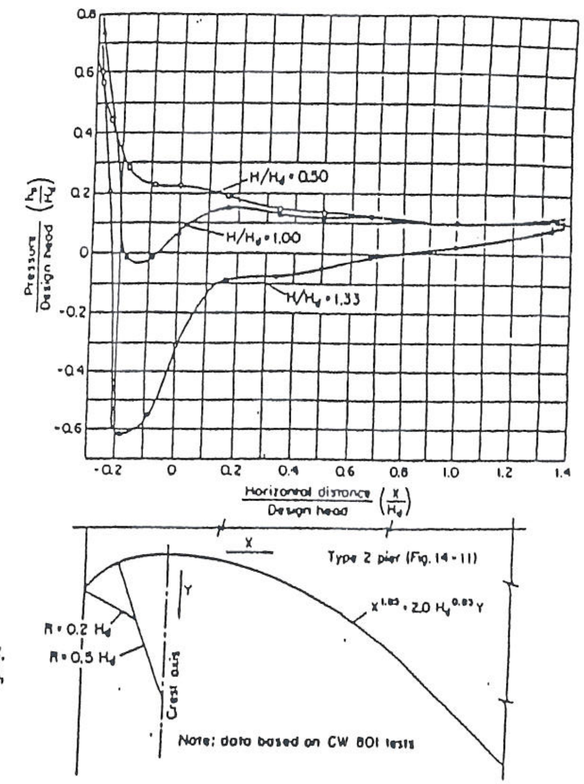


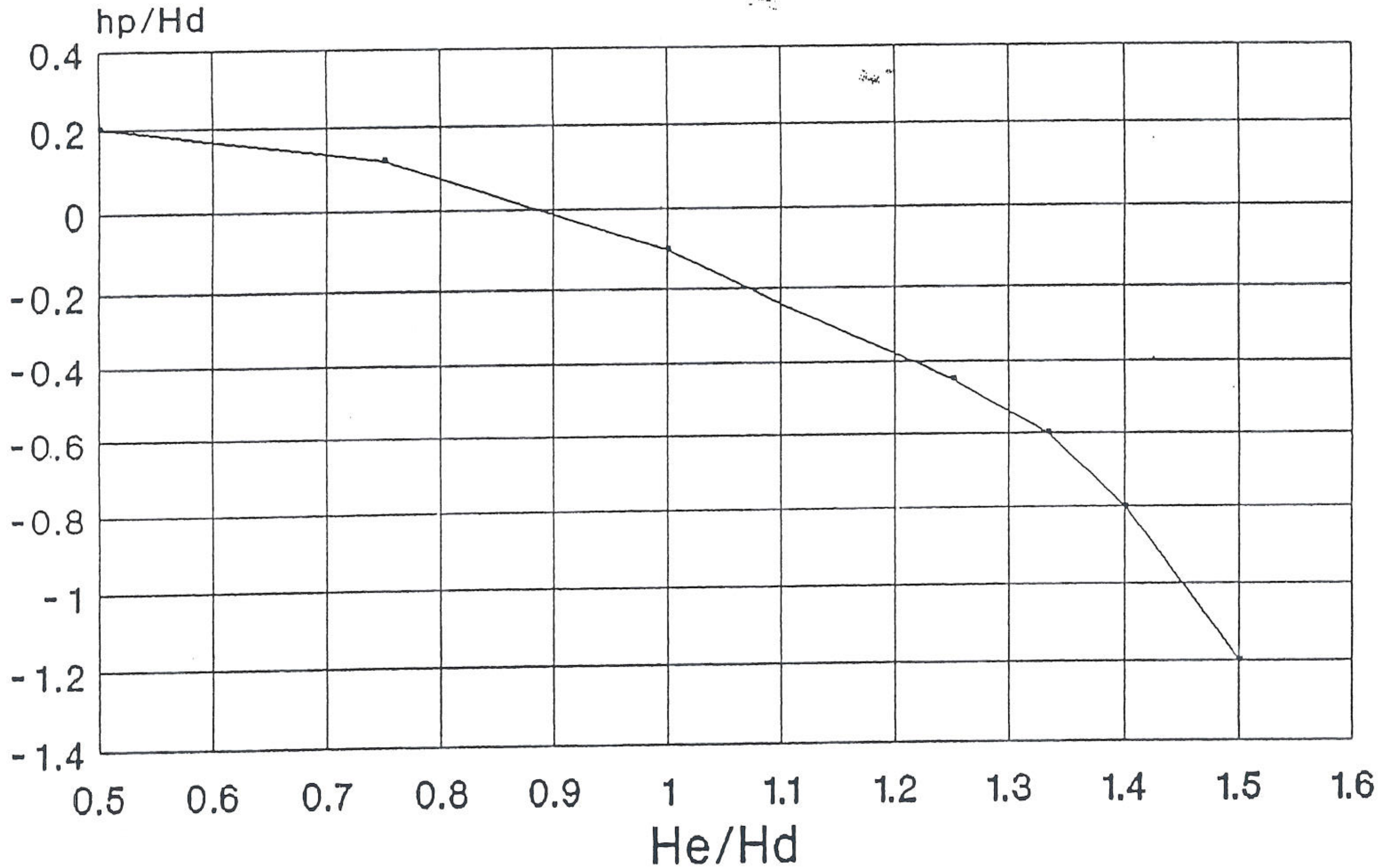
FIG. 14-13. Crest pressures on WES high overflow spillways (*continued*). (*c*) Along piers. (*U.S. Army Engineers Waterways Experiment Station* [20], *Hydraulic Design Chart* 111-16/2, *WES* 3-55.)

w/piers ; pressure along piers

FIG 26-1

Minimum Crest Pressure Head

After Ven te Chow



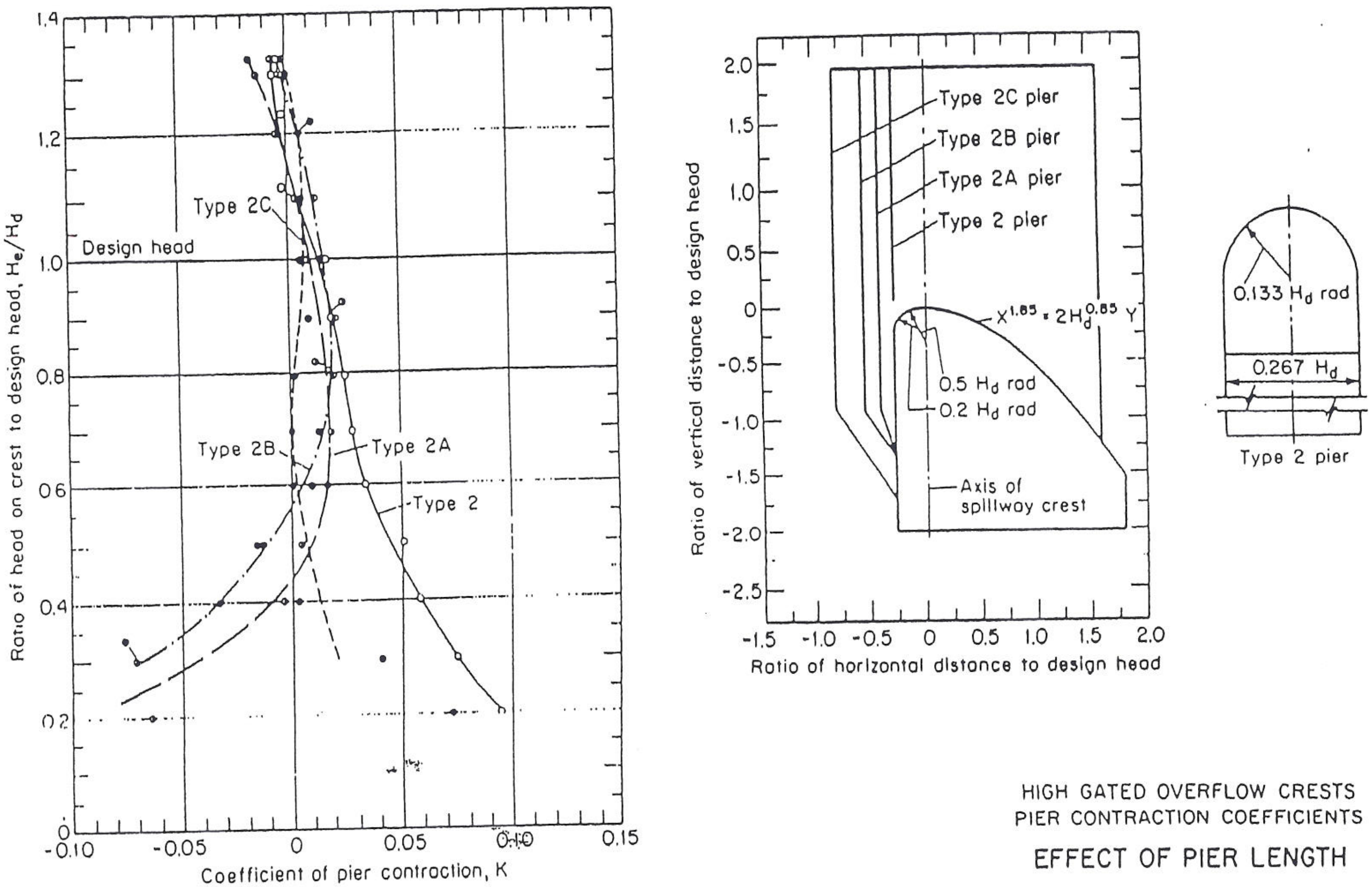


FIG. 11-10. Coefficient of contraction for the round-nose pier in high dams. (*U.S. Army Engineers Waterways Experiment Station* [20], *Hydraulic Design Chart* 111-6, *WES* 4-1-53.)

Determination of the Maximum Energy Head $H_{e\,max}$

The maximum head on the crest of the spillway during the passage of the PMF is

$$H_{e\,max} = (\text{Maximum Pond Level - Crest Elevation}) + V_a^2/2g$$

where V_a =approach velocity $= Q_{max}/A_{forbay}$;the Crest Elevation; usually the normal pond level.

Discharge Equation

The discharge over a WES spillway is given by

$$Q = C_d\, L\, H_e^{\ 1.5} \qquad 27\text{-}2$$

can use to get 1st est. of length of spillway
actual = Le + width of piers + contraction from piers...
"effective L=Le"

where H_e = the energy head above the spillway crest; L = effective length of the spillway crest; C_d = discharge coefficient which is a function of the ratio of $\{H_e /H_d\}$, e.g.

reference, emperical from lab

$$C_d = C_{do}\, \{H_e/H_d\}^{0.12} \qquad 27\text{-}3$$

where C_{do} = the discharge coefficient for $H_e = H_d$. In U.S. units C_{do} = 3.97. From COE

Selection of Design Head H_d

The design head H_d is the scaling parameter for all of the elements of the spillway crest. It is selected to reduce the concrete in the crest section, to maximize the Q but to do this without causing cavitation due to low negative pressures on the crest. Since the size of the crest increases with H_d, the larger the H_d the more concrete that will be needed.

From the discharge coefficient it can be seen that there is an advantage of increase Q due to selecting

$$H_d < H_{e\,max}. \qquad 27\text{-}4$$

However, since the radii of curvature are proportional to H_d, as H_e increases relative to H_d the negative pressure on the crest also increases, as indicated by

$$p/\gamma \sim d(1 - V^2/(g\,k\,H_d) \sim d(1 - C\,H_e/H_d) \text{ since } V_c^2/2g \sim H_e/3$$

Figure 27-1 (attached) was developed using experimental data on the lowest pressure head on a WES spillway with different ratios of $\{H_e /H_d\}$. Figures 26-14 a,b,c (from ven te Chow) show some of the dimensionless WES experimental plots of pressure head along the bed of the crests for different ratios of $\{H_e /H_d\}$ with and without piers. As a guide the lowest pressure should be

COE deffinitions: $H_e = H + \frac{Va^2}{2g}$ (energy head; Va — velocity approaching spillway)

$He_{max} = \{\text{max pond level} - \text{Crest el.}\} + \frac{Va^2}{2g}$

>> the vapour pressure of approximately **- 33 ft** for sea level installations. Due to irregularities in the concrete bed and walls a safe negative pressure is approximately, **- 18 to -20 ft.**

The design H_d that will give the highest discharge coefficient and still be safe from cavitation is the one that gives $p_{min}/\gamma \sim -18$ ft at the maximum head on the crest, $H_{e\,max}$.

$$H_d.\ H=H_{e\,max}.\ /\{1- h_p./(1.35\ H_{e\,max}.)\} \qquad 27\text{-}5$$

Example:
Given: $H_{e\,max} = 60$ ft; use $p_{min}/\gamma \sim -20$ ft.
Find H_d.

Selection of Piers
The pier width and nose are determined based on $\boldsymbol{H_d}$. For example a Type II WES Pier has a thickness of $0.266\ \boldsymbol{H_d}$ and Radius of $0.133\ \boldsymbol{H_d}$.

Crest Length
The effective crest length L is

$$Le = \boldsymbol{L_a} - \boldsymbol{N\, K_p\, H_e} \qquad 27\text{-}6$$

where $\boldsymbol{L_a}$ = actual (clear) crest length; $\boldsymbol{N}$ = number of pier contractions; $\boldsymbol{K_p}$ = pier contraction coefficient. The effective length is found from

$$Le = Q/\{\ C_d\ H_e^{\ 1.5}\} \qquad 27\text{-}7$$

and then $\boldsymbol{L_a} = Le + \boldsymbol{N\, K_p\, H_e}$ 27-8

Start of Chute

The Chute starts when the slope of the crest function = the assigned chute slope (1/m). The minimum value of m depends to some extent the stability analysis of the gravity section of the entire spillway.

$$dY/dX = 1/m$$

Bucket Radius

The bucket radius depends on the velocity and flow. Chow gives an empirical equation for

$$R_b =$$

Velocity at Entrance to Stilling Basin

The energy principle along with an appropriate friction equation can be used to estimate the velocity at the bottom of the spillway:

$$V_1 = [2g\,(Z - y_1 - h_f)]^{1/2} \qquad 27\text{-}9$$

where Z = [TEL in pond - Stilling Basin Floor Elevation]; y_1 = depth at start of stilling basin; h_f = energy loss from pond to stilling basin entrance. Note: $y_1 = Q/(V_1\,W)$ where W is the width of the stilling basin.

The USBR developed an alternative to the above equation:

$$V_1 = [2g\,(Z - H_e/2]^{1/2} \qquad 27\text{-}10$$

which is explicit and eliminates the friction term.

Lecture 28
Design of Stilling Basins

Function of a Stilling Basin

The function of a stilling basin is to dissipate the excess kinetic energy at the toe of the spillway to avoid damage to the downstream channel or property.

Types of Stilling Basins

1. Hydraulic Jump,
2. Impact,
3. Flip-bucket
4. Plunge Pool,
5. Submerged Bucket and Roller,
6. Stepped spillway,
7. Baffled Chute,
8. Raft.

Design Criteria

1. The stilling basin must protect the dam and spillway from failure for all floods up to the PMF.
2. The stilling basin should protect the downstream channel and property for serious damage for the regional flood, e.g. 1:100 year flood.

Theory

Hydraulic Jump Stilling Basins: An hydraulic jump is the transition for supercritical to subcritical flow. Due to the inherent instability of the decelerating flow, a large portion of the kinetic energy is converted to turbulent energy and subsequently lost as heat energy. A portion of the kinetic energy is also converted tp potential energy. There are several types of hydraulic jumps. For stilling basin design, two of these are very important, i.e. the free jump and the forced jump. The former occurs when there are no appurtenances to aid in the formation of jump and the latter refers to jumps that have appurtenances such as baffle blocks, chute blocks and sills, to assist in the formation of the jump.

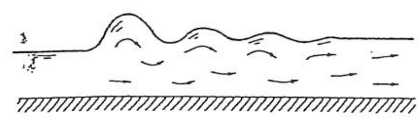
The characteristics of a free hydraulic jump depends on the Froude Number at the start of the jump. The table below summarizes 5 phases of the free jump. The US Bureau of Reclamation (USAR), St. Anthony Fall Laboratory (SAF) and WES have designed stilling basins especially for each phase (see USAR, "Design of Small Dams"; USCOE, "Hydraulic Design Criteria" and EM1100-1602 & 1603; ven te Chow, "Open Channel Hydraulics").

The momentum principle gives the downstream (sequent depth) of a free jump,

$$y_2 = y_1 \{[1 + 8N_{F1}^2]^{1/2} - 1\}/2 \qquad 28.1$$

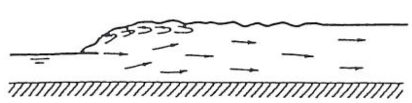
and the energy equation gives the energy loss as

$$\Delta E = E_1 - E_2 = (y_2 - y_1)^3/\{4\, y_1\, y_2\} = [V_1^2/2g].(y_2 - y_1)^3/\{2N_{F1}^2\, y_1^2\, y_2\} \qquad 28.2$$

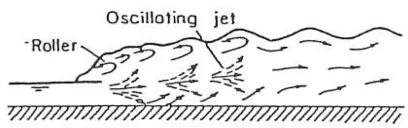


$F_1 = 1-1.7$ Undular jump

v.t. Chow

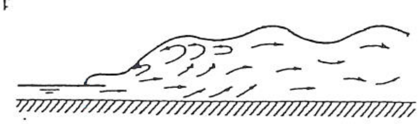


$F_1 = 1.7-2.5$ Weak jump



$F_1 = 2.5-4.5$ Oscillating jump

$F_1 = 4.5-9.0$ Steady jump



$F_1 > 9.0$ Strong jump

Stilling Basin Design

The velocity at the toe of the spillway can be estimated from the energy and continuity principles applied between the forebay and the toe, i.e.,

$$V_1 = [\, 2g\,(Z - y_1)]^{1/2} \qquad \& \qquad y_1 = Q/(W\, V_1) \qquad 28.3$$

or the USBR empirical equation can be used to obtain V_1 directly,

$$V_1 = [\, 2g\,(Z - H/2)]^{½} \qquad \& \qquad y_1 = Q/(W\, V_1) \qquad 28.4$$

Froude No. at the toe is $N_{F1} = V_1/(g\, y_1)^{1/2}$ is supercritical.

The purpose of the stilling basin is the dissipation of the excess energy at the toe of the spillway.

One means of doing this is to force a hydraulic jump to occur before the flow re-enters the river channel.

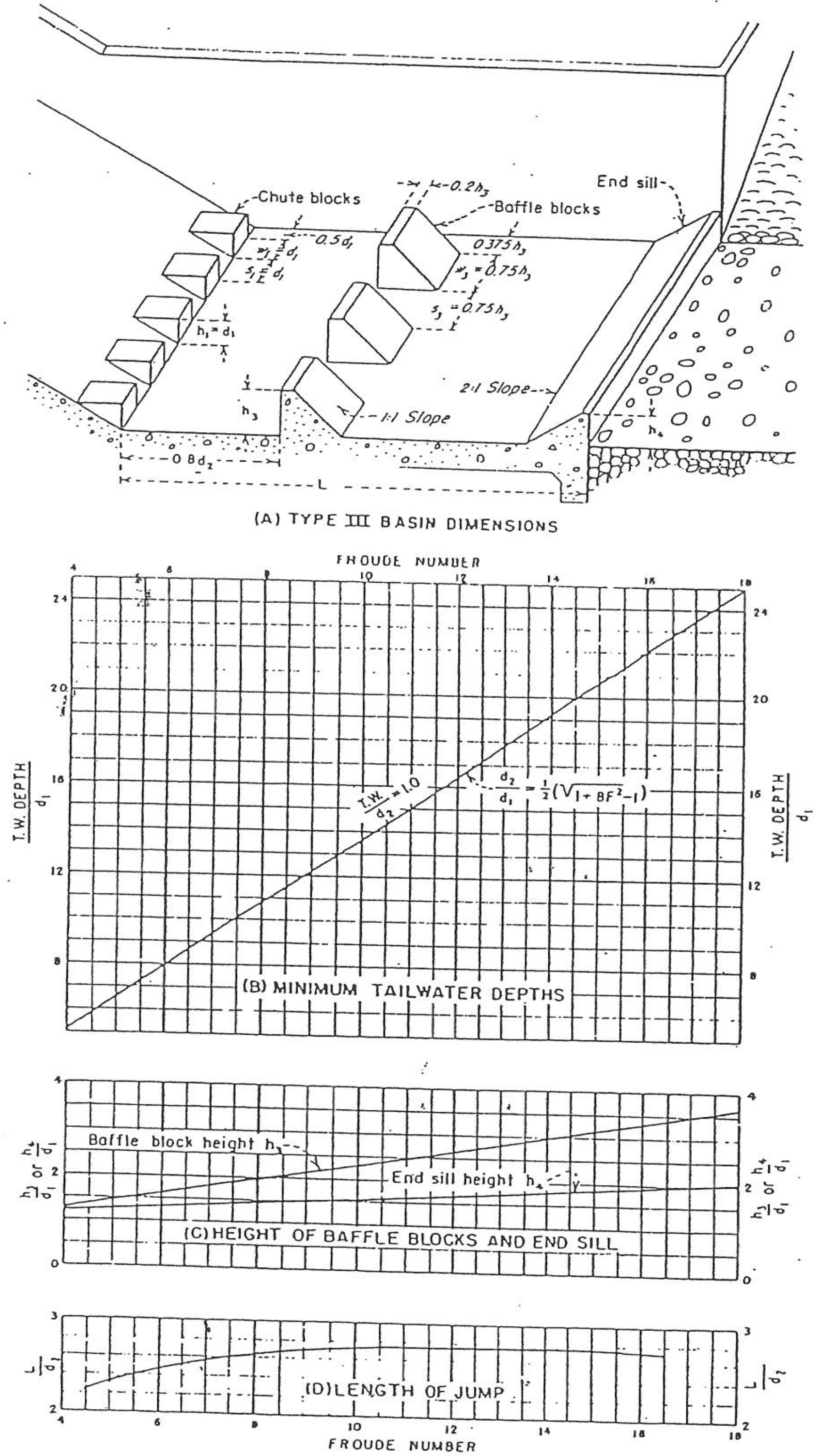


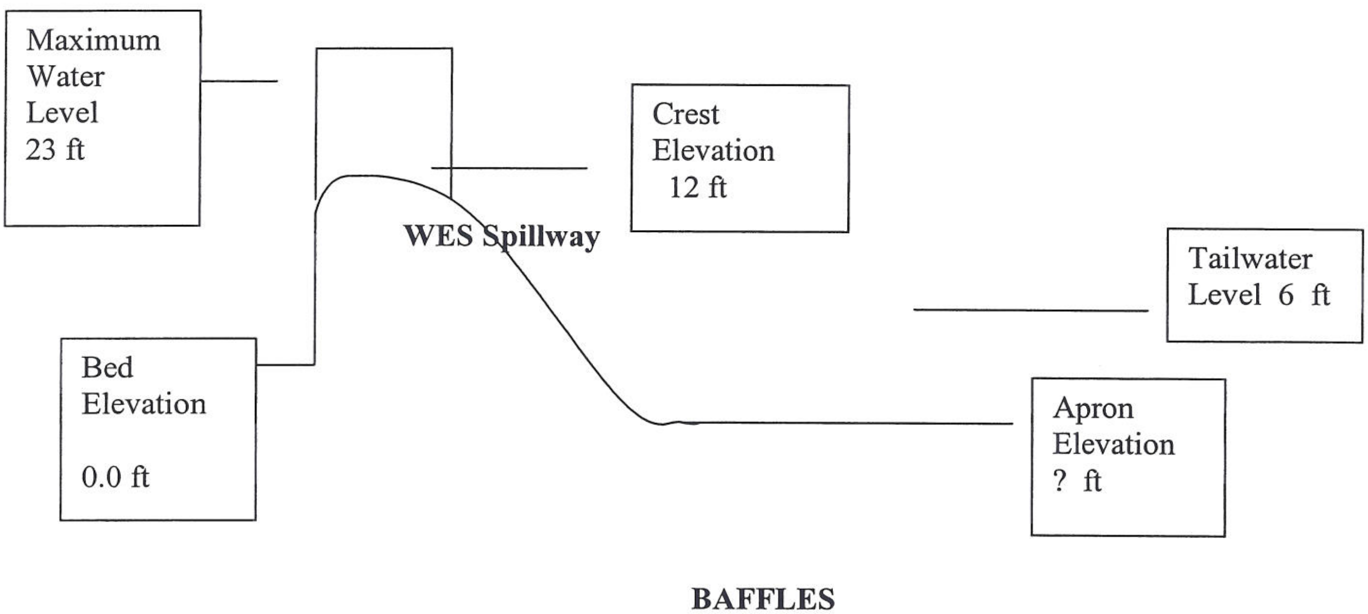
Figure 266. Stilling basin characteristics for use with Froude numbers above 4.5 where incoming velocity (V_1) does not exceed 50-60 feet per second. 288–D–2426.

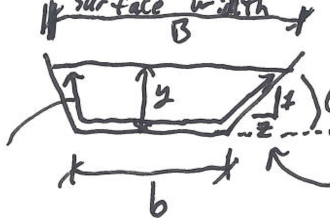
In-Class Tutorial

Complete the design of the Spillway and Stilling Basin in the problem statement below. WES = waterways experimental station

A typical cross-section of the spillway is shown below. The crest follows the WES standard design. There are 100 piers each 6 ft wide. Assume that the minimum pressure head on the spillway is -20 ft.

a) The maximum discharge from the spillway is 385,000 cfs.
b) Determine if a hydraulic jump conditions on the apron.





"z" is cotan of $\theta = \frac{1}{\tan\theta} = \frac{Run}{Rise}$

P_w "wetted Perimeter"

$A = y\underbrace{(b + zy)}_{\text{avg. width}}$ $B = b + (2zy)$ $P_w = b + (2y\sqrt{1+z^2})$

UNIVERSITY OF NEW ORLEANS
DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

ENCE 4318 & 4318G HYDRAULIC ENGINEERING SYSTEMS

Professor A. McCorquodale, P.E.

Outlet Structure at a Small Dam Showing a Radial Flow Stilling Basin with Baffles

D (mean depth) $= \frac{A}{B}$ R (Hydraulic Radius) $= \frac{A}{P_w}$

Fundemental	Derived
ft, lbf, sec	mass : $\frac{W}{g} = \frac{lb\,sec^2}{ft}$ = slug
m, kg, s	Force : $W = m \cdot g$ = N

Common Symbols in Hydraulics

A = Area normal to flow

b = Bottom width

B = Top width [also T]

c = Celerity = ˜(gD) [also c = head due to centrifugal force]

C_c = Contraction coefficient [e.g. 0.5 for re-entrant case; 0.61 for flush opening]

d = Depth of flow normal to the bed

D = Hydraulic mean depth = A/B or A/T

E = Specific Energy $= y + \alpha V^2/2g$

f = Friction factor

F = Force [sometimes Froude number = **F**, e.g. French and ven te Chow]

▯ = Specific force = ▯ A + $\beta Q^2/(gA)$

g = Acceleration due to gravity [use 32.2 ft/sec^2 or 9.81 m/s^2]

h = Energy head [energy per unit weight]

h_T =Total mechanical energy head at a point [Pressure head+elevation+velocity head]

h_v = Discharge averaged velocity head $= \alpha V^2/2g$

h_z = Elevation

H_T = Discharge averaged total mechanical energy head over a cross-section
[Pressure head+elevation+velocity head]$_{average}$
$= p/\gamma + h_z + \alpha V^2/2g$

L = Length [length of channel or length of a reach]

m = Mass

m= = Mass flow or mass flux.

M= = Momentum flux or momentum flow

M$_a$= = Angular Momentum flux or angular momentum flow

n = Manning=s roughness factor

N_F = Froude Number = V/̃(gD) [also **F**] $V/\sqrt{gD}$

N_R = Reynolds Number = VR/ν [also $\mathbf{R}_N$]

p = Pressure

P = Wetted perimeter [also pressure force]

q = Discharge per unit width

Q = Discharge = VA

r = Radius of curvature

R = Hydraulic radius = A/P

S_e = Energy slope

S_f= Friction slope

S_o= Bed slope

t = Time

T = Top width in French and ven te Chow [also B in other references]

T = Wave period.

{u,v,w} = Velocity components in {x,y,z} Cartesian coordinates

V = Average velocity over a cross-section of area A

W = Width

x = Distance measured along the bed. Commonly taken in the direction of flow

y = Vertical depth of flow at deepest portion of cross-section

▯ = Depth to the centroid of A

α = Kinetic energy correction factor

β = Momentum correction factor

γ = Specific freshwater weight = g ρ (62.4 lbs/ft^3; 9810 N/m^3)

δ = Boundary layer thickness

ε = Roughness height

κ = von Karman universal constant = 0.4

λ = Wave length

θ = Bed slope angle

θ_f = Friction angle

μ = Dynamic viscosity

ν = Kinematic viscosity (typical 10^{-5} ft^2/sec; 10^{-6} m^2/s)

σ = Surface tension (typical 5*10^{-3} lbs/ft; 7.3*10^{-2} N/m)

φ = Side slope angle

τ = Shear stress

ρ = Density (Freshwater 1.94 slugs/ft^3; 1000 kg/m^3)

Some Useful Conversion Factors (About 3 significant figures)

Multiply Value in [---]	**by**	**To Get Value in [--]**
[ft]	**0.3048**	**[m]**
[acre]	**43560**	**[ft^2]**
[gal]	**3.785**	**[L]**
[gal/min] = [gpm]	**0.00223**	**[ft^3/sec] = [cfs]**
[million gal/day] = [MGD]	**1.547**	**[ft^3/sec]**
[m^3/s] = [cms]	**35.3**	**[ft^3/sec]**
[horse power] = [hp]	**0.746**	**[kW]**
[Joule] = [J]	**1.000**	**[N.m]**
[lbs force] = [lbs]	**4.448**	**[N]**
[lbs mass] = [lbm] **The weight of [1 lbm] is very close to [1 lbs force] at sea level. Use W[lbs force]/g[ft/sec^2] to get value of mass in [slugs]**	**1/32.2**	**[Slug]**
[lbm] **[slug]**	**1/2.205 = 0.4535** **14.6**	**[kg]** **[kg]**
[lbs mass/ft^3] density **[slugs/ft^3] density** **[lbs force/ft^3] specific weight**	**16.02** **515.7** **157.1**	**[kg/m^3]** **[kg/m^3]** **[N /m^3]**
[mile] = [mi]	**5280**	**[ft]**
[psi]	**144**	**[lbs/ft^2] = [psf]**
[psi]	**6894.8**	**[N/m^2] = [Pa]**
[psf]	**47.88**	**[N/m^2] = [Pa]**
[N.m]	**0.7376**	**[ft.lbs]**
[atm]	**33.9**	**[ft of water]**
[atm]	**101.3**	**KPa**

[inch] = [in]	**2.54**	**[cm]**
[inch]	**1/12**	**[ft]**

Note:

1. In the U.S. system the fundamental units are ***ft, lbs* force and *sec*** while mass is derived and is expressed as *slugs* = *lbs.sec²/ft*. For example mass of a certain weight = {W/g}.
2. In the S.I. system the fundamental units are ***m, kg* mass and *s*** while force is derived and is expressed as $N = kg.m/s^2$. For example weight of a certain mass is W = {m.g}.

Definitions, Terminology and Classification of Flows

Incompressible Flow means that the fluid has a constant density.

Free surface flow: ***Flow where one surface has a constant pressure; usually this is an air-water interface with the constant pressure being atmospheric pressure.***

Prismatic channel: ***A channel with constant geometry, including fixed cross-sectional shape, constant slope as well as constant boundary roughness.***

Flow boundary

Pipe flow – *flow entirely constrained by solid boundaries with no constant pressure surface.*

Open channel flow – *flow with a free surface with constant pressure, e.g. atmospheric pressure.*

Time

Steady – *depth and velocity or pressure and velocity do not change with time at any given location.*

Unsteady– *depth and velocity or pressure and velocity change with time at any given location.*

Space

Uniform– *depth and velocity do not change with location along a channel at any instant in time in a prismatic channel.*

Varied: – *depth and velocity change with location along a channel at any instant in time.*

Gradually Varied *–gradual change in depth with distance along the channel* $|dy/dx| < 1/20$

Rapidly Varied *–large in depth with distance along the channel* $|dy/dx| >/20$

Force

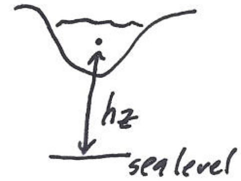
Viscous: Reynolds Number = $N_R = R_h V/\nu$

Laminar	*Turbulent*
$N_R < 500$	$N_R > 500$

Gravity: Froude Number = $N_F = V/\sqrt{(gD)}$ where $\sqrt{(gD)} = c$ = Celerity (surface gravity waves) and V is the average water velocity.

Critical	*Subcritical*	*Supercritical*
$N_F = 1$ and $V = \sqrt{(gD)}$	$N_F < 1$ and $V < \sqrt{(gD)}$	$N_F > 1$ and $V > (gD)$

Head: energy per unit weight



$\gamma_{H_2O} = 62.4 \frac{lb}{ft^3}$

h_T = press. energy + PE + KE — local velocity

$= \left(\frac{p}{\gamma}\right) + h_z + \frac{1}{2}\frac{v^2}{g}$

Lecture 1

Definitions, Terminology and Classification of Flows

Q, discharge $\left[\frac{ft^3}{s}\right]$
V = mean velocity
Q = V·A

Incompressible Flow means that the fluid has a constant density.

Free surface flow: ***Flow where one surface has a constant pressure; usually this is an air-water interface with the constant pressure being atmospheric pressure.***

Prismatic channel: ***A channel with constant geometry, including fixed cross-sectional shape, constant slope as well as constant boundary roughness.***

Incompressible fluid : ρ = const.

Flow boundary — "pressure flow"

Pipe flow – *flow entirely constrained by solid boundaries with no constant pressure surface.*

Open channel flow – *flow with a free surface with constant pressure, e.g. atmospheric pressure.*

Time

Steady – *depth and velocity or pressure and velocity do not change with time at any given location.*

Unsteady– *depth and velocity or pressure and velocity change with time at any given location.*

Space

$\frac{dy}{dx} = 0$

Uniform– *depth and velocity do not change with location along a channel at any instant in time in a prismatic channel.*

Varied: – *depth and velocity change with location along a channel at any instant in time.*

Gradually Varied *–gradual change in depth with distance along the channel |dy/dx|<1/20*

Rapidly Varied*–large in depth with distance along the channel |dy/dx|>/20*

Force

$R_n(\text{pipe}) = \frac{\text{Dia.}\,V}{\nu}$ > 2000 is turbulent flow

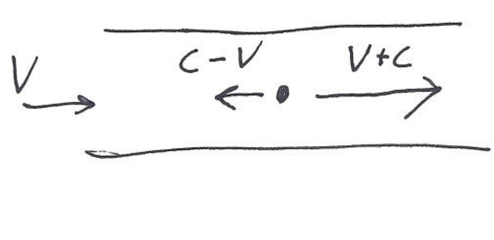
Viscous: Reynolds Number = $N_R = R_h V/\nu$

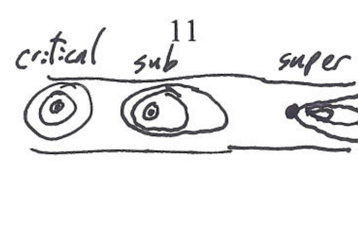
Laminar	*Turbulent*	Full turbulent
$N_R < 500$	$N_R > 500$	$N_R \geq 2500$

Gravity: Froude Number = $N_F = V/\sqrt{(gD)}$ where $\sqrt{(gD)}$ = c = Celerity (surface gravity waves) speed and V is the average water velocity. (D: mean depth)

Critical Flow	*Subcritical*	*Supercritical*
$N_F = 1$ and $V = \sqrt{(gD)}$	$N_F < 1$ and $V < \sqrt{(gD)}$	$N_F > 1$ and $V > \sqrt{(gD)}$

GVF (Gradually Varied Flow) $\left|\frac{dy}{dx}\right|$ small < $\frac{1}{20}$





Lecture 2
Basic Principles of Hydraulic Engineering

The following are the fundamental principles of hydraulic engineering:

22380. Conservation of Mass (also called Continuity);
22381. Conservation of Energy;
22382. Conservation of Linear Momentum;
22383. Conservation of Angular Momentum;
22384. Thermodynamics (internal and friction losses of mechanical energy);
22385. Dimensional analysis;
22386. Hydrostatics (special case of momentum and/or energy conservation).

Concept of Flow and Continuity

Flow is the amount of a species (scalar or vector quantity) passing through an area per unit time. Types of flow used in hydraulics include:

volume
mass
mechanical energy
momentum
heat.

Volume flow = discharge = $Q = \int_A \underline{v} \, .d\underline{A} = V A$ **1.1**

Mass flow = $m = = \int_A dm' = \int_A \rho \, \underline{v} \, .d\underline{A} = \rho V A$ **1.2**

Flow of kinetic energy = $KE = = \frac{1}{2} \int_A v^2 \, dm' = \frac{1}{2} \int_A \rho v^2 \, \underline{v} \, .d\underline{A} = \frac{1}{2} \alpha \rho V^3 A$ **1.3**

α **= Kinetic energy correction factor =** $\int_A v^3 dA \Big/ (V^3 A) \approx \Sigma v_i^{\,3} A_i / [V^3 A]$ **1.4**

Flow of momentum = $\underline{M} = = \int_A \underline{v} \, dm' = \int_A \rho \underline{v} \;\; \underline{v} \, .d\underline{A} \rightarrow \beta \rho V^2 A$ --- *{Scalar approximation}* *1.5*

β **= Momentum correction factor =** $\int_A v^2 dA \Big/ (V^2 A) \approx \Sigma v_i^{\,2} A_i / [V^2 A]$ **1.6**

Definitions:

System – ***Fixed set of particles.***

Control volume – ***Fixed volume in space.***

Conservation of Mass (Continuity)

General Statement:

{Rate of change of Mass in a control volume c.v.} = Mass flow into the c.v – Mass flow out of c.v

$$\int_{c.v} \frac{\partial \rho}{\partial t} dV_{ol} + \oint_{c.s} \rho \underline{v} \bullet d\underline{A} = 0 \qquad 1.7$$

Special case:

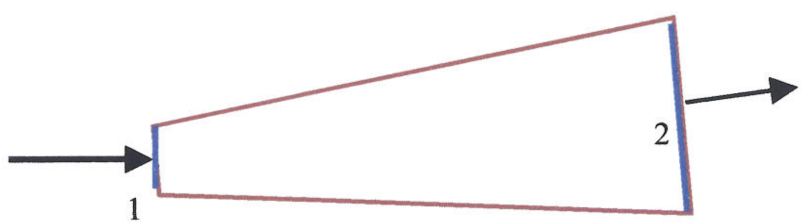
ρ = constant and steady flow:

$$\oint_{c.s} \rho \underline{v} \bullet d\underline{A} = 0$$

Therefore for a inflow/outflow c.v. we get

$\rho V_1 A_1 = \rho V_2 A_2 = \rho Q_1 = \rho Q_2$

or $V_1 A_1 = V_2 A_2 = Q = \text{constant}$ **1.8**



Lecture 3
Energy Principle for Steady Flow

General Statement for a Control Volume:

{Rate of change of Energy in a control volume c.v.} = Flow of Energy into the c.v – Flow of Energy out of c.v

$$\int_{c.v} \frac{\partial I}{\partial t} \gamma dV_{ol} + \oint_{c.s} \gamma H \underline{v} \bullet d\underline{A} + P_{out'} - P_{in'} + Heat_{out'} = 0 \qquad (3.1)$$

I = internal energy per unit weight.
H = energy per unit weight.
P_{out}' = Rate of Mechanical Energy extracted by a turbine
P_{in}' = Rate of Mechanical Energy input by a pump
$Heat_{out}$' = Net Rate Heat Energy lost by conduction and radiation through the control surface.

Special case:
Assumptions: 1) steady state flow
2) incompressible flow
3) hydrostatic pressure
4) no chemical or nuclear reactions.

Concept of energy head

Energy head is the energy per unit weight of water flowing.

At a point:

Mechanical energy head = $h_T = p/\gamma + h_z + v^2/2g$

Cross-sectional average:

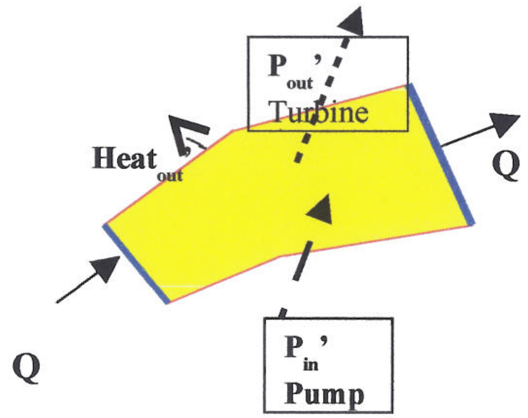
Mechanical energy head = $H_T = p/\gamma + h_z + \alpha V^2/2g$

Other forms of energy:

Heat energy per unit weight = h_{SH}

Turbulence kinetic energy = h_{Tu}

Construction an Energy Balance for a Conduit for the Special Case
Steady: $Q_1 = Q_2$ ***&*** $\partial I/\partial t = 0$***; incompressible flow:*** $\rho = \rho_1 = \rho_2 = constant$.



Defining Sketch

Word statement:-

Rate of change of energy in c.v. = Energy inflow - Energy outflow = 0

or ***Energy inflow = Energy outflow*** ******* Note the prime indicates a rate.***

$\gamma Q \{ H_{T1} + h_{SH1} + h_{Tu1} \} + P_{in}' = \gamma Q \{ H_{T2} + h_{SH2} + h_{Tu2} \} + P_{out}' + Heat_{out}'$

Divide by γQ

$\{ H_{T1} + h_{SH1} + h_{Tu1} \} + P_{in}' / \gamma Q = \{ H_{T2} + h_{SH2} + h_{Tu2} \} + P_{out}'/ \gamma Q + Heat_{out}'/ \gamma Q$

Now $P_{in}' / \gamma Q = H_p$ = **Pump Head and** $P_{out}' / \gamma Q = H_{tb}$ =**Turbine Head (extracted)**

Collect all terms related to non-mechanical energy and define as h_L

where $h_L \equiv [\{ h_{SH2} + h_{Tu2} \} - \{h_{SH1} + h_{Tu1} \} + Heat_{out}'/ \gamma Q]$

Now $H_{T1} + H_p = H_{T2} + H_{tb} + h_L$ *_(3.2)*

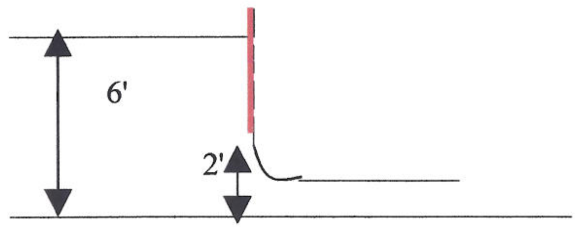
The Claussius Theorem in Thermodynamic requires that $h_L > 0$; i.e. in real fluids there must always be a lost of energy in the direction of the flow. Therefore,

$p_1/\gamma + h_{z1} + \alpha_1 V_1^2/2g + H_p = p_2/\gamma + h_{z2} + \alpha_2 V_2^2/2g + H_{tb} + h_{L\,1\text{-}2}$ ***_(3.3 pipe)***

$y_1 + h_{z1} + \alpha_1 V_1^2/2g = y_2 + h_{z2} + \alpha_2 V_2^2/2g + h_{L\,1\text{-}2}$ ***_(3.4 open channel)***

Tutorial Assignment 1.

1. Using energy and continuity principles estimate the flow under the sluice gate shown in the sketch below. Assume: no head loss, $\alpha = \beta = 1$, W = 6 ft, Gate opening = 2 ft; y_o = 6 ft and Cc = 0.61.



Assumptions:

$\checkmark$ $Q = 3.64(36) = 131.2$

$Q = 17.91(6)(1.22) = 131.10$

Equations:

Continuity:

Energy:

$V_o = \frac{1.22(17.91)}{6} = 3.64$

Solution:

$307.8 = V_1^2 - 0.0413 V_1^2$

$307.8 = (1 - 0.0413) V_1^2$

$321.1 = V_1^2$

$17.91 = V_1$

Q = ______________

$(1.22 V_1)(1.22 V_1)$ $\quad V_0 = \frac{1.22(V_1)}{6}$

~~386.4~~

$307.8 + \frac{1.488 V_1^2}{36}$ $\quad (2g)6 + \frac{\left(\frac{1.22 V_1}{6}\right)^2}{~~2g~~} - 1.22(2g) = V_1^2$

$307.8 + 0.0413 V_1^2$

Donald Jerolleman, Jr., S.M. ASCE

"Continuity" $Q = VA$

Lecture 4
Momentum Principle for Steady Flow

There are two momentum principles:
1. "In a system with no external forces, the linear momentum is conserved."
2. "In a system with no external moments, the angular momentum is conserved."

Linear Momentum

General Equation

steady flow : $\frac{d()}{dt} = \emptyset$

System: $\Sigma \underline{\mathbf{F}}_{ext} = \mathbf{m}\, \underline{\mathbf{a}}_{system}$ **(4.1)**

Control volume form:

acceleration of centroid

$$\sum \underline{\mathrm{F}}_{\mathrm{ext}} = \int_{c.v} \rho \frac{\partial v}{\partial t} dV_{ol} + \oint_{c.s} \rho \underline{v}\underline{v} \bullet d\underline{A} \qquad (4.2)$$

temporal acceleration — spacial acceleration

$$\sum \underline{\mathrm{F}}_{\mathrm{ext}} = \int_{c.v} \rho \frac{\partial v}{\partial t} dV_{ol} + \Delta\{\rho Q \underline{V})_{cs}$$

cancels out for S.S.

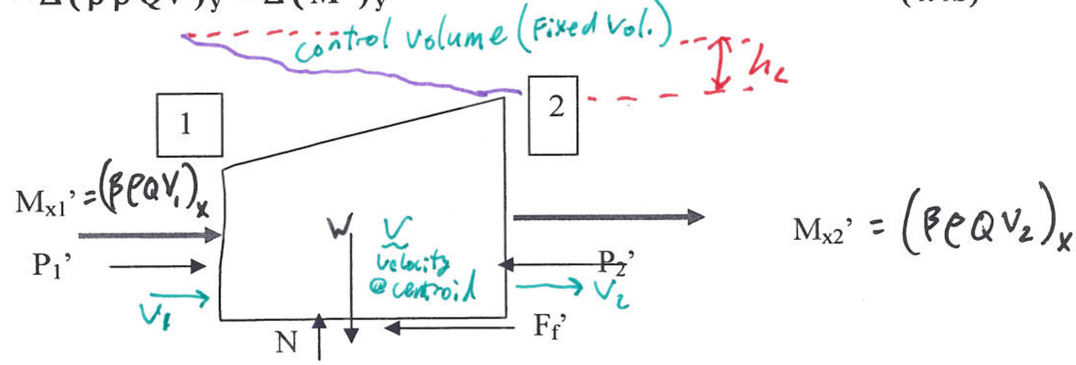
Simplified Equation for Steady State

$\Sigma \underline{\mathbf{F}}_{ext} = \Delta(\beta \rho Q \underline{\mathbf{V}}) = \Delta(\underline{\mathbf{M'}})$ **(4.3)**

Component Equations

$\Sigma \mathbf{F}_x = \Delta(\beta \rho Q V)_x = \Delta(\mathbf{M'})_x$ **(4.4a)**

$\Sigma \mathbf{F}_y = \Delta(\beta \rho Q V)_y = \Delta(\mathbf{M'})_y$ **(4.4b)**



w = width

Simplified Control Volume showing Component Forces and Momentum Flow

$\overset{+}{\rightarrow} \Sigma F_x = P_1 - P_2 - F_f = \Delta(M')_x = M_2' - M_1' = \beta_2 \rho Q V_{2x} - \beta_1 \rho Q V_{1x} = \frac{1}{2}\gamma y_1^2 w_1 - \frac{1}{2}\gamma y_2^2 w_2 - F_f$

① ② $Q = V_1 A_1 = V_2 A_2$ } use to solve unknowns.

to get head loss

$h_L = H_{T_1} - H_{T_2}$ (energy equ)

Apply Mom. in y-direction

Force balance

$$\Sigma F_y = N - W = \emptyset \rightarrow N = W$$

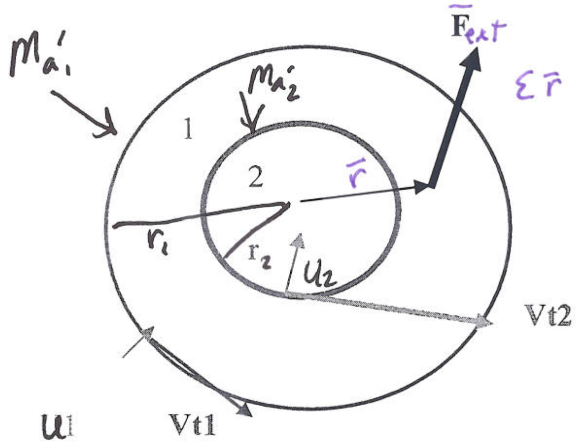
Lecture 5
Angular Momentum Principle for Steady Flow

An external moment applied to a body will increase its annular velocity. If no external moment is applied the angular moment of the body will remain constant. This is also true of fluid particles, e.g.

The angular momentum flux of a mass flow m' is

$$M_a' = m'(\underline{r} \times \underline{v}) \qquad (5.1)$$

Therefore for the c.v. shown below, we have



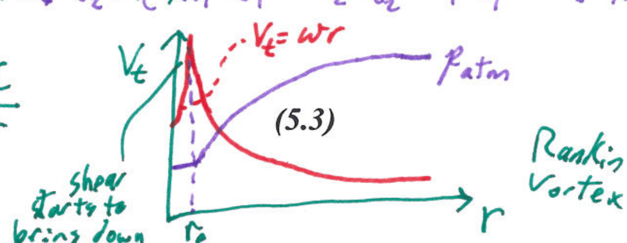
External Moment $= \Sigma(\underline{r} \times \underline{F}) = \Delta(m'\underline{r} \times \underline{v}) \rightarrow \Delta(\rho Q r V_t) = \Delta(M_a')$ **(5.2)**

Scalar

If the external moment ≅ 0

Then $m'(\underline{r} \times \underline{v}) = constant$

If m' = constant then $r V_t = C = constant$ **(5.3)**



$$r_1 V_{t1} = r_2 V_{t2}$$

where r = the perpendicular distance to the tangential velocity V_t. This is the free vortex equation.

Sep 14

Lecture 6
Application of Hydrostatics

Principles of Hydrostatics

1. **Hydrostatic pressure acts equally in all directions.**
2. **Hydrostatic pressure varies directly with the depth below the free surface.**
3. **The pressure acts normal to the boundary.**
4. **The pressure force is equal to the liquid specific weight times the volume under the pressure loading curve.**
5. **The pressure force acts at the centroid of the pressure loading curve.**

The equations of Hydrostatics are derived from a special case of the momentum and energy principles in which there is no loss of energy and no acceleration of fluid particles, i.e.,

$$p/\gamma + h_z = \text{Constant} \quad \text{.................... 6.1}$$

and

$$\Sigma F_{ext} = 0 \quad \text{.......................... 6.2}$$

$$\Sigma (r \times F)_{ext} = 0 \quad \text{............................ 6.3}$$

Also absolute and gage pressure are related by:

$$p_{abs} = p_{gage} + p_{atm} \quad \text{.............................. 6.4}$$

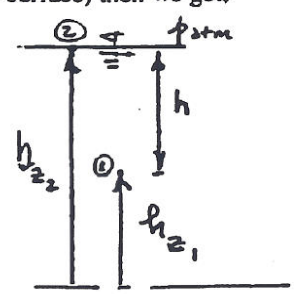
If Eq. 6.1 is applied in a reservoir at elevation h_{z1} and h_{z2} (at the surface) then we get,

$$p_{1abs}/\gamma + h_{z1} = p_{atm}/\gamma + h_{z2}$$

$$(p_{1gage} + p_{atm})/\gamma + h_{z1} = p_{atm}/\gamma + h_{z2}$$

or $\quad p_{1gage}/\gamma = (h_{z2} - h_{z1})$

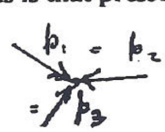
or $\quad p_{1gage} = \gamma (h_{z2} - h_{z1}) = \gamma h \quad$ 6.5



where h is the depth below the surface. It is customary to drop the subscript "gage" and use only p for gage pressure, giving

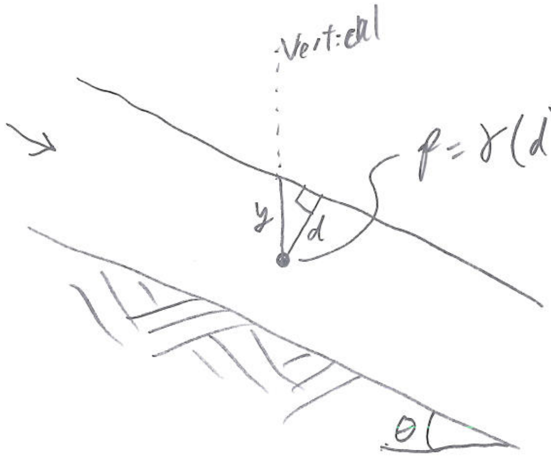
$$p = \gamma h \quad \text{...................................} \textit{6.5}$$

Another important concept in hydrostatics is that pressure acts equally in all directions at any point in the water.



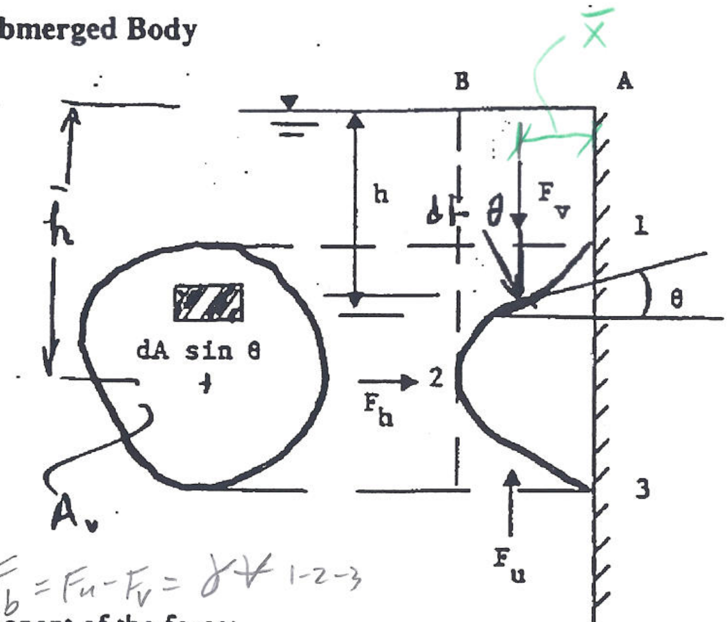
Also pressure acts normal to a wall or solid surface in the water.

$P_{wall} = \gamma h \perp_{(\text{to wall})}$



$p = \gamma(d)(\cos\theta)$ (or) $= \gamma y \cos^2\theta$

Hydrostatic Forces on a Submerged Body



$F_V = \gamma \forall$ of water above it $[1\text{-}2\text{-}B\text{-}A]$ acting @ centroid ($\bar{x}$)

$F_u = \gamma \forall [3\text{-}2\text{-}B\text{-}A)$ @ centroid of 3-2-B-A

$F_b = F_u - F_V = \gamma \forall$ 1-2-3

Let F_h = horizontal component of the force;
F_v = vertical downward component of the force;
F_h = vertical upward component of the force;
F_b = net vertical force or buoyant force.

y_p (center of pressure) $= \bar{y} + \frac{I_{NA}}{A_v \bar{y}}$ (moment of Inertia)

The pressure force on an area dA is

$$dF = p\, dA = \gamma\, h\, dA$$

For the horizontal component we take the projection of dA on a vertical plane, so

$dA_v = dA \sin\theta$ and

$$F_h = {}_{Av3\text{-}1}\int \gamma\, h\, dA \sin\theta = \gamma\, \hat{h}\, A_v \qquad \text{.................. 6.6}$$

(aka $\bar{y}$) centroid of area, normal to it, but does not act @ centroid; it acts at center of pressure (y_p)

Similarly

$$F_v = {}_{Ah1\text{-}2}\int \gamma\, h\, dA \cos\theta = \gamma\, h\, A_{h1\text{-}2} = \gamma\, \forall_{1\text{-}2} = \gamma\{\textit{Volume above } A_{h1\text{-}2}\} \text{ ...... 6.7}$$

$$F_u = {}_{Ah3\text{-}2}\int \gamma\, h\, dA \cos\theta = \gamma\, h\, A_{h3\text{-}2} = \gamma\, \forall_{2\text{-}3} = \gamma\{\textit{Volume above } A_{h\,3\text{-}2}\} \text{ ...... 6.8}$$

$$F_b = F_u - F_v = \gamma\,(\forall_{1\text{-}2} - \forall_{3\text{-}2}) = \gamma\{\textit{Volume of body}\} \qquad \text{..........6.9}$$

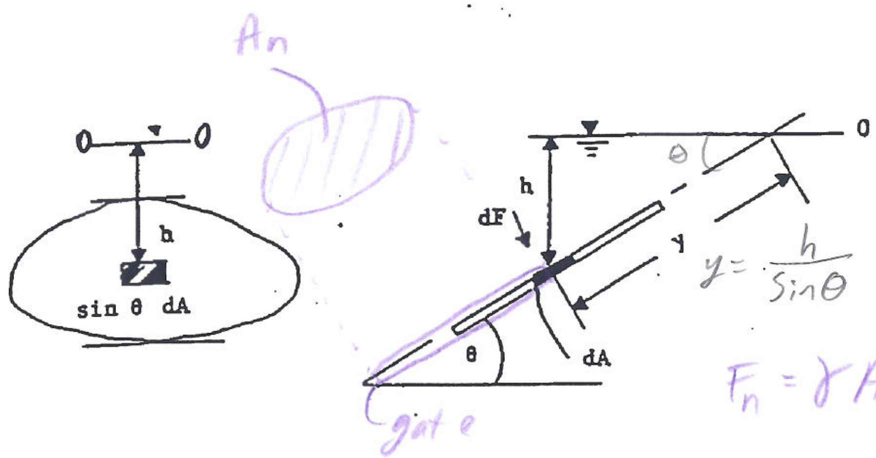
ε (ecentricity) $= \frac{I}{A_n \bar{y}}$

$M = \varepsilon F_n$

Hydrostatic Forces and Moments on Plane Surfaces

Consider the inclined flat submerged surface shown below. We want to find the hydrostatic force and its effective location (y_p = centre of pressure).

$p = \gamma h = \gamma (y)(\sin\theta)$



$y = \frac{h}{\sin\theta}$

normal area of gate

$F_n = \gamma A_n \bar{y} \sin\theta$ ← centroid of gate

$y_{p_{gate}} = \bar{y} + \frac{I}{A_n \bar{y}}$

From Eq. 6.5 we have

$$dF = p\, dA = \gamma\, h\, dA$$

and $F = {}_A\int \gamma\, y\, dA \sin\theta = \gamma \hat{h} A = \gamma \hat{y}\, dA \sin\theta$ 6.10

where $\hat{y}$ = centroid of area A from origin O and $\hat{h}$ = centroid of area A_v.

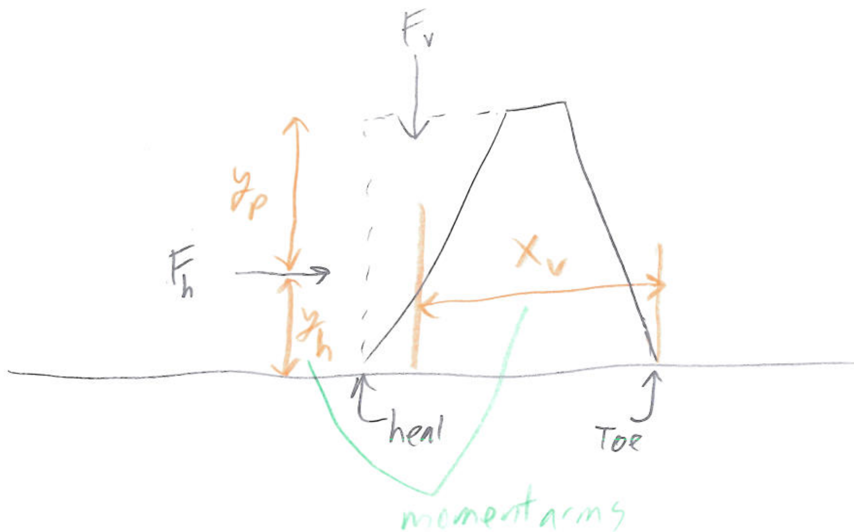
The moment about O due to the distributed force and the concentrated force F should be equal,

$${}_A\int y\, dF = {}_A\int \gamma\, y^2\, dA \sin\theta = y_p F$$

Therefore,

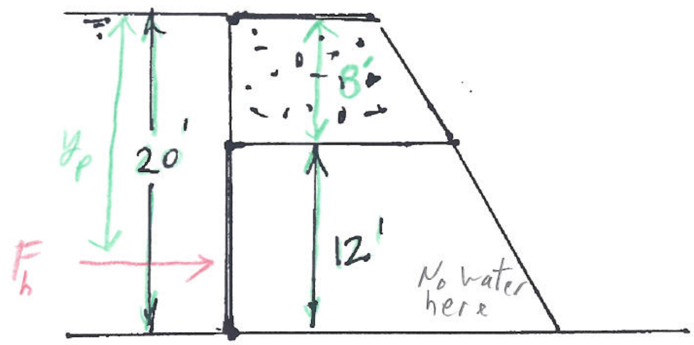
$$y_p = \{ {}_A\int \gamma\, y^2\, dA \sin\theta \}/F = \{ {}_A\int \gamma\, y^2\, dA \sin\theta \}/\{ {}_A\int \gamma\, y\, dA \sin\theta \}$$

$$= \{ {}_A\int y^2\, dA \}/\{ {}_A\int y\, dA \} = I_o/\{\hat{y}A\} = \hat{y} + I_{NA}/\{\hat{y}A\}$$ 6.11



F_v
y_p
F_h
x_v
y_h
heal
Toe
momentarms

Problem: Find the hydrostatic force and the center of pressure on the sluice gate shown below.

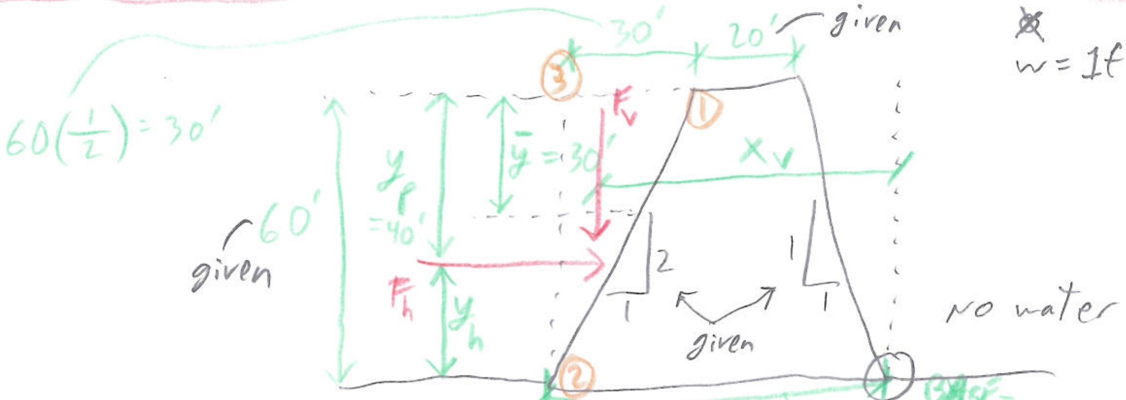


W = 16 ft

$F_h = \gamma A_v \bar{y}$; $A_v = 12'(16') =$ ____ ; $\bar{y} = 8' + 6' = 14'$

$F_h = 167.7$ kips

$$y_p = \bar{y} + \frac{I}{A_v \bar{y}} = 14' + \frac{12(b)(h)^3}{12(16)(14)} = 14.86 \text{ ft}$$



BASE = 30' + 20' + (½(60)) = 110'

$$F_h = \gamma A_v \bar{y} = 62.4\left[1ft(60ft)\right]\left[\frac{60ft}{2}\right] = 112.3 \text{ kips}$$

$$y_p = \bar{y} + \frac{I}{A_y \bar{y}} = 30' + \frac{\frac{1}{12}(1ft)(60ft)^3}{(60ft)(1ft)(\frac{60ft}{2})} = 40 \text{ ft}$$

$$F_v = \gamma V_{1-2-3} = 62.4\left[\frac{1}{2}(60)(30)(1)\right] = 56.16 \text{ kips}$$

X_v = Base − distance before where force acts $= 110' - [30'(\frac{1}{3})] = 100'$

$$\circlearrowleft + \sum M_{TOE} = F_v(X_v) - F_h(y_h) = 56.16^k(100') - 112.3^k(20') = 3370 \text{ k-ft}$$

– slip circle failure (shear failure) * hoover is an arch/gravity dam
– piping failure

Lecture 6 (continued)

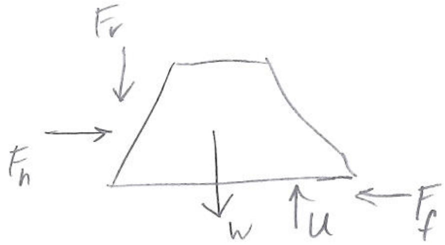
Reading: Handouts & Design of Small Dams & US Corps of Engineers Manuals.

Stability of Dams - Gravity Dams

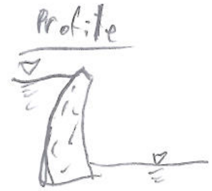
Common Types of Dams

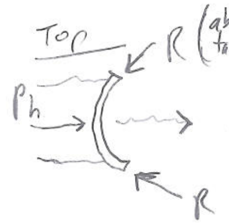
Type ***Mechanism of stability***

- **Gravity Dam** uses its weight for stability to prevent sliding & overturning



- **Arch Dam**





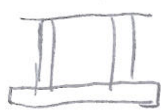
good as long as prevent tention in concrete

☆☆ [Thus very important abutment remain stable]

Transfer of Applied forces thru compression along the arch to abutments

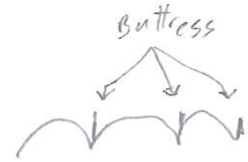
- **Buttress Dam**

usually not heigh (uses slab



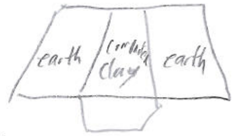
benefit (uses less concrete)

Arched-Buttress

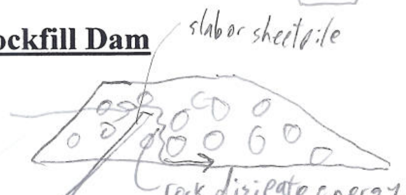


- **Earth Dam**

TYPICALLY: TRAPAZOID

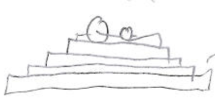


- **Rockfill Dam**



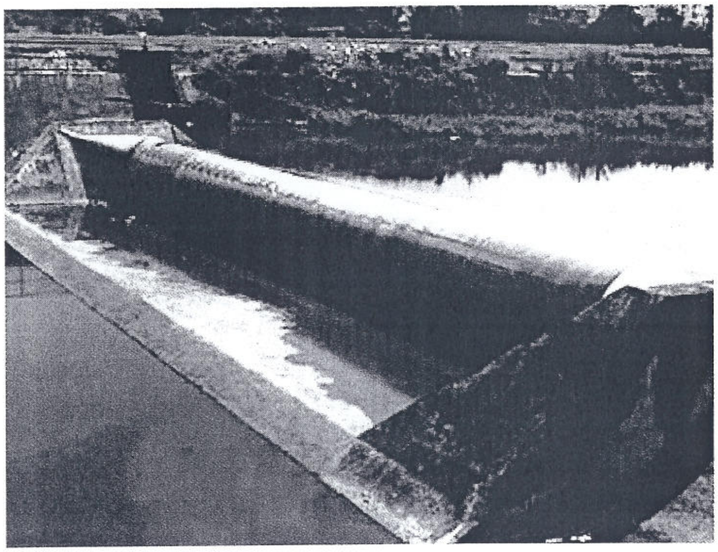
- **Roller Compacted Concrete**

Helps control heat disipation so can be constructed quiker

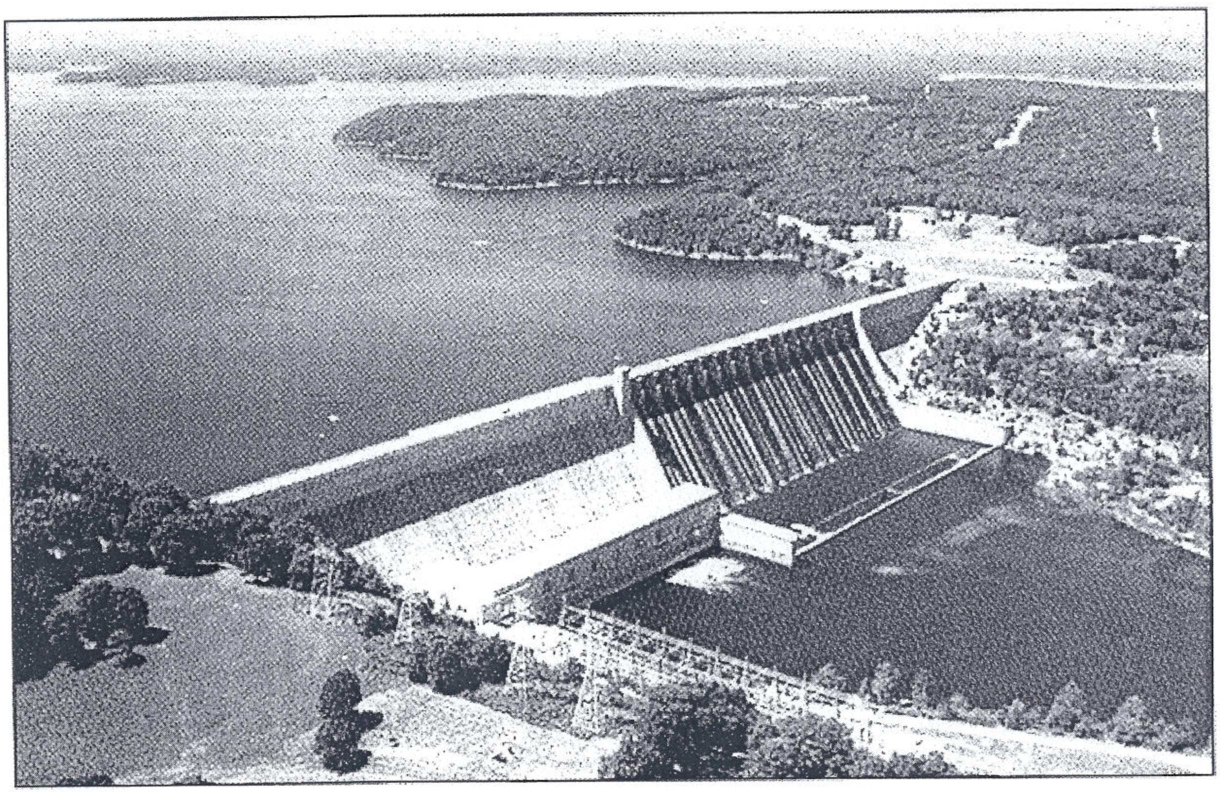


– steping disipates excess energy if there is over flow

– use low slup in thin layers

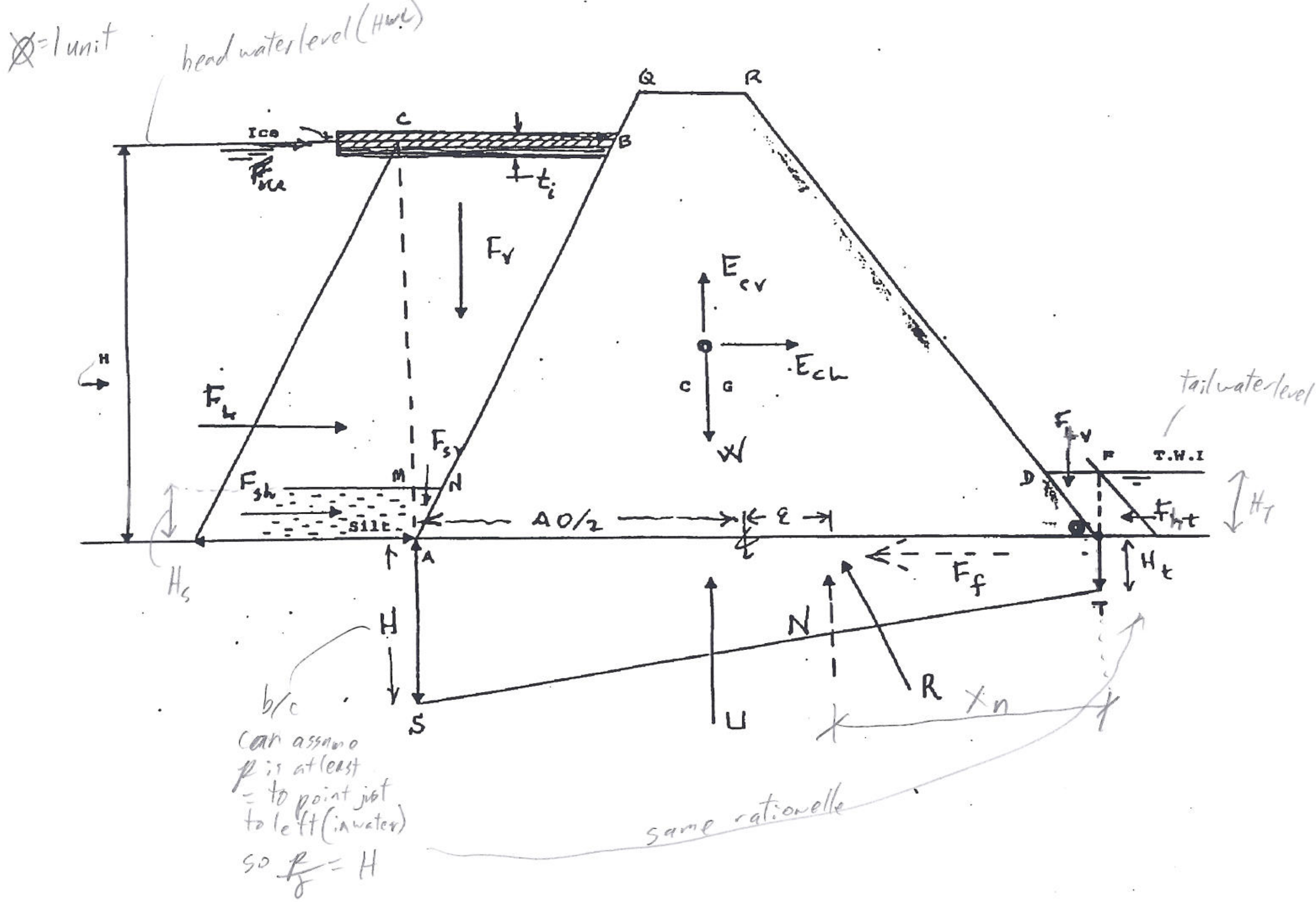


Rubber Dam



Bull Shoals, Little Rock District

Gravity Dam



Free Body Diagram of a Vertical Slice of a Gravity Dam

m is correction for voids

$G_s - 1 \Rightarrow$ the 1 is G_s of water — this is a gamma correction

$\gamma_c \sim 150\ lb/ft^3$ $\qquad \tau_{ice} = 5000\ lb/ft^2$

Forces on Gravity Dams ***See attached Free Body Diagram***

Force Type	Force	Direction	Magnitude	Point of application
Hydraulic	F_h	→	$\frac{1}{2}\gamma H^2$	$y_h = \frac{H}{3}$
	F_v	↓	γA_{ABC} (1[ø])	@ centroid of ABC
	F_{th}	←	$\frac{1}{2}\gamma H_t^2$	@ $H_t/3$ * Very Small
	F_{tv}	↓	γA_{DFO} (1[ø])	Very small
Seepage	U	↑	γA_{OTS}	X_u is to centroid of AOTS
Earthquake on water	E_w	→ / ← - - -	$\frac{5}{9}\gamma H^2 \frac{a}{g}$	$y_w = \frac{4}{3\pi} H$ above A_0
Ice	F_I	→	$t_{ice}(\tau_{ice})$ (← thickness of ice; ← crushing strength of ice)	@ $y_I \sim H$
Silt	F_{sh}	→	$\frac{1}{2}\gamma H_s^2 (G_{s_{silt}} - 1)(1-m)$	@ $H_3/3$ of Anm
	F_{sv}	↓	$\gamma A_{Amn}(G_{s_{silt}} - 1)(1-m)$	@ centroid
Waves	F_w	→	See (SPM) shore protection manual	near $y = H$
Concrete (stabilizing) (weight)	W	↓	$\gamma_c A_{AORQ}$ (1[ø])	@ centroid of AORQ
Earthquake on concrete	$E_{c_{horztl}}$	→ / ← - - -	$W\frac{a}{g}$	@ $\bar{y}$ of concrete
	E_{c_v}	↑↓	$W\frac{a}{g}$	@ $\bar{x}$ " "
Foundation	R	↖	$\sqrt{F_f^2 + N^2}$ (Friction force, Normal force)	@ X_n
Shear & Friction	F_f		$\Sigma F_x'$ (sum of all x-forces, except F_f); $F_f = \min(\Sigma F_x', \mu N)$	
Normal	N		$\Sigma F_y'$ (sum of all y-forces, except for N)	@ $X_n = \frac{G\,\Sigma M_O}{N}$

other forces: shrinkage stress, temp change

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Seepage Forces

Approximate Solution of Seepage Force on a Dam

Problem: Consider the dam shown below. $K = 10^{-8}$ ft/sec.

Find the uplift on the base after decades of operation.

The approximate method assumes that the loss of piezometric head varies linearly with the seepage path (s), i.e.

$$\phi = \phi_1 + (\phi_2 - \phi_1)s/S_B$$

$\phi_1 = H_1 + h_{z_1} = 106' + 100' = 206'$

$\phi_2 = H_2 + h_{z_2} = 100 + 10 = 110'$

$S_B = 20' + 50' + 50' + 105' = 225'$

$\Delta\phi = (-96')$

where s = distance along the seepage path starting at the u/s end; ϕ_1 = upstream piezometric head; ϕ_2 = downstream piezometric head; S_B = total length of seepage path.

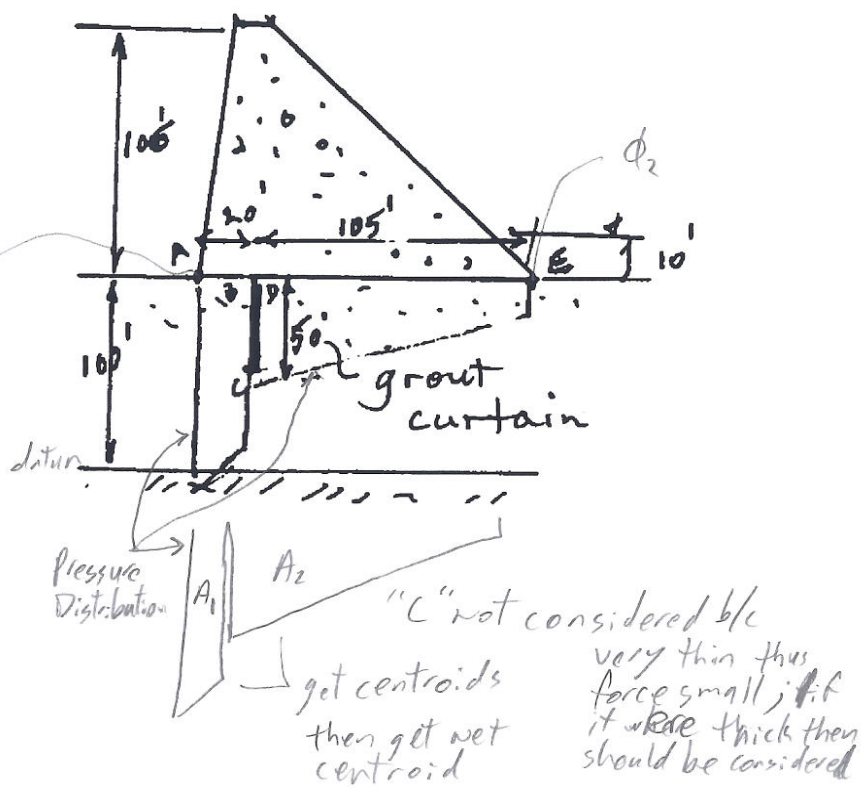
The pressure head at any location (s) is found from

$$p/\gamma = \phi - h_z$$

Pt	h_z(ft)	s	ϕ	p/γ
A	100	Ø	$=\phi_1=$ 206'	106
B	100	20'	197.5	97.5
C	50	20+50 70'	176.1	126.5
D	100	20+50+50 120'	154.8	54.8
E	100	225'	110	10

$\phi_B = 206 + \frac{20}{225}(-96) = 197.5$

$\phi_C =$



$U = \gamma(A_1 + A_2)$

$q = -\frac{\Delta\phi}{S_B}k$

Earthquake Forces

For preliminary analysis we can take the earthquake force on the mass of concrete as

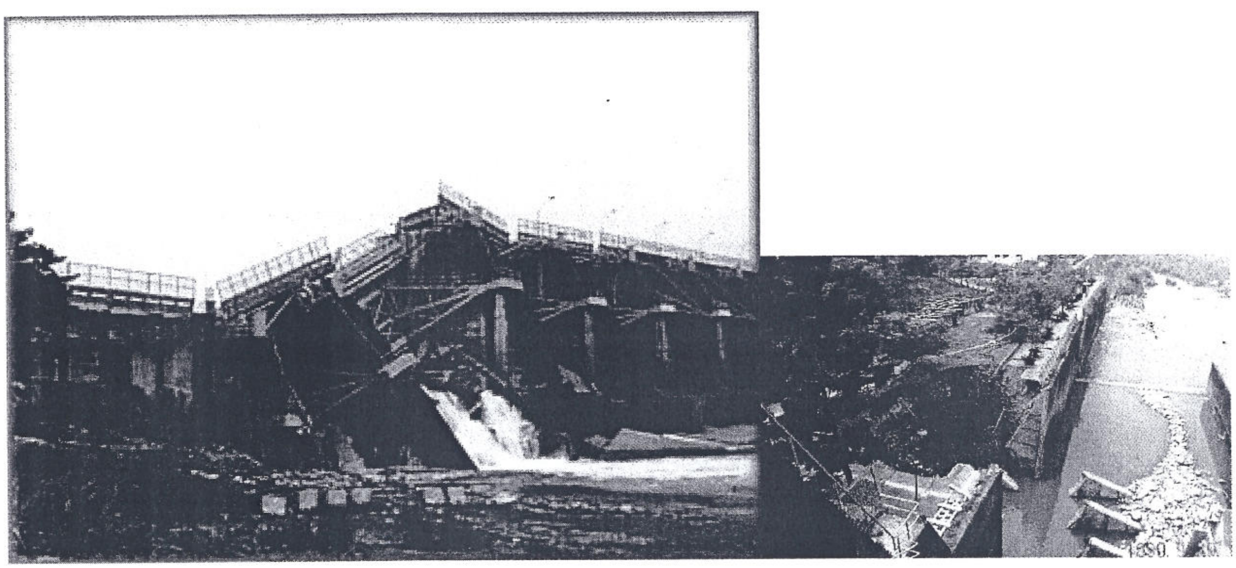
$$E_c = W\, a/g$$

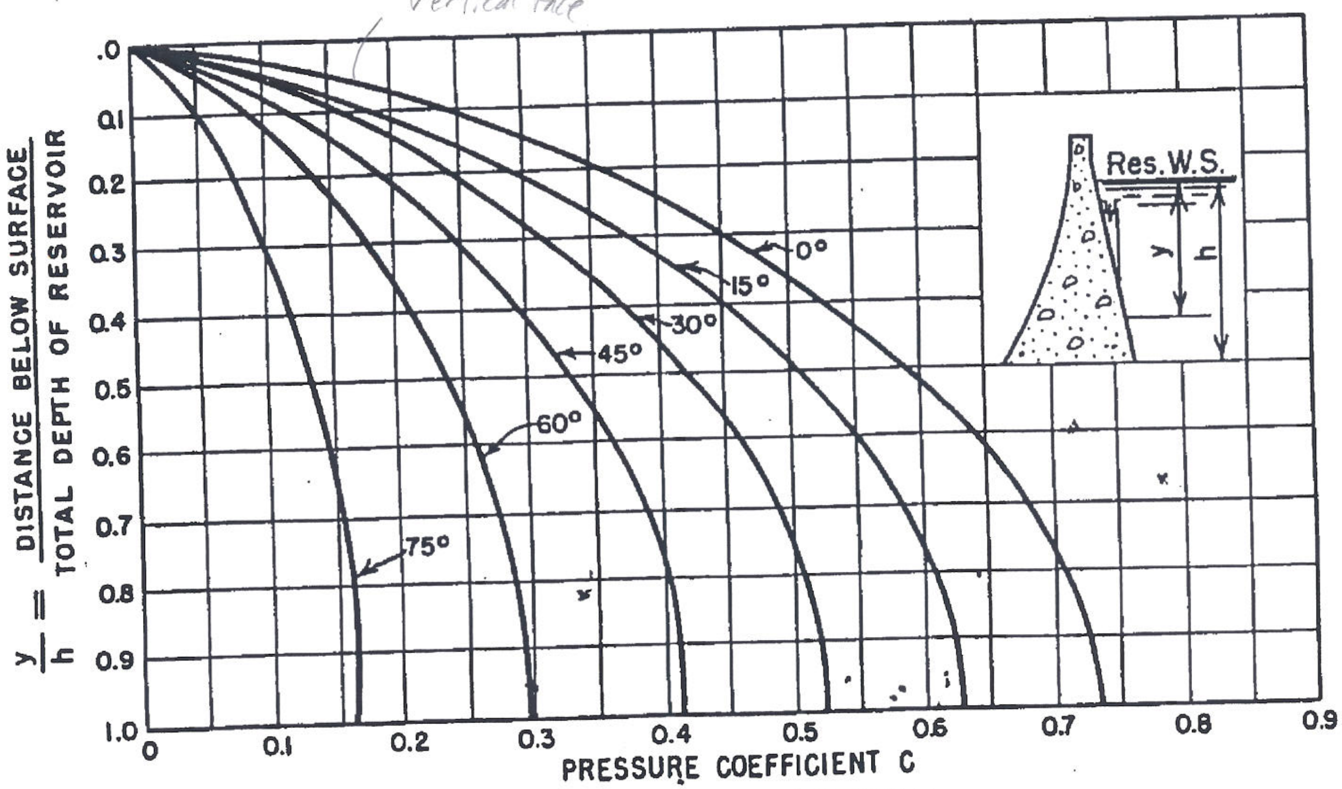
Where W= weight of the dam; a = the acceleration due to the earthquake and g is the acceleration due to gravity. The earthquake acceleration can be 3-dimensional.

In addition there can be a wave-like force induced in the water behind the dam; the attached Figure "8.6" shows the pressure distribution due to the earthquake on the water (***h*** is the height of the water and ***y*** is measured from the water surface). For a vertical face, this force is given approximately by

$E_w \sim 5/9\ \gamma H^2\ a/g$ and acts at $4H/(3\pi)$ from the base where H is the depth of water.

The photographs below show earthquake damage to dams in Taiwan.





$$C=\frac{1}{2}C_m\left[\frac{y}{h}\left(2-\frac{y}{h}\right)+\sqrt{\frac{y}{h}\left(2-\frac{y}{h}\right)}\right]$$

Figure 8-6.—Coefficients for pressure distribution for constant sloping faces. 288–D–2509.

$p = C\gamma y$

Stability Criteria

1. **Safety Factor Against Overturning**

2. **Safety Factor Against Sliding**

3. **No tension in the foundation/concrete**

4. **Other considerations:**

NO TENSION ANYWHERE

Design Criteria

1. Criteria for Overturning:

Definition:

$$F_{os\ ot} = \{\text{Uprighting moments}\}/\{\text{Overturning moments}\}$$

The table below for suggested factors of safety for different loading combinations.

2. Criteria for Sliding

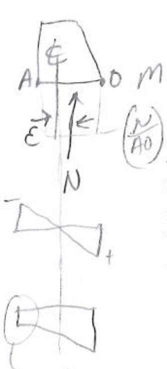
Definition:

$$F_{os\ sl} = \{\text{Resisting Forces in Horizontal plane}\}/\{\text{Downstream Forces}\}$$

The table below for suggested factors of safety for different loading combinations.

Table of Typical Factors of Safety against Overturning and Sliding

Mode	Normal	Unusual	Extreme
Overturning	>1.5 >2 to 3 for some structures	>1.25 >1.5 some structures	>1
Sliding	Depends on adverse bed angle! Horizontal case: > 1.5 earth foundation >1.5 with shear for rock >1 neglecting shear for rock	Depends on adverse bed angle! Horizontal case: >1.25 earth foundation >1.25 with shear for rock >1 neglecting shear for rock	Depends on adverse bed angle! Horizontal case: >1 neglecting shear for rock Earth foundations in earthquake zones need special attention.



3. Foundation Criteria:

a) The most import foundation criteria is that there should be no tension in the foundation under all loading conditions.

This can be achieved by designing the dam so that the foundation reaction always falls in the kern or middle third of the base of the dam.

b) The second criterion for the foundation is that the compressive stress does not exceed the bearing strength of the foundation, e.g. rock crushing strength of approximately 1000 psi is typical. A safety factor of 2 is often applied; thus reducing the actual estimated strength by 50% to obtain the allowable bearing strength.

c) The third criterion is that the shear in the concrete or Rock should not exceed the allowable shear strength. A safety factor of 2 is often applied; thus reducing the actual estimated shear strength by 50% to obtain the allowable shear strength.

d) Excessive seepage that can lead to a piping failure. This is particularly important for structures built on gravel, sand or silt. A limit is often placed on the maximum piezometric gradient (H/B < 1/7 to 1/8 for silts and sands) or the maximum Reynolds number at the exit (e.g. for sand $Re = V_{Darcy} \times D_{50}/\nu < 1$).

middle third rule — "Kern rule"

4. Concrete Stresses.

a) There should be no tension in the concrete.

b) The compressive strength of he concrete should not be exceeded. (Factor of Safety ~ 2). Typical 3000 psi concrete.

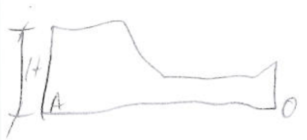
c) The shear strength of the concrete should not be exceeded. (Factor of Safety ~ 2). Typical shear strength is 250 psi.

5. Over topping.

The dam and spillway should design to pass the Probable Maximum Flood (PMF) without overtopping. This criterion is satisfied on the hydraulic design of the Spillway.

6. Stilling Basin.

The stilling basin must dissipate or deflect the high kinetic energy at the toe of the spillway so that the structure is not undermined by excessive scour.



Tutorial No. 4

1. Evaluate the safety of Shasta Dam section shown on the attached Figure 1.

Assume:

- **neglect ice**
- **neglect waves**
- **neglect silt**
- **full uplift**
- **earthquake a/g ~ 0.15**
- **maximum friction factor at base, $\mu_s = 0.65$**
- **Ss = 2.40 for concrete**
- **SF against sliding $\geq$ 1.5**
- **SF against overturning $\geq$ 2.0**
- **no tension in the base**
- **Sound greenstone foundation rock (1500 psi).**
- **Concrete strength (3000 psi)**
- **linear seepage uplift variation from heel to toe of dam.**

Use simple shapes to approximate the concrete section.

<u>Calculation of Reactions</u>

For static equilibrium of the vertical slice, we have:

Normal Load Condition (excluding Silt, Ice, Earthquake etc)

$$\Sigma F_x = 0 : -F_f - P_{TW} + P_h = 0$$

$$\mathbf{F_{f\ reqired} = P_h - P_{TW}}$$

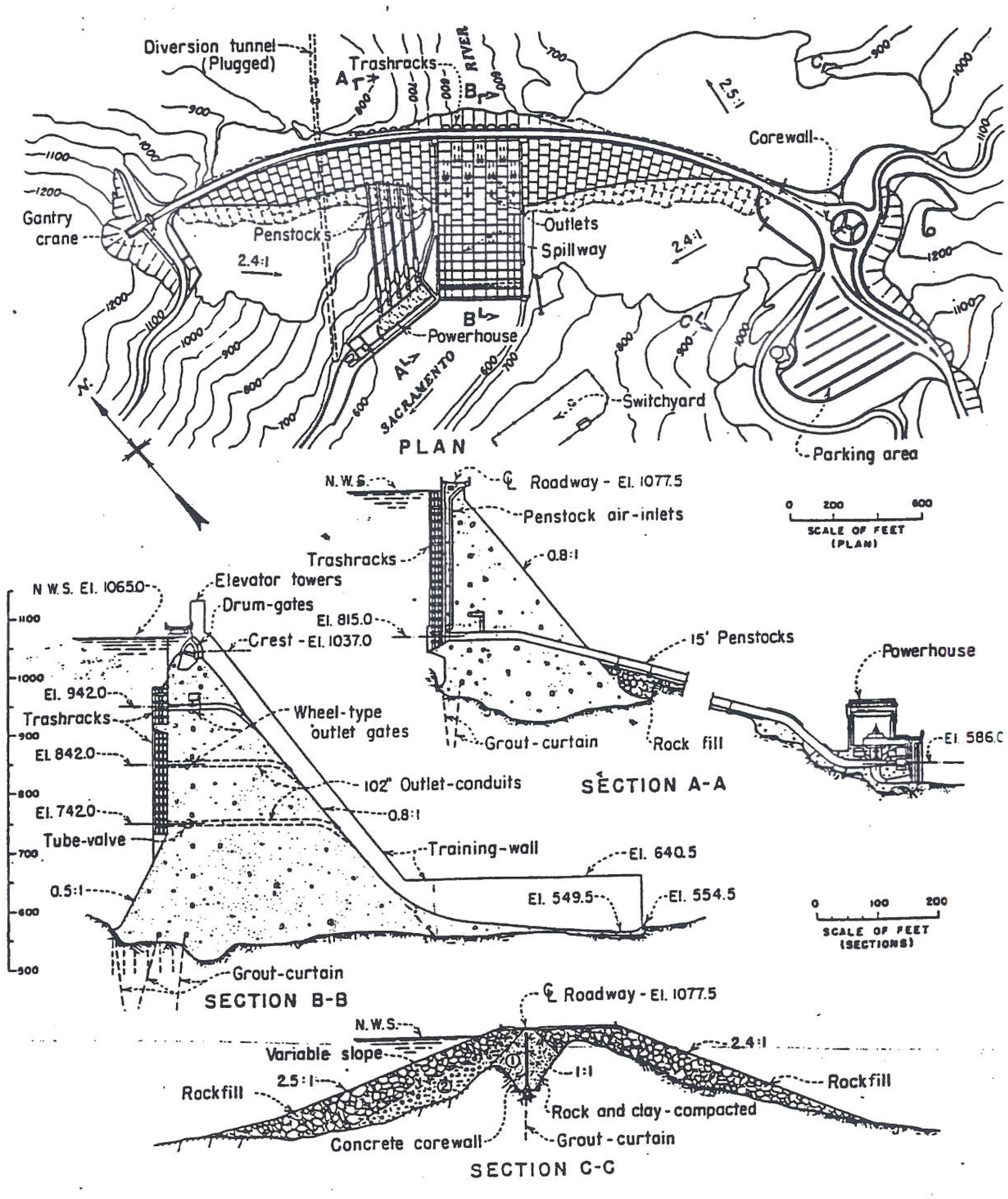
$$\textbf{Maximum Friction } \mathbf{F_{f\ available} = N\, \mu_s}$$

$$\Sigma F_y = 0 : \ N + U - W - P_v = 0$$

$$\mathbf{N = W - U + P_v}$$

$$\Sigma M_o = 0: \ - x_R\, N - H/3\, (P_h) - x_u\, U + x_{cg}\, W + x_v\, (P_v) + y_{tw}\, P_{tw} = 0$$

$$\mathbf{x_R = \{ H/3\, (P_h) + x_u\, U - x_{cg}\, W - x_v\, (P_v) - y_{tw}\, P_{tw} \}/N}$$



Shasta Dam, Plan and Sections

Sep 14 Hydraulics ① Lec. 6 cont.

2 methods to calc. seepage

1) Flow Net

$$\nabla^2 \phi = 0 \quad [\phi = \text{Piezometric head} = h_z + \frac{p}{\gamma}]$$

$\rightarrow$ get pressure : $\frac{p}{\gamma} = \phi - h_z$

2) Approx. method (quick answer)

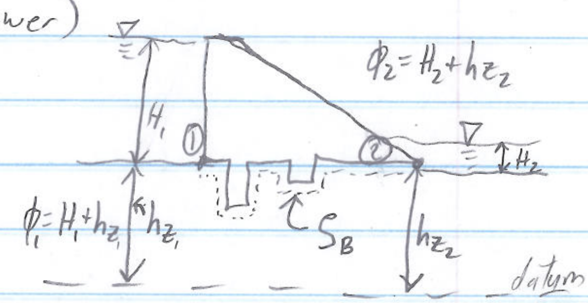
$\Delta\phi = \phi_2 - \phi_1$ (Force pushing water b/t ① & ②)

ϕ along seepage path

$$\phi = \phi_1 + \Delta\phi \frac{S}{S_B}$$

S ← distance to point of interest

S_B ← total seepage path



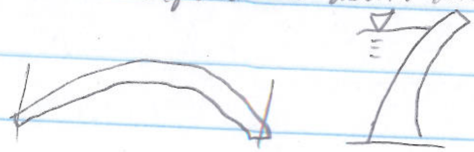
$f'_c = 6ksi \quad 6000\ lb/in^2 = h\left(150\ \frac{lb}{ft^3}\right)\frac{1\ ft^3}{(12^3)\ in^3}$

$\rightarrow h = \quad in = \quad ft$

Hydraulics Sep 16 ①

Arch Dams   S.F. typically = 3, & even as heigh as 5

* need dependable abatments & really need to watch fault lines



(No fix for a fault, will need to abadon site)

Types see handout

Hydro Sep 16

6. Experiments and theoretical studies can be used to find the actual form of the above dimensionless function.

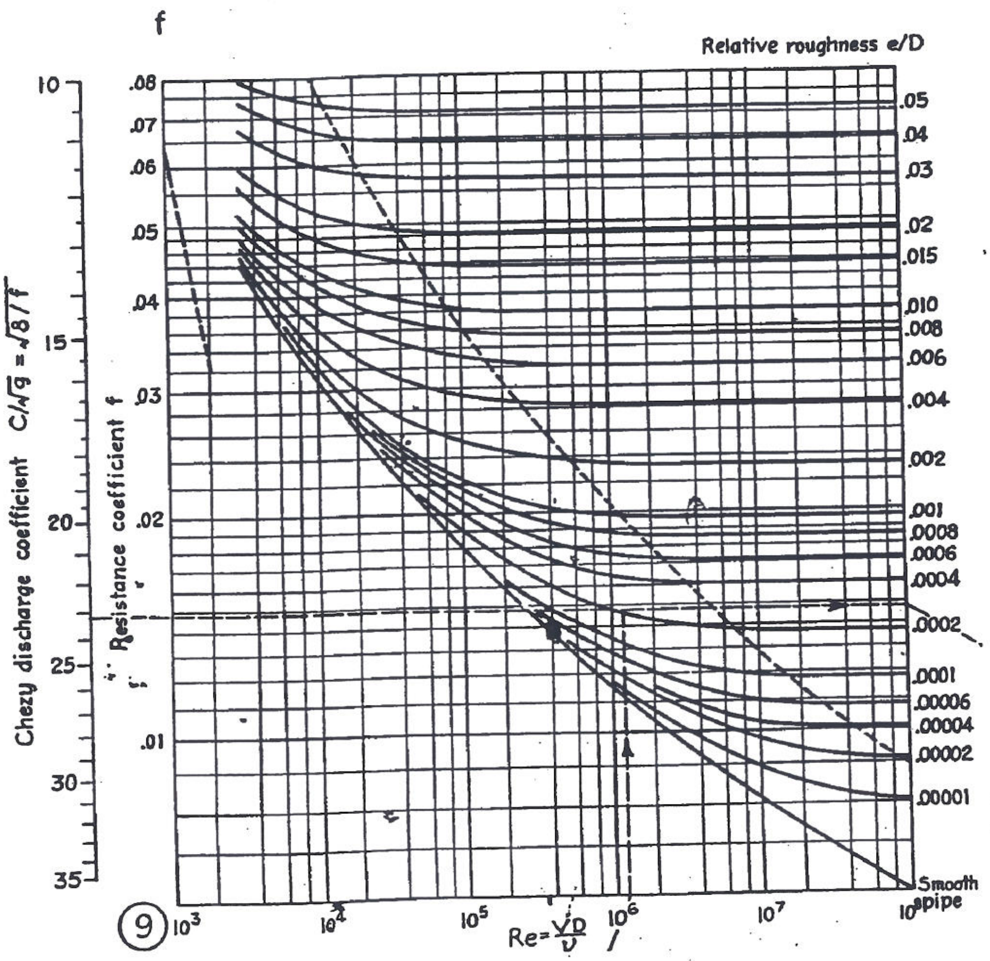
e.g. i) Experiments show that h_L is proportional to L;
ii) Gravity is included in N_E ;
iii) If there is no air/water surface N_W , has no effect;
iv) also if only circular pipes are used P/D is constant;

This leaves

$$N_E = h_L/\{V^2/2g\} = \{L/D\}\, f_c\,(N_R ,\ \varepsilon/D)$$

$$h_L = f_c\,(N_R ,\ \varepsilon/D)L\, V^2/\{2gD\}$$

which is the Darcy friction equation. The function f_c (N_R , ε/D) is given in the Moody's diagram.



There are several other friction equations that are used in pipe flow, e.g.

Hazen-Williams

The head loss formulas we have presented up to now are general because they are applicable for any fluid and any system of units. Other more restrictive empirical equations are also useful for their limited range of application. The most notable one, used for decades by waterworks engineers in the United States, is the Hazen-Williams formula. In English units, the formula is given in Eq. (5-12):

$$V = 1.318 C_h R^{0.63} S^{0.54} \tag{5-12}$$

where V = mean velocity in ft/s
C_h = Hazen-Williams friction coefficient (depends on pipe roughness)
R = hydraulic radius in ft
$S = h_f/L$ (slope of energy grade line)

To solve for head loss using the Hazen-Williams equation, a little algebraic manipulation of Eq. (5-12) yields

$$h_f = 3.02 L D^{-1.167} \left(\frac{V}{C_h}\right)^{1.85} \tag{5-13}$$

The resistance coefficient C_h depends on the surface characteristics of the pipe

Table 5-2 Hazen-Williams C_h Values for Different Kinds of Pipe (5)

Character of Pipe	C_h
New or in excellent condition cast-iron and steel pipe with cement or bituminous linings centrifugally applied, concrete pipe centrifugally spun, cement-asbestos pipe, copper tubing, brass pipe, plastic pipe, and glass pipe	140
Older pipe listed above in good condition, and cement mortar-lined pipes in place with good workmanship, larger than 24 in. in diameter	130
Cement mortar-lined pipe in place, small diameter with good workmanship or large diameter with ordinary workmanship; wood stave; tar dipped cast-iron pipe new or old in inactive water	120
Old unlined or tar-dipped cast-iron pipe in good condition	100
Old cast-iron pipe severely tuberculated, or any pipe with heavy deposits	10–80

Manning's Equation

Civil Engineers commonly use the Manning's Equation to computer friction losses in open channels and storm sewers. This equation relates the velocity V to the friction slope (S_f) and the section geometry by:

$$V = c' R^{2/3} S_f^{1/2} / n$$

where c' is a conversion factor =1 in SI units and 1.486 in US units; n is the Manning's roughness coefficient and R = hydraulic radius of the section.

Now the head loss due to friction is

$$h_f = S_f L$$

Recall that Q = V A

Therefore a common form of the Manning's Equation is

$$Q = c' A R^{2/3} S_f^{1/2} / n$$

Typical n values are:

Smooth Concrete 0.013
Rough (old) Concrete 0.015
CMP 0.024
Mississippi River ~ 0.025
Grass lined channels ~ 0.03
Natural Rivers 0.02 to 0.04

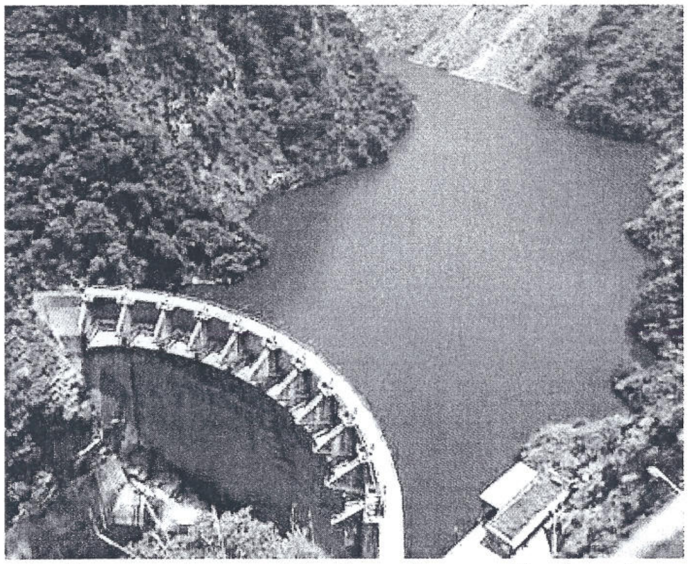
Strickler Equation gives an approximate relation between the grain roughness (D50 = median grain size) and n:

$$n = 0.034 \, (D50 \, ft)^{1/6}$$

Hydro Sep 16

Lecture 7
Stability of an Arch Dam

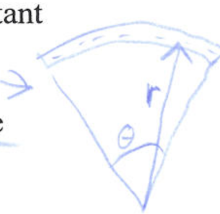
The stability of an arch dam depends on the safe transfer of the applied loads from the point of application to the abutments by an arching mechanism, i.e. via compressive forces acting along the arch. The integrity of the abutments is absolutely critical to the stability of the arch dam.



The arch dam is subject to the same types of loads as the gravity dam; however, the relative importance of the loads is different. A gravity dam is mainly supported at its base while an arch dam is largely supported at its sides. In addition, thermal stresses which are not very important in a gravity dam are considered in the detailed design of an arch dam.

usually small dams

There are two common types of arch dams: a) constant radius and b) constant angle variable radius.

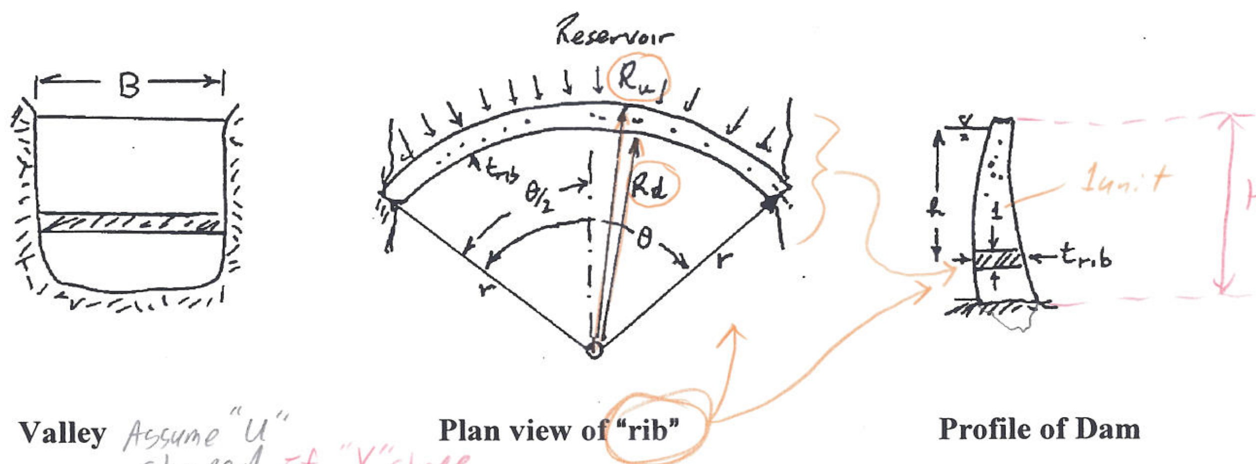


Three analysis procedures are used in the design of arch dams: a) ring theory, b) trial load analysis and c) 3-D finite element analysis. Only the simplest of these, the ring theory will be considered. This method is often used to obtain a preliminary design that can be used as the basis for the other methods. we will cover ring theory

To illustrate the mechanism of support of an arch dam we will consider a simplified analysis of a horizontal strip 1 unit in height; this slice is called a "rib". As indicated the slice is taken at some depth, h, below the reservoir surface. "thickness"

$$\frac{t_{rib}}{r} < \frac{1}{25}$$

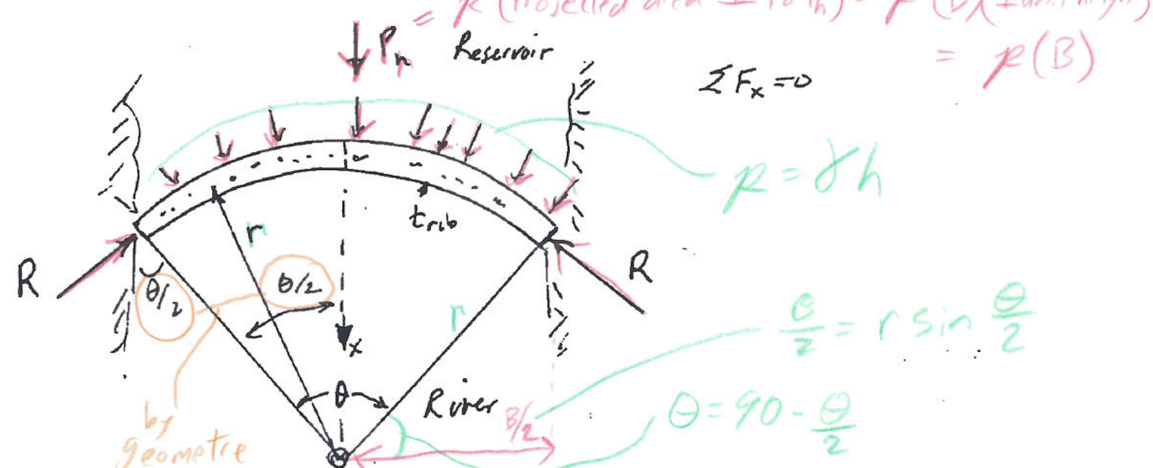
each rib must be independently stable



Valley Assume "U" shaped. If "V" shape the 'B' will change w/ h **Plan view of "rib"** **Profile of Dam**

Simplified Ring Analysis

This approach assumes: a ring of unit height subjected to a constant pressure, as shown in the figure below and a very thin rib thickness compared to the radius.



Defining Diagram and FBD of Ring (Rib) of Arch Dam

Case 1. Consider the hydrostatic loads only:-

$$\mathbf{p = \gamma\, h}$$

resolve forces on Dam in direction of R to find $R = \frac{P_H}{2\sin\frac{\theta}{2}} = \frac{\gamma h B}{2\sin\frac{\theta}{2}}$

From hydrostatic loading on a curved surface we obtain,

$$\mathbf{P_H = \gamma\, h\, B = 2\, \gamma\, h\, r \sin(\theta/2)}$$

where $\mathbf{r = B/\{2 \sin(\theta/2)\}}$

From the FBD and $\Sigma F_x = 0$ we get $\mathbf{R = \gamma\, h\, r}$

Now Determine rib thickness $\sigma_w = \sigma_c / F_{os}$

If the allowable compressive stress is σ_w then the ring or rib thickness is

$$\mathbf{t_r = R/\sigma_w = \gamma\, h\, r/ \sigma_w}$$

b/c t_r varies linearly w/ h we should have linear increase/decrease in t_r as we move up & down the dam

Typically $\sigma_w = \sigma_{concrete}$ /FOS

where the FOS ~ 3 to 5.

The volume of the rib is

$$V_{rib} = t_r \theta r$$

or $$=(\gamma h/\sigma_\omega)\theta r^2 = (\gamma h/\sigma_\omega)\theta [B/\{2 \sin(\theta/2)\}]^2$$

The minimization of this volume gives an arch angle of 133.5°.

If $r/t_r < 25$ then an abutment stress factor should be applied to get the abutment stress. Furthermore the rib thickness may need to be increased at the abutment in order to better distribute the stress to meet the allowable stress for the rock or the reduce the deformation of the rock. If the allowable abutment rock stress is σ_r then the thickness at the abutment should be,

$$\mathbf{t_{ra} = t_r \sigma_w / \sigma_r}$$

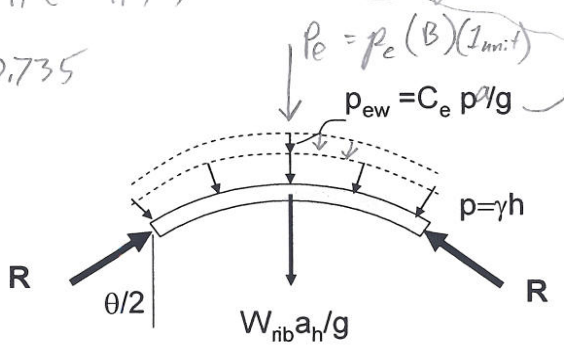
Other Loads

Ice can be the governing load in the upper elevations of an arch dam:-

Assume that the ice acts as a pressure p_I on the whole rib. The rib thickness to withstand the ice loading is

$$\mathbf{t_{rI} = p_I r / \sigma_w}$$

Earthquake: Arch Dams are not suitable for zones with Strong Earthquake Loads. It may be necessary to include an allowance for a small earthquake force (a/g ~ 0.05 to 0.1). In this case the following approximate method can be used to estimate the thickness to resist the added compressive stresses.



FBD of Rib with Water and Earthquake Acting

Maximum Compression:
The added pressure from the earthquake at depth h on the water is

$$\mathbf{p_{ew} = C_e \gamma h \, a/g}$$

or a horizontal force of

$$\mathbf{P_e = B \, C_e \gamma h \, a/g}$$

The added force on the concrete mass is,

$$\mathbf{F_{ceH} = \rho_c V_{rib} \, a = \gamma_c \, r \, t_{rib} \theta \, a/g}$$

Combining the water and earthquake forces, we get (see FBD)

$$\mathbf{2 R \sin(\theta/2) = \{P_e + F_{ceH} + P_H\},}$$

or $\quad R = t_{rib} \sigma_w = [\gamma_c r \, t_{rib} \theta \, a/g + B \, C_e \gamma h \, a/g + \gamma h B]/[2 \sin(\theta/2)]$

which gives

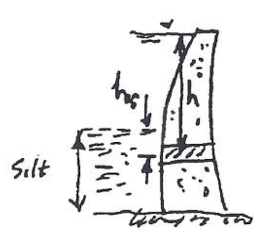
$$\mathbf{t_{rib} = \{[B \, C_e \gamma h \, a/g + \gamma h B]/[2 \sin(\theta/2)]\} / [\sigma_w - \gamma_c r \, \theta (a/g)/(2 \sin(\theta/2))] > t_{rI}}$$

$$\mathbf{t_{rib} = r \, [C_e \gamma h \, a/g + \gamma h] / [\sigma_w - \gamma_c r^2 \, \theta (a/g)/B] > t_{rI}}$$

Silt Load:
This is similar to the silt load for a gravity dam. The added pressure due to silt if the friction angle is assumed to be zero is:

$$p_{silt} = \gamma (S_s - 1)(1 - \text{porosity}) h_s$$

Therefore $\mathbf{t_{rib} = r \, [C_e \gamma h \, a/g + \gamma h + \gamma (S_s - 1)(1 - porosity) h_s] / [\sigma_w - \gamma_c r^2 \, \theta (a/g)/B] > t_{rI}}$



Definition Diagram for Silt Load

Corrections for Thermal Stress: Stresses may develop in the arch due to changes in temperature. The temperature in the concrete depends on the upstream (water) and downstream (air and solar radiation effects) temperatures as well as the stored heat in the concrete. The following is an approximate formula for the added load due to a change in temperature of DT with respect to the base temperature (e.g. mean construction temperature after cooling of the reaction heat).

The added effective pressure load is,

$$p_{thermal} \sim E_c C'' t_{rib} \Delta T / R_u$$

where $p_{thermal}$ = effective added pressure due to thermal stress; E_c = Young's modulus of concrete;
C'' = thermal expansion coefficient; t_{rib} = rib thickness; R_u = u/s radius; ΔT = change in temperature. Thus

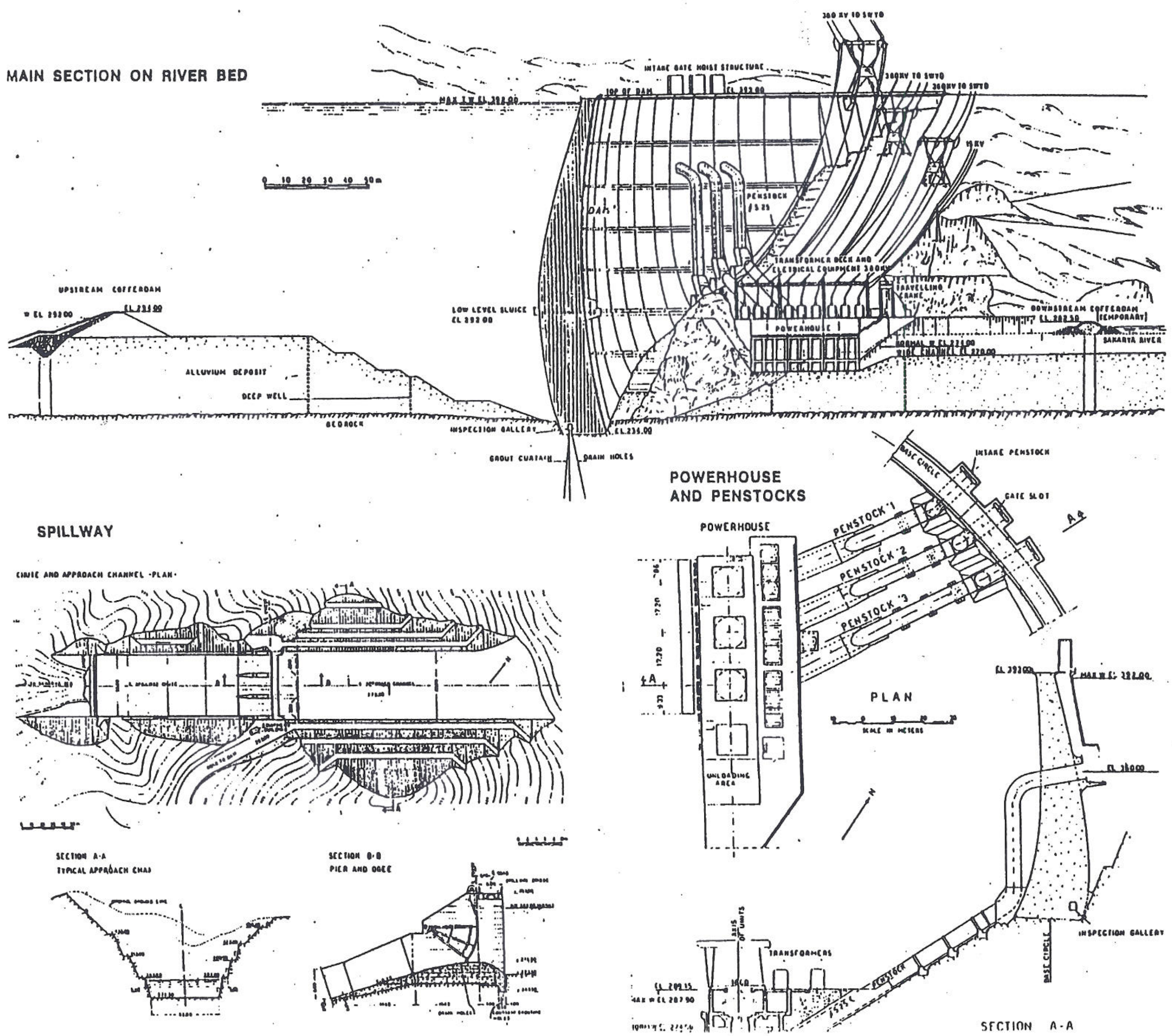
(upstream)

$$t_{rib} = r\,[C_e \gamma h\, a/g + \gamma h + \gamma (S_s - 1)(1 - porosity)h_s] \,/\, [\sigma_w - \gamma_c r^2\, \theta\, (a/g)/B - E_c C'' \Delta T\, r/R_u] > t_{rI}$$

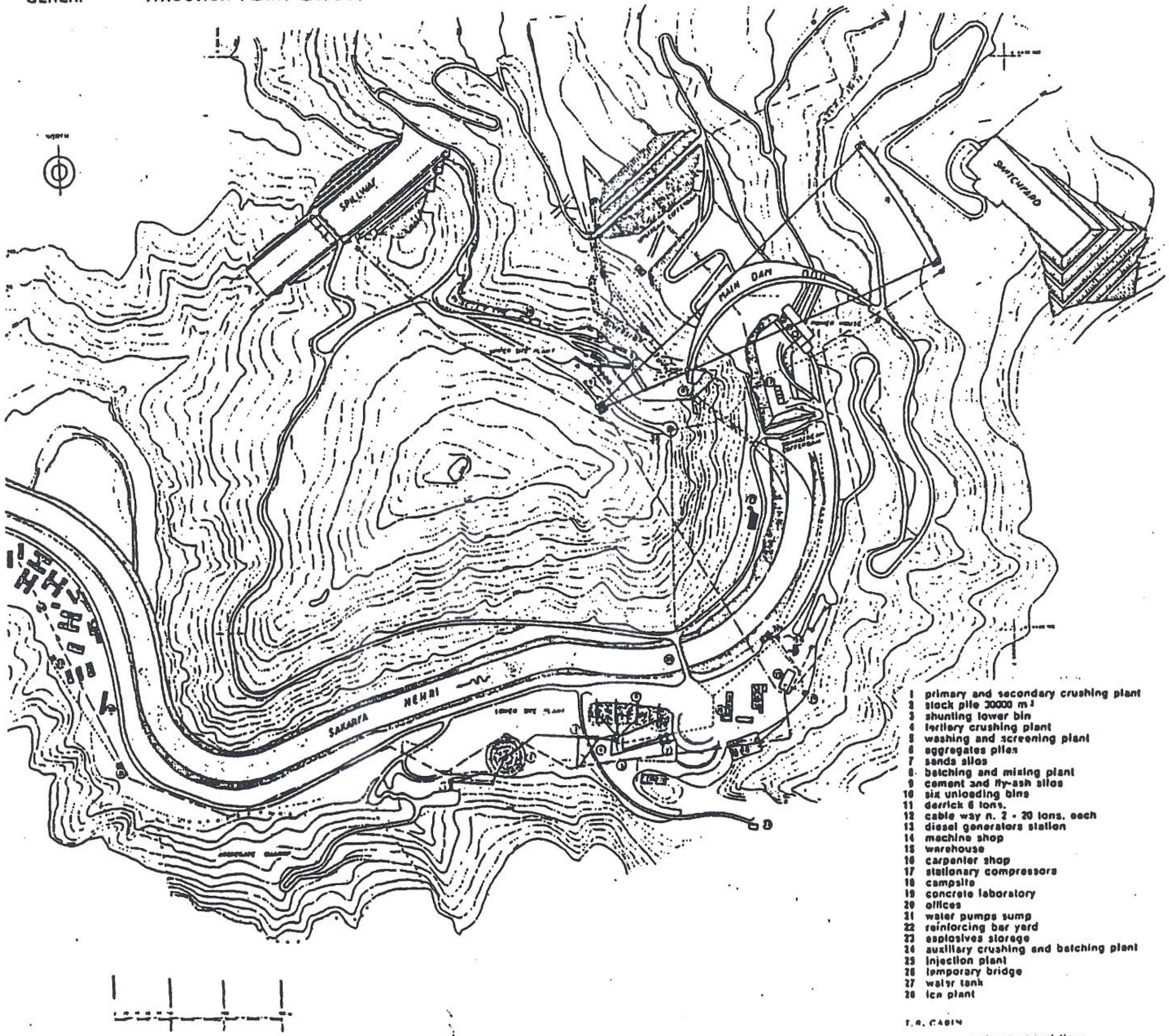
If the ratio $r/t_{rib} < 25$ then a correction should be added for the rib thickness, i.e.

$$t_{rib} = \{R_u^2/r\}\,[C_e \gamma h\, a/g + \gamma h + \gamma (S_s - 1)(1 - porosity)h_s] / [\sigma_w - \gamma_c r^2\, \theta (a/g)/B - E_c C'' \Delta T\, r/R_u] > t_{rI}$$

since $R_u = r + \frac{t_{rib}}{2}$ assume t is small
get answer for t_{rib}; put back in &
solve for t_{rib} again.


MAIN SECTION ON RIVER BED
UPSTREAM COFFERDAM
ALLUVIUM DEPOSIT
DEEP WELL
BEDROCK
INSPECTION GALLERY
GROUT CURTAIN
DRAIN HOLES
LOW LEVEL SLUICE
DAM
PENSTOCK
INTAKE GATE HOIST STRUCTURE
TOP OF DAM
TRANSFORMER DECK AND ELECTRICAL EQUIPMENT
POWERHOUSE
DOWNSTREAM COFFERDAM (TEMPORARY)
SAKARYA RIVER
POWERHOUSE
AND PENSTOCKS
POWERHOUSE
INTAKE PENSTOCK
GATE SLOT
BASE CIRCLE
PENSTOCK 1
PENSTOCK 2
PENSTOCK 3
UNLOADING AREA
PLAN
SCALE IN METERS
SPILLWAY
SECTION A-A
TYPICAL APPROACH CHAN
SECTION B-B
PIER AND OGEE
TRANSFORMERS
AXIS OF UNITS
PENSTOCK
INSPECTION GALLERY
BASE CIRCLE
SECTION A-A

GENER/ ;TRUCTION PLANT LAYOUT



1 primary and secondary crushing plant
2 stock pile 30000 m³
3 shunting lower bin
4 tertiary crushing plant
5 washing and screening plant
6 aggregates piles
7 sands silos
8 batching and mixing plant
9 cement and fly-ash silos
10 six unloading bins
11 derrick 6 tons.
12 cable way n. 2 - 20 tons, each
13 diesel generators station
14 machine shop
15 warehouse
16 carpenter shop
17 stationary compressors
18 campsite
19 concrete laboratory
20 offices
21 water pumps sump
22 reinforcing bar yard
23 explosives storage
24 auxiliary crushing and batching plant
25 injection plant
26 temporary bridge
27 water tank
28 ice plant

T.R. CABIN

main electrical lines

Hydro sep 21 ①

~~H.C. sep 20 ②~~

Lecture 8 - minor losses
Lecture 9 - friction in pipes } on B.B. not covering in class, expected to know

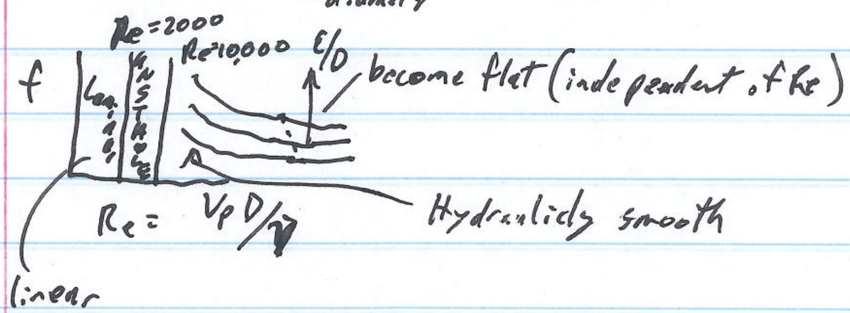
Friction in Pipes

Darcy equ. most common — equ. 1 Flow in pipes; — equ 2 " through porous medium

Flow in pipes

$$h_f = \frac{fL}{D}\left(\frac{V^2}{2g}\right) \quad [\text{ft or m}]$$

$$f = f\left(Re, \frac{\varepsilon}{D}\right)$$ (ε = roughness, D = diameter) summarized in Moody's Diagram



$$h_f = \frac{fL}{D}\left(\frac{\left(\frac{Q}{A}\right)^2}{2g}\right) = \frac{fL}{2gA^2}Q^2 = K_p Q^2$$

$$K_p = \frac{fL}{2gA^2D} = \left(\frac{16fL}{2g\pi^2D^5}\right)$$

minor losses

$$h_{L_m} = K_m \frac{V^2}{2g} \quad (\text{entrence, exit, bend, valve})$$

$$= \frac{K_m}{2g}\left(\frac{Q}{A}\right)^2 = \frac{16 K_m Q^2}{2g\pi^2 D^4}$$

Total K → $K_p = K_{p_f} + K_{p_m}$

Hydro Sep 21 (2)

Manning's Equ (open channel / storm surge > often use)

$$V = \frac{c'}{n} R^{2/3} S_f^{1/2}$$

$c' = 1.486$ us $= 1$ metric (conversion coefficient)

R = hydraulic radius $= \left(\frac{D}{4}\right)$

n = roughness coef. $\sim 0.034 \left(\epsilon_{\text{in feet}}\right)^{1/6}$

For PVC $n \sim 1$, concrete $n \sim 0.012 \rightarrow 0.015$

S_f (friction slope) $= \frac{h_f}{L}$

$\therefore \; h_f = \left(\frac{n V}{c' R^{2/3}}\right)^2 L$

Hazing/william equ (will not cover) but prefered by Pipe net

---

sump $\equiv$ wet well

Sep. 21 Pump Stations

Lecture 10
Computations with Pumps

Typical Types of Pumps

- **Centrifugal** (large stations) (aka radial flow) (ie. Archamedies skrew)
- **Axial flow** (large stations)
- **Mixed Flow** (between Centrifugal & Axial flow)
- **Archimedes Screw**
- **Positive displacement**

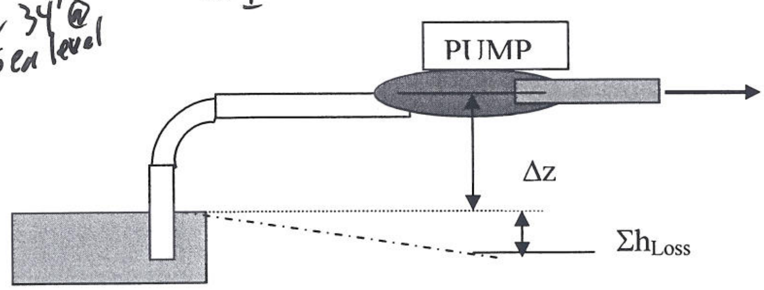
Pump Characteristics

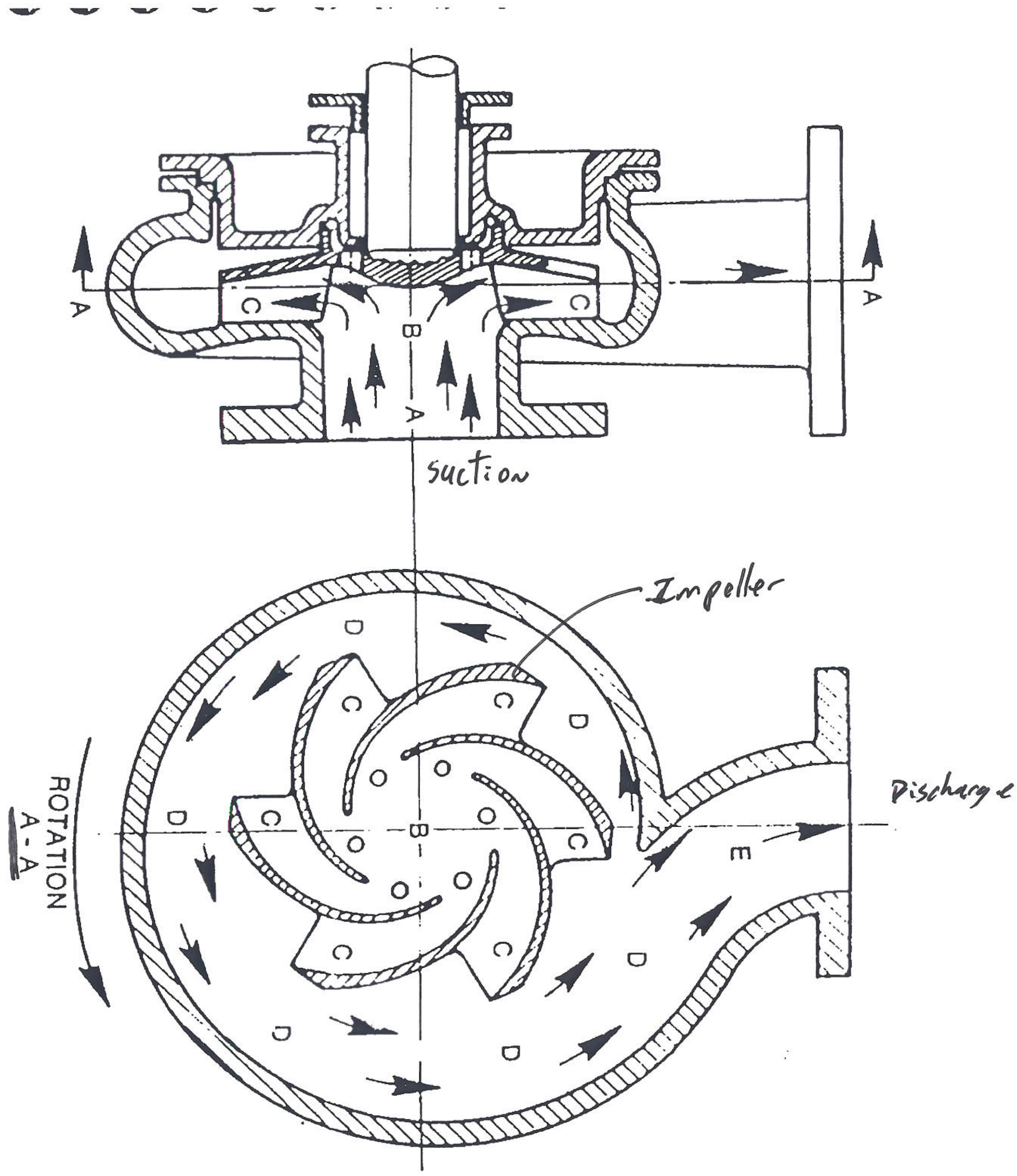
- **Head - Discharge Curve**
- **Efficiency**
- **Net Positive Suction Head Requirement (NPSH)**

This is a requirement that the manufacturer specifies in order to avoid cavitation on the pump impeller. For a given pump placement the available NPSH is defined by:

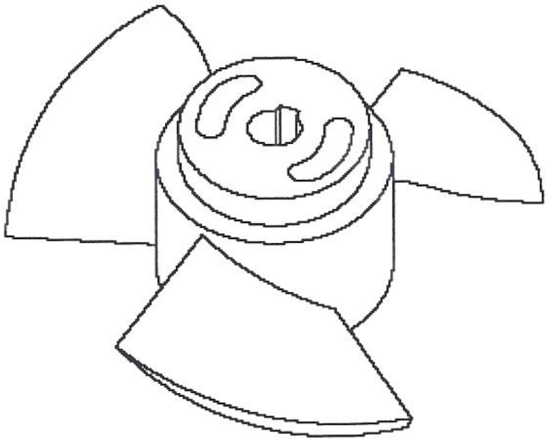
NPSH (available) $= \{p_{atm}/\gamma - p_{vap}/\gamma - \Delta z - \Sigma h_{loss}\} \geq$ **Required NPSH (manufacturer)**

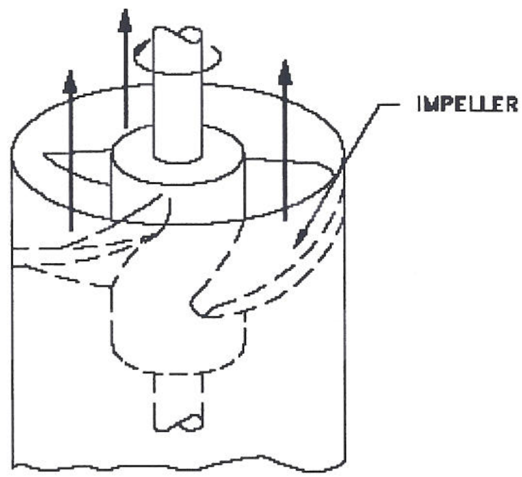
p_{atm}/γ ~ 34' @ sea level; p_{vap}/γ ~ 1; Σh_{loss} = $k_p Q_s^2$



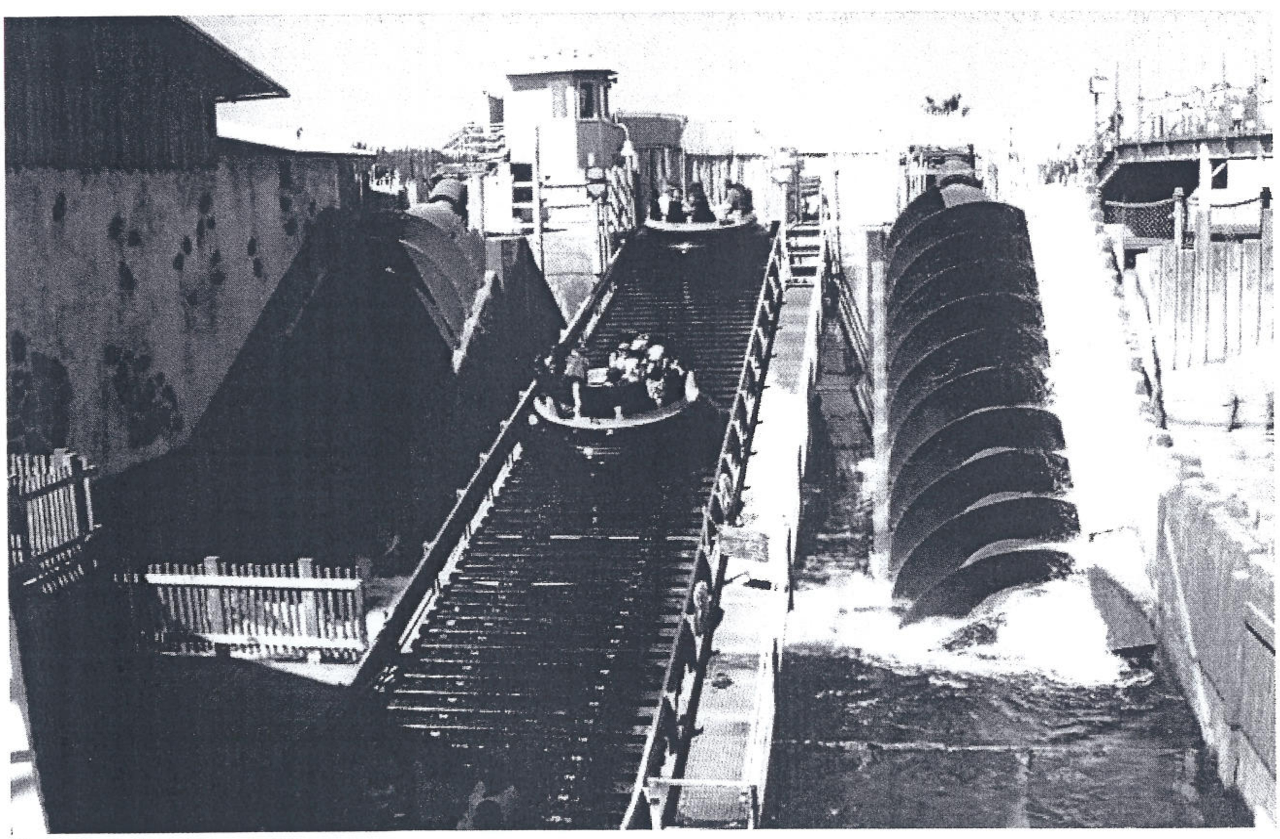

Suction
Impeller
Discharge
ROTATION
A - A
A
B
C
D
E

Centrifugal Pump

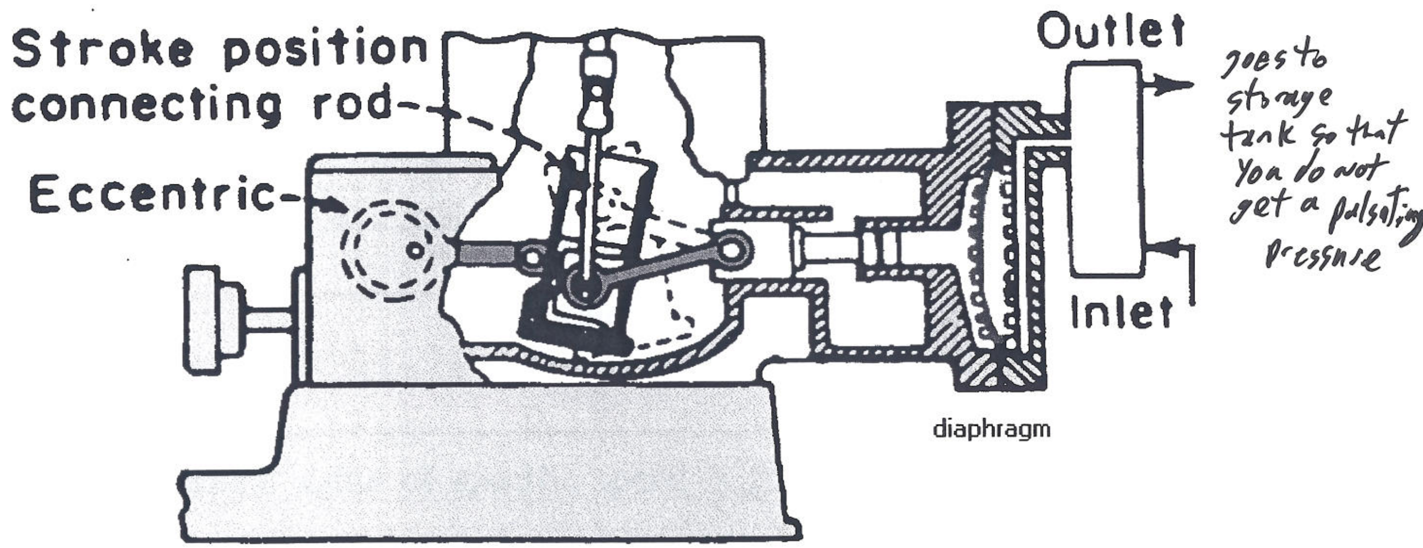




Axial Flow Pump



Archimedes Screw Pump



Positive Displacement Pump

http://www.rpi.edu/dept/chem-eng/Biotech-Environ/PUMPS/reciprocating.html

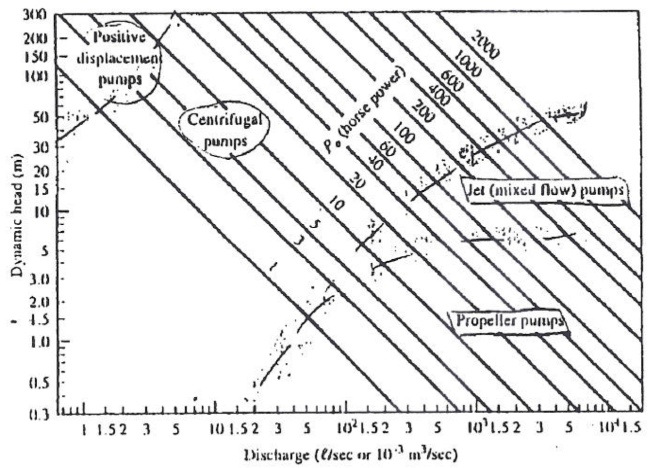


Figure 5.8 Head, discharge, and power requirement of different types of pumps.

matched to the pump performance chart (e.g., Figures 5.9 and 5.10) provided by the manufacturer. The matching point, *M*, indicates the actual working conditions. The selection process is demonstrated in the following example.

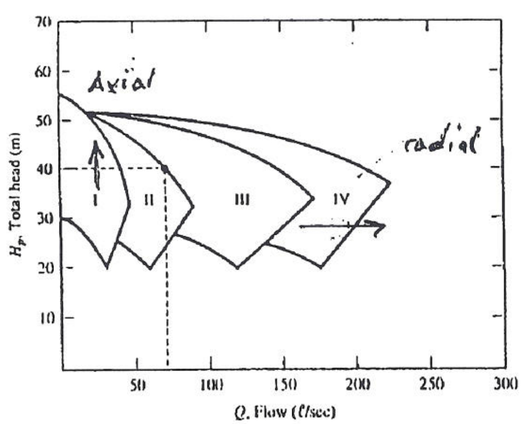


Figure 5.9 Pump selection chart.

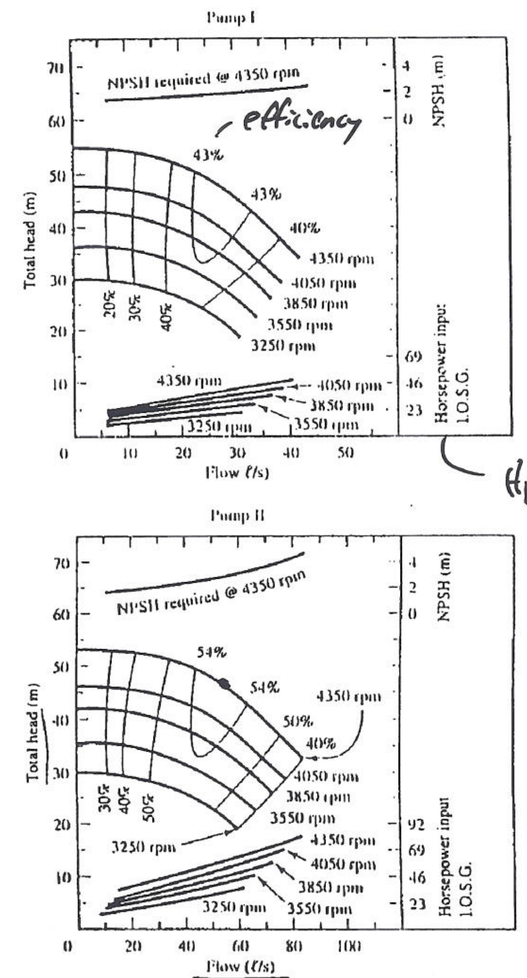


Figure 5.10 Characteristic curves for several pumps.

Hp given

NPSH (net positive suction head)
Tells if pump will cavitate

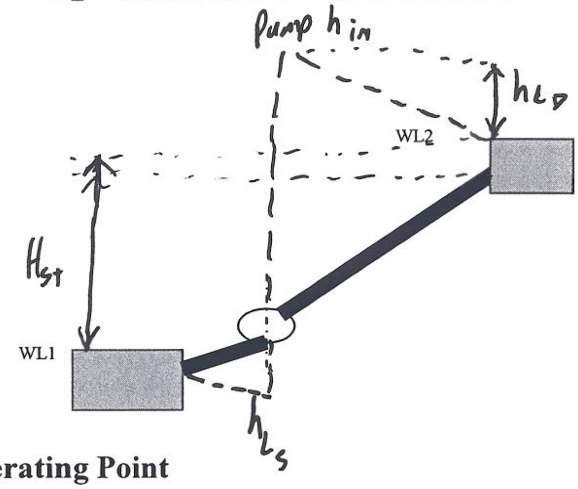
Pipeline System Curve and Pump Operating Point

The System Curve for a Pipeline is defined as the head to lift the water from the wet well to the outlet water level and overcome all of the losses in between,

sys head $= H_{st} + h_{L_S} + h_{L_D} \rightarrow = H_{st} + K_{PS}(Q_s^2) + K_{PD}(Q_d^2)$ at some point $H_{sys} = H_o = H_{pump}$

$$H_{sys} = H_{st} + \Sigma h_L = H_{st} + f(Q^2)$$

where H_{st} **= Static Lift = WL1 – WL2**
h_L **= minor losses + friction losses**

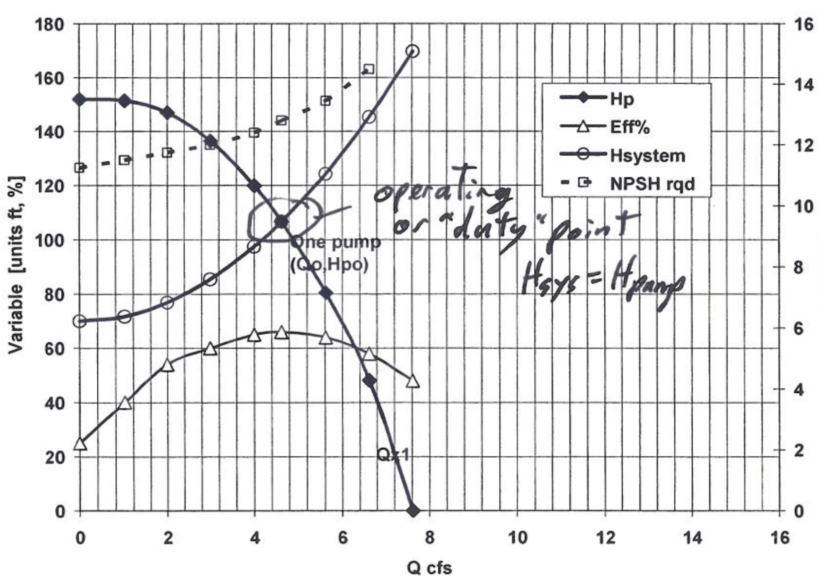


Operating Point

The pump operating point (Qo , Hpo) is at the intersection of the Pump Curve and the System Curve, i.e.

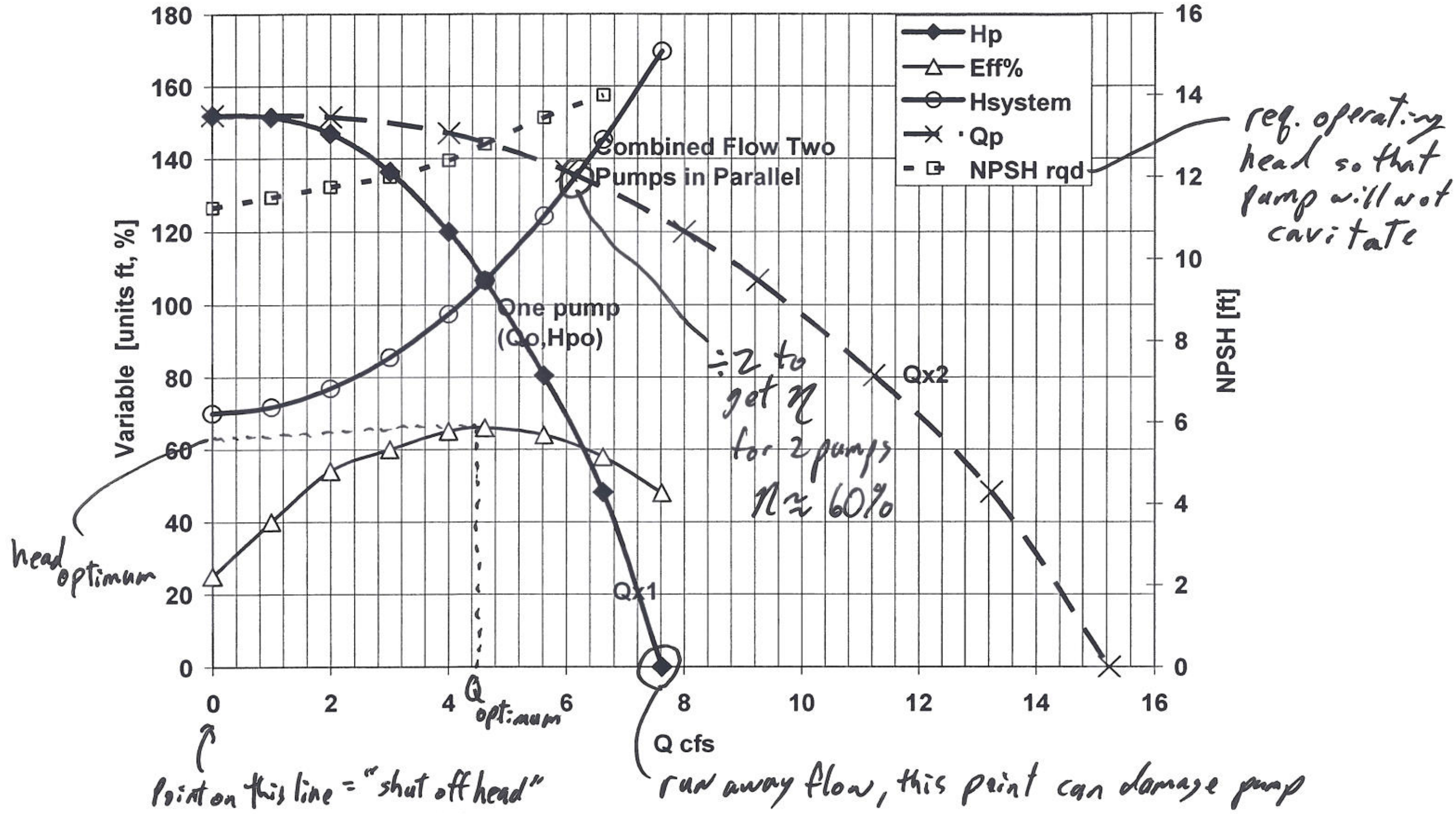
$$H_{sys} = H_p$$

This is shown graphically in the figure below



Theoretical Power $P_w = \gamma Q H$ $\qquad P_{w_{H.P.}} = \frac{\gamma Q H}{550}$ [hp]

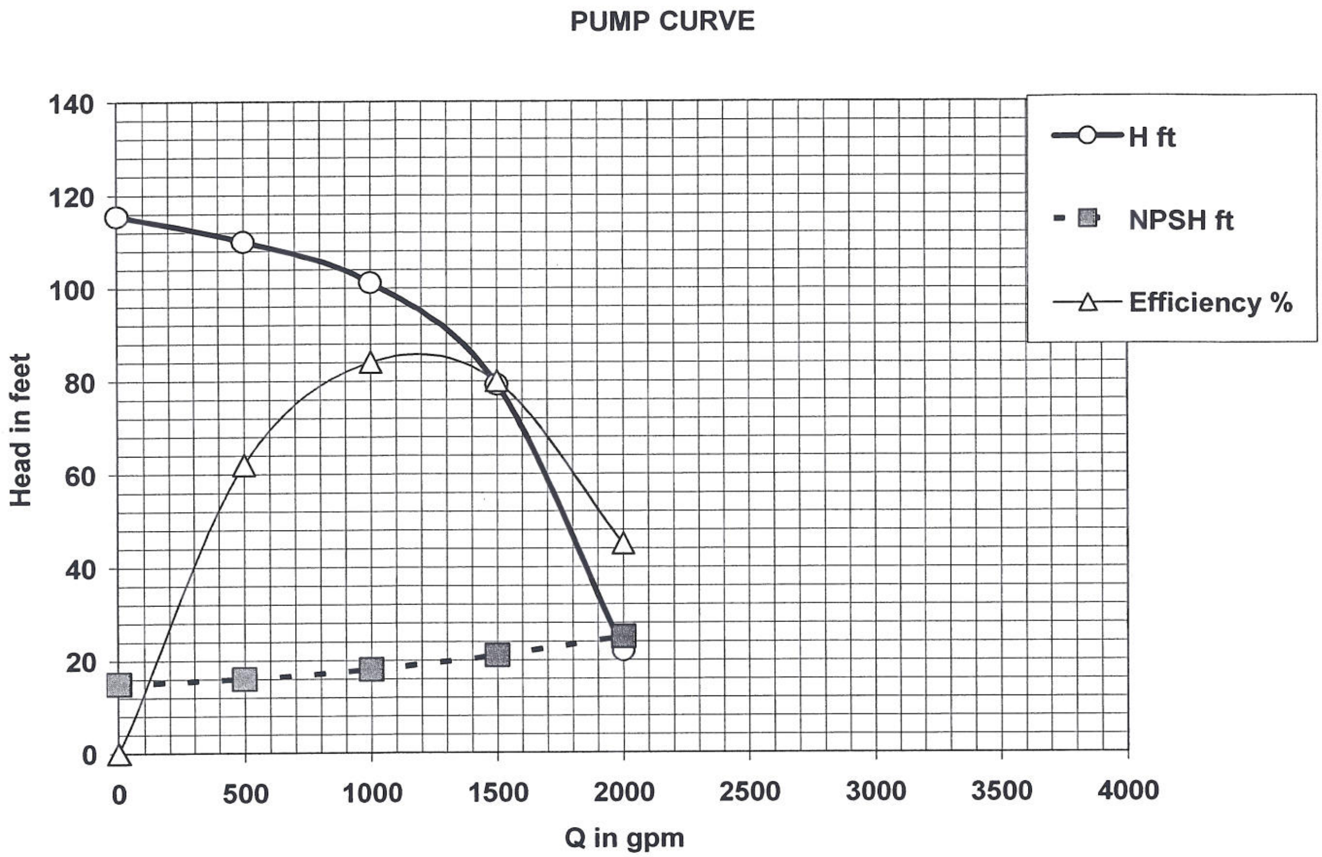
Power needed from motor $P_{motor} = \frac{\gamma Q H}{\eta\,(\text{efficiency})}$



Example of Pumps in Parallel

* These curves correspond to a specific pump rpm 'N'

$$N_s \text{ (specific speed)} = \frac{N_{\text{rpm}}\left(Q_{opt}^{1/2}\right)}{\left(H_{opt}\right)^{3/4}}$$



PUMP CURVE
140
120
100
80
60
40
20
0
Head in feet
0
500
1000
1500
2000
2500
3000
3500
4000
Q in gpm
H ft
NPSH ft
Efficiency %

Sep. 28 Hydro.

Lecture 11
Analysis of Pipe Networks

Equivalent Pipes

Define $h_{Li} = K_{pi} Q_i^2$ where $K_{pi} = 16 f_i L_i / (2 g D_i^5 \pi^2)$ for the Darcy friction equation.

Pipes in Series

Multiple pipes (segments) of different size and/or friction factor, connected end to end.
By continuity: $Q_1 = Q_2 = Q_3 = Q_i$
By Energy: The total head loss = The sum of the head losses in each segment

$h_L = \Sigma h_{Li}$

Equivalent pipe: $\mathbf{K_{pT} = \Sigma K_{pi}}$

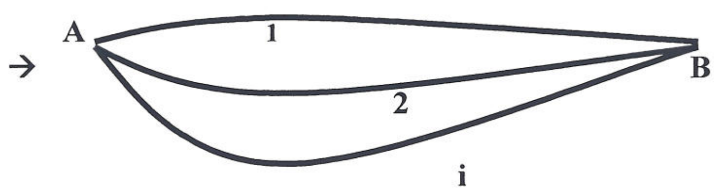
1 K_{p1} **2** K_{p2} **i** K_{pi}

A === **B**

Pipes in Parallel

Multiple pipes (segments) of different size and/or friction factor, a connected at a common starting and ending point.
By continuity: $Q_T = \Sigma Q_i$
By Energy: The total head loss = The same in each segment

$h_L = h_{Li}$



Equivalent Pipe: $\mathbf{K_{pT} = 1 / \{\Sigma 1 / K_{pi}^{1/2}\}^2}$

Examples

1a) Parallel: Q = 5 cfs. Find Equivalent Kp , head loss and flow in each pipe.
1b) Series: Q = 5 cfs. Find Equivalent Kp , head loss each pipe and total head loss.

Parallel **Q** **5** cfs 8fL/{g pi^2 D^5}

pipe	D	D	f	L	Ki	1/sqrt(Ki)	Qi cfs
1	12"	1	0.025	1200	0.755	1.151	3.67
2	8"	0.66667	0.025	1200	5.735	0.418	1.33

$$Q_i = Q_T \sqrt{\frac{K_{pi}}{K_{pT}}}$$

Sum	1.568	5	cfs
Sq	2.459599		check
KT rec	**0.40657**		
Hloss	**10.1643**	**ft**	

Series **Q** **5** cfs 8fL/{g pi^2 D^5}

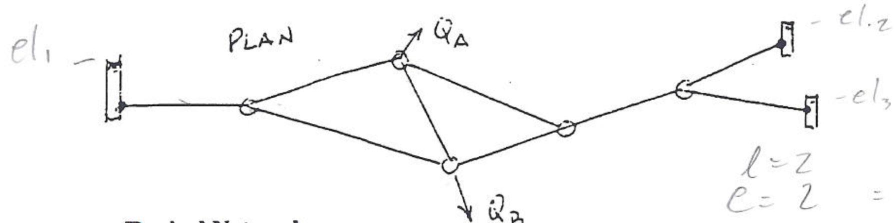
pipe	D	D	f	L	Ki		Qi cfs	hfi
1	12"	1	0.025	1200	0.755		5.00	18.87972
2	8"	0.66667	0.025	1200	5.735		5.00	143.3679

Sum	6.490	162.25	ft
			check
KT	**6.490**		
Hloss	**162.248**	**ft**	

Branching Systems

Pipe Networks

A pipe network is a system of interconnected pipes. The following is an example:



Typical Network

l = 2
e = 2 = (t-1)
j = 5
Np = 9

The components of a simple network are:

1. Terminal Energy point *(t)* = point with known energy or pressure

2. Loop (*l*)= set of connected pipes that form a close loop. The head loss around the loop is always = 0.

3. Path (*e*)= set of connected pipes between terminal energy points

4. Junction (*j*) = node where two or more pipes meet (excluding terminal energy points). pressure unknown

5. Pipes (*Np*) = number of pipes in the network —> usually denotes the number of flows to be computed.

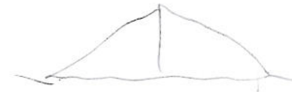
For consistency we must have:

$$Np = \text{Loops} + \text{Paths} + \text{Junctions}$$

e = Paths= (Terminal Energy points - 1)

$$Np = l + (t - 1) + j$$ = 9 = 2 + (3-1) + 5 = 9

Example: Check Consistency requirements for network shown above.



Differentiating $\mathbf{h_{Lp} = K_p\, Q_p\, |Q_p|}$ we get

$\Delta\, \mathbf{h_{Lpi} = (dh_{Lp}/dQ)\, \Delta\, Q}$

or

$\Delta\, \mathbf{h_{Lpi}\; = 2\, K_p\, |Q_{pi}|\Delta Q}$ **EQ E**

where Δ Q is the correction must be added to the flow to make $\Sigma\, \mathbf{h_{Lpi}} + \Delta\, \mathbf{h_{Lpi}} \rightarrow \mathbf{0}$
& note that Δ Q carries the sign of the correction.

Therefore

$\Sigma\, \mathbf{(h_{Lpi} + \Delta\, h_{Lpi}\,) = \Sigma\, (K_p\, Q_{pi}\, |Q_{pi}| + 2\, K_p\, |Q_{pi}|\Delta Q) \rightarrow 0}$

This gives

$\mathbf{\Delta Q = -\, \Sigma\, (K_p\, Q_{pi}\, |Q_{pi}|)/\{\Sigma\, (\, 2\, K_p\, |Q_{pi}|\}}$ **EQ F**

For a Path this becomes,

$\mathbf{\Delta Q = \{-\, \Sigma\, (K_p\, Q_{pi}\, |Q_{pi}|) + E_d\}/\{\Sigma\, (\, 2\, K_p\, |Q_{pi}|\}}$ **EQ G**

Assignment Problem 11.2
Find the flow in all of the pipes in the network shown below.
Assume: $\mathbf{h_f = Kp\; Q|Q|}$ $t=2,\ l=1,\ e=t-1=1,\ j=3,\ N_p=5$ el. of all j = 10'

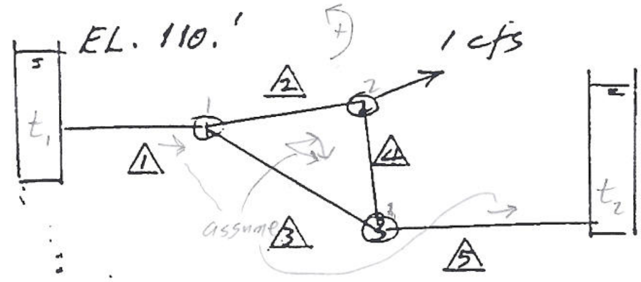
$\mathbf{K_1\; = 2;\ K_2 = 1;\ K_3\; = 1;\ K_4\; = 1;\ K_5\; = 2}$

consistancy equ. $5 = 1+1+3 = 5$

Continuity equ
$Q_{in} - Q_{out} = \emptyset$

Energy equ: loop
$h_{L_3} - h_{L_4} - h_{L_2} = \emptyset \; (\Sigma h_{L_l} = 0)$

Energy equ: path
$\Sigma h_L = E_{drop} = WL_1^{t_1} - WL_2^{t_2}$



(A) guess @ Q_{pi} ; $\Sigma Q_{pi} = \emptyset$ @ $j = 1, 2, 3$
* assume $Q_1 = 3$
j_1 * assume $Q_2 = 2$, $Q_3 = 1$
j_2 ∴ $Q_4 = 1$
j_3 ∴ $Q_5 = 2$

Put in terms of Q
$h_{Lp} = k_p Q_p^2 = k_p Q_p|Q_p|$

Loop: $\Sigma K_p Q_p|Q_p| = 0$
Path: $\Sigma K_p Q_p|Q_p| = E_d$

(B) Q_{pi}
loop
$\Sigma h_{Lp} = 0 \rightarrow \Sigma K_p Q_p|Q_p| = \emptyset$

over

(B cont)

account for error

$$\Sigma\left(h_{L_{p_i}} + \Delta h_{L_{p_i}}\right) = \emptyset = \Sigma\left(k_p Q_{p_i}|Q_{p_i}| + \Delta h_{L_{p_i}}\right) = \emptyset$$

$h_L = k_p Q_p^2$

$d(h_L) = 2k_p|Q_p|\Delta Q$

$\Delta h_{L_{p_i}} = 2k_p|Q_{p_i}|\Delta Q$

$$\Sigma\left(k_p Q_{p_i}|Q_{p_i}| + 2k_p|Q_{p_i}|\Delta Q\right) = \emptyset$$

correction: $$\Delta Q = \frac{-\Sigma k_p Q_{p_i}|Q_{p_i}|}{\Sigma\ 2k_p|Q_{p_i}|}$$

given: $k_{p_1} = 2 = k_{p_5}$, $k_{p_2} = k_{p_3} = k_{p_4} = 1$

Path

$$\Delta Q_{path} = \frac{-\left(\Sigma k_p Q_{p_i}|Q_{p_i}|\right) + E_{drop}}{\Sigma\ 2k_p|Q_{p_i}|}$$

$E_{drop} = E_d = WL_1 - WL_2 = 110 - 60 = 50'$ (water level)

approx correction

$$\Delta Q = \frac{-\left((2\times3\times3) + (1\times1\times1) + (2\times2\times2)\right) + 50}{(2\times2\times3) + (2\times1\times1) + (2\times2\times2)} = \frac{23}{22} \sim 1$$

(k_p Q $|Q|$)

$Q_1 = 3 \to 4$, $Q_3 = 1 \to 2$, $Q_5 = 2 \to 3$

Loop

3 - 4 - 2 (+)

$$\Delta Q = \frac{-\left((1\times2\times2) - (1\times1\times1) - (1\times2\times2)\right)}{(2\times1\times2) + (2\times1\times1) + (1\times2\times2)} = \frac{1}{10} = +0.1$$

(k_p Q(new value) $|Q|$)

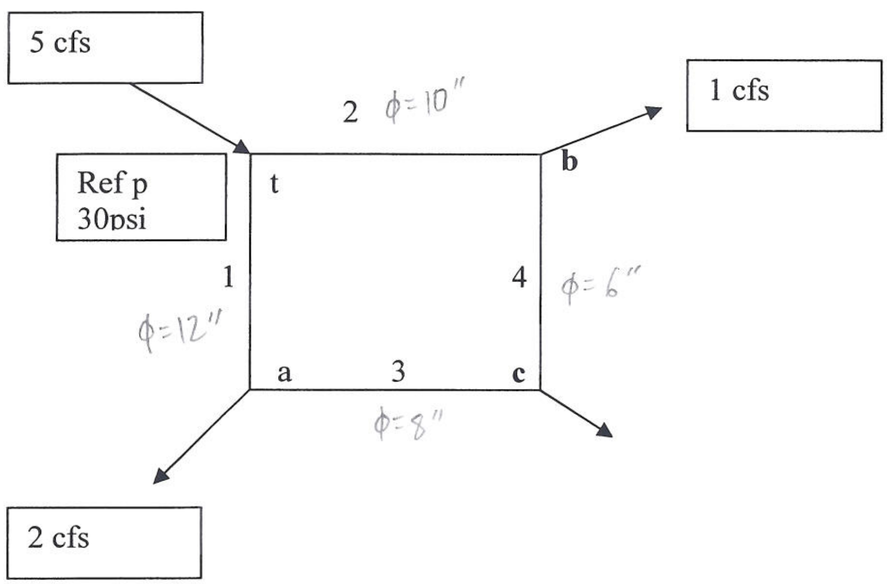
$Q_3 = 2 \to 2.1$, $Q_4 = 1 \to 0.9$, $Q_2 = 2 \to 1.9$

Tollerence = 1% of given Q out (0.01) given by path & loop
so, reiterate path & loop (aka $\Delta Q_{\to\emptyset}$) w/ new values

$$\frac{P_1}{\gamma} = WL_1 - k_{p_1} Q_{p_1}^2$$

Assignment Problem 11.3

Find the flow in all of the pipes in the network shown below.
Assume: f = 0.025 and L = 2000 ft in all pipes & K_m = 0. All junctions at 0 elevation.
D1 = 12 inches
D2 = 10 inches
D3 = 8 inches
D4 = 6 inches



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Lecture 12
Energy Principle Applied to Open Channel Flow
Reference : Lecture 3

Energy Equation

General

$$H_{T1} + H_{Pump} == H_{T2} + H_{Turbine} + \Sigma h_{loss} \qquad 12.1$$

Open Channel

$$H_{T1} = H_{T2} + \Sigma h_{loss} \qquad 12.2$$

$$H_{T1} = y_1 + h_{z1} + \alpha_1 V_1^2/2g = H_{T2} + \Sigma h_{loss} = y_2 + h_{z2} + \alpha_2 V_2^2/2g + h_f + h_e \qquad 12.3$$

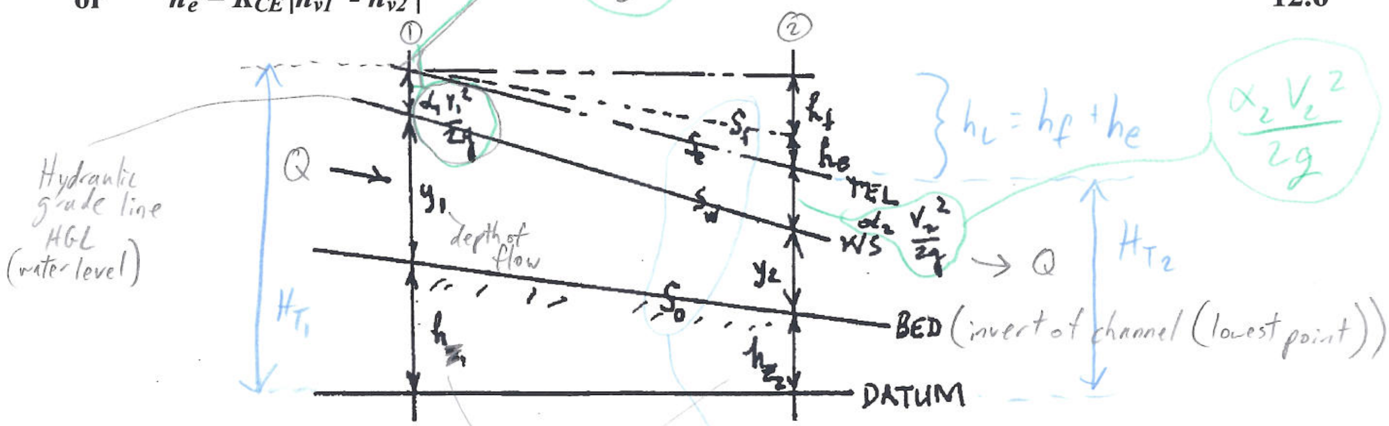
Friction Loss = h_f

e.g. $$h_f = L\,S_f = L\,\{V\,n/(c'R^{2/3})\}^2_{\;average} \qquad 12.4$$

Eddy Loss = h_e

$$h_e = K_{ce}(V_1 - V_2)^2/2g \qquad 12.5$$

or $$h_e = K_{CE}|h_{v1} - h_{v2}| \qquad 12.6$$



Defining Diagram

Pressure term in open channels:

$\Sigma F_{y'}' = N - W\cos\theta = \cancel{0} = pL - W\cos\theta$

$\rightarrow p = \frac{W\cos\theta}{L} = \frac{\gamma(L\cdot d\cdot 1)\cos\theta}{L}$

$p = \gamma d \cos\theta$

or pressure head @ bed: $\frac{p}{\gamma} = d\cos\theta$

Pressure term in open channels:

Sloping channels

"steep channel"

normal force — pressure — Area

$$N = p\,L\times 1 = W\cos\theta$$

$$p = \frac{W\cos\theta}{L} = \frac{\gamma L d\cos\theta}{L}$$

$$\frac{p}{\gamma} = d\cos\theta$$

$\Sigma F_x = P_1 - P_2 - F_f + W\sin\theta$
$= M_2' - M_1'$

If $d_1 = d_2$ then P_1 & P_2; M_1' & M_2' cancel out → $F_f = W\sin\theta$

needs to be accounted for

y_1 becomes: $d_1\cos\theta_1$
y_2 ": $d_2\cos\theta_2$

Convex vertical curve

$F_c = ma_c$ — centripetal force = mass × rotational accel.

$$N = p\,A_n = W - \frac{WV^2}{gr}$$

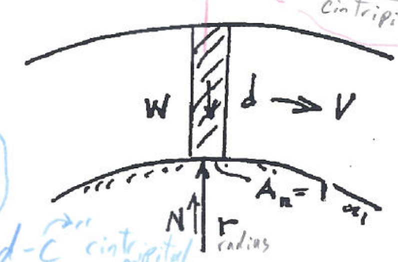
$$p\times 1 = \gamma d\times 1 - \frac{(\gamma d\times 1)}{g}\frac{V^2}{r}$$

$$\frac{p}{\gamma} = d - \frac{dV^2}{gr} = d\left(1-\frac{V^2}{gr}\right)$$

"$L = d - C$" "centripetal effect"

$\Sigma F_y = \cancel{0} = F_c + N - W$
$= \frac{W}{g}\frac{V^2}{r} + p - W$
$= \frac{\gamma d V^2}{gr} + p - \gamma d$ →

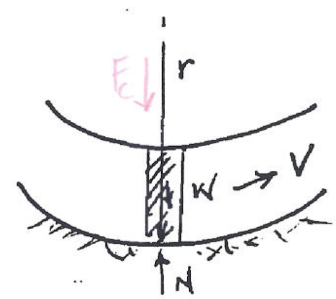
$F_c = m\frac{V^2}{r}$
$= \frac{W}{g}\frac{V^2}{r}$



Concave vertical curve

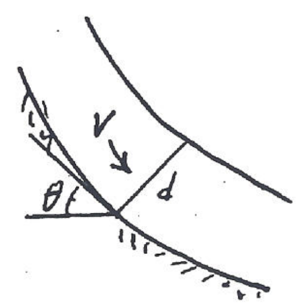
$$N = p\,A_n = W + \frac{WV^2}{gr}$$

$$\frac{p}{\gamma} = d + \frac{dV^2}{gr}$$



Combined slope and vertical curve

$$\frac{p}{\gamma} = d\cos\theta \pm \frac{dV^2}{gr}$$

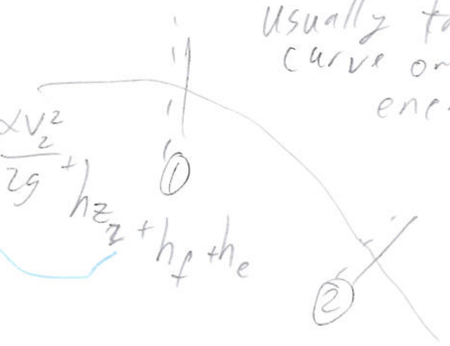


general formula for curved bed $\left(\frac{p}{\gamma}\right)_{bed} = d\cos\theta \pm \frac{dV^2}{gr}$ (+) (−)

Energy Equ.

$$H_{T_1} = H_{T_2} + h_e + h_f$$

$$\underbrace{\overbrace{d_1 \cos\theta_1}^{y_1} + \frac{\alpha V_1^2}{2g} + h_{z_1}}_{H_{T_1}} = \underbrace{\overbrace{d_2 \cos\theta_2}^{y_2} + \frac{\alpha V_2^2}{2g}}_{H_{T_2}} + h_{z_2} + h_f + h_e$$



usually take curve out of energy equ.

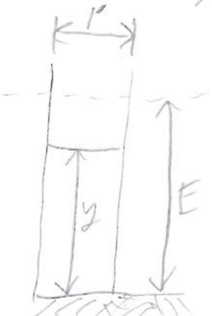
Use E as specific energy
E at local bed

$$E = H_T - h_z = d\cos\theta + \frac{\alpha V^2}{2g}$$

* Assume $\cos\theta \simeq 1$ & $\alpha = 1$

$$\rightarrow E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2} \quad \left(\begin{matrix}\text{if } Q = \\ \text{cost.}\end{matrix}\right) = f(y)$$

Flow in rectangular channel 1' wide:



flow per unit weight $\left[\frac{ft^3}{s \cdot ft}\right]\left[cfs/ft\right]$

$A = 1y = y$; $Q = q$; $E = y + \frac{q^2}{2gy^2}$

$\rightarrow y^2 E = y^3 + \frac{q^2}{2g} \rightarrow \boxed{y^3 - y^2 E + \frac{q^2}{2g} = \emptyset}$

use Picard's method (interative method) or goal seek

if $q = 3$ cfs/ft

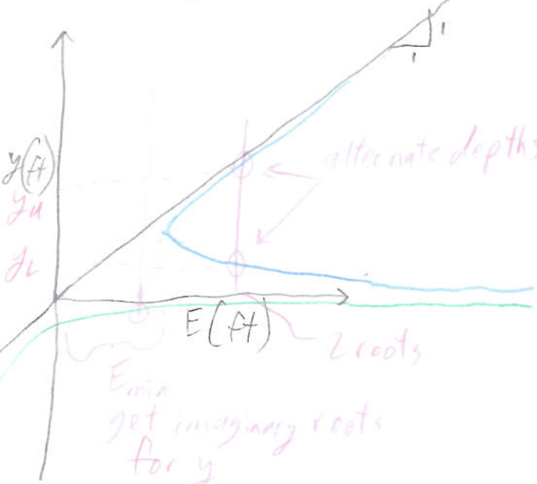
---

how E varies w/ y

$E = y + \frac{q^2}{2gy^2}$

Positive y
Neg y

$E_1 \sim E_0$ even though different depths



Minimum E corresponds to critical flow

$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2}$ {Q = const} $\frac{dA^{-2}}{dy} = \frac{-2}{A^3}\frac{dA}{dy}$

$\frac{dE}{dy} = \frac{dy}{dy} + \frac{Q^2}{2g}\left(\frac{dA^{-2}}{dy}\right) = 1 - \frac{2}{A^3}\frac{Q^2}{2g}\frac{dA}{dy} = \varnothing$

mean depth $= D = \frac{B}{A}$

$dA = B\,dy$ or $\frac{dA}{dy} = B$

$1 = \frac{Q^2}{A^2}\frac{B}{Ag} = \frac{V^2 B}{gA} = \frac{V^2}{gD}$ (root both sides)

$1 = \frac{V}{\sqrt{gD}} = N_F$ for critical flow

Re-statement of energy equation:

(See defining diagram. Neglecting curvature effects.)

$$d_1 \cos\theta_1 + hz_1 + \alpha_1 V_1^{\,2}/2g = d_2 \cos\theta_2 + hz_2 + \alpha_2 V_2^{\,2}/2g + h_f + h_e \qquad \textbf{12.11}$$

Critical Velocity $= V_c = \sqrt{gD_c}$

$E_{min} = E_c = y_c + \frac{V_c^2}{2g} = y_c + \frac{(\sqrt{gD_c})^2}{2g} = y_c + \frac{D_c}{2}$

$Q_c = V_c A_c = \sqrt{gD_c}\,A_c$

Simplifications:

<u>Specific Energy</u>

Specific Energy (E) is the energy head with respect to the bed.

E $= d\cos\theta + \alpha V^2/2g$

Assuming $\alpha \sim 1$ and θ very small, we have

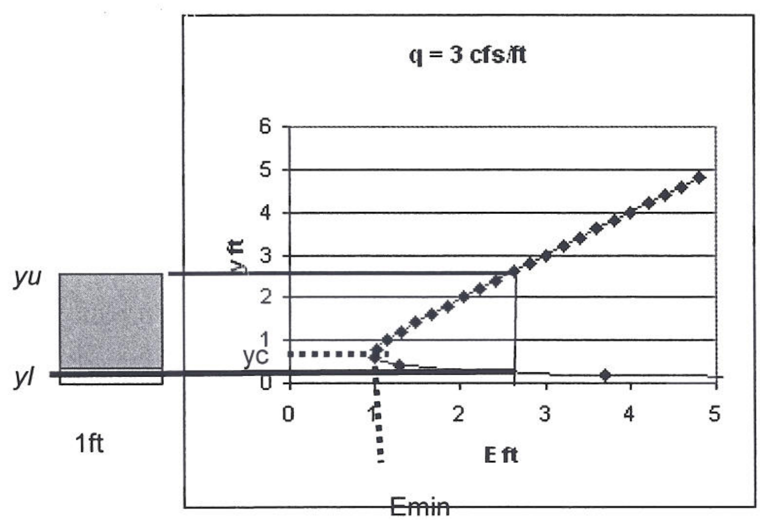
$E = y + V^2/2g$

or $E = y + (Q/A)^2/2g$ $E = y + \frac{q^2}{2gy^2}$

For a channel of unit width A→y and Q→ q then $E = y + (q/y)^2/2g$

This is a cubic equation with 3 roots (y_u, y_l and y-), Only the two real roots are of interest roots (y_u, and y_l-) .

Note: No real roots exist for E< Emin. At E = Emin there is only one real root (roots (yc-).



Example of a Specific Energy Curve for q = 3 cfs/ft in a 1 ft wide channel.

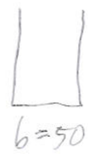
(given) Const. Q — critical — given Geometry

Find y_c, E_c

$Q_c = Q = \sqrt{g D_c}\, A_c \rightarrow \sqrt{D_c}\, A_c = \frac{Q_c}{\sqrt{g}}$

For rectangular chanel $d_c = y_c$ & $A_c = y_c b$

given: $Q = 1000$ cfs

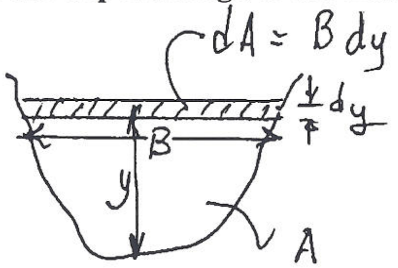


Find y_c: $\frac{Q_c}{\sqrt{g}} = \frac{1000}{\sqrt{32.2}} = \sqrt{y_c}\, y_c\, b \rightarrow y_c =$

$E_c = y_c + \frac{D_c}{2} = $ (b/c rect. chanel) $= y_c + \frac{y_c}{2} = \frac{3}{2} y_c$

Minimum E:

Find the depth that gives the minimum E for a given Q and cross-section.



Alternate depths:

Subcritical (y_u): $N_F < 1$ and $y > h_v$

***Picard Method*:**

Assume $y^{(0)} \sim E$ then $V^{(0)} \sim [Q/A]$

$y^{(1)} \sim E - V^{(0)\,2}/2g$ and $V^{(1)} \sim [Q/A]$

$y^{(2)} \sim E - V^{(1)\,2}/2g$ and $V^{(2)} \sim [Q/A]$

etc until $y^{(n+1)} \sim\rightarrow y^{(n)}$

Supercritical (y_l): $N_F > 1$ and h_v *tends to dominate over y*

***Picard Method*:**

Assume $y^{(0)} \sim$ between 0 and yc , e.g. yc/2 where yc ~ $(q\text{\^{}}2/g)^{1/3}$ and q ~ Q/B

$V^{(0)} \sim [2g(E - y^{(0)})]^{1/2}$ and $A^{(1)} \sim Q/V^{(0)}$ and $y^{(1)} \sim fcn(A^{(1)})$

$V^{(1)} \sim [2g(E - y^{(1)})]^{1/2}$ and $A^{(2)} \sim Q/V^{(1)}$ and $y^{(2)} \sim fcn(A^{(2)})$

etc until $V^{(n+1)} \sim\rightarrow V^{(n)}$

Critical Flow Concepts:

Constant Q see Back of p. 91

Constant E $E_c = E$ = given @ 9'

geometry

b=50

Find y_c & Q_c

$$E_c = y_c + \frac{D_c}{2} = (\text{b/c rectangular}) = \frac{3}{2} y_c = 9 \rightarrow y_c = 6'$$

$$Q_c = \sqrt{g}\sqrt{D_c}\, A_c = \sqrt{32.2}\sqrt{6}\,(6)(50) = 4170$$

(D_c = y_c b/c rectangle)

need to maintain a slope to prevent backwater buildup that would raise water level upstream

Critical Slope (to maintain critical flow (aka not let it slip back to subcritical))

Critical flow is max amt. of flow you can have for a given amt of energy (why you want to be in critical)

Manning's equ: $Q_c = \frac{c'}{n} A_c R_c^{2/3} S_c^{1/2} \rightarrow S_c = \left(\frac{Q_c n}{c' A_c R_c^{2/3}}\right)^2 = \left(\frac{n\sqrt{g}\sqrt{D_c}}{c' R_c^{3/2}}\right)^2$

$Q_c = A_c \sqrt{D_c}\sqrt{y_c}$ (plug in)

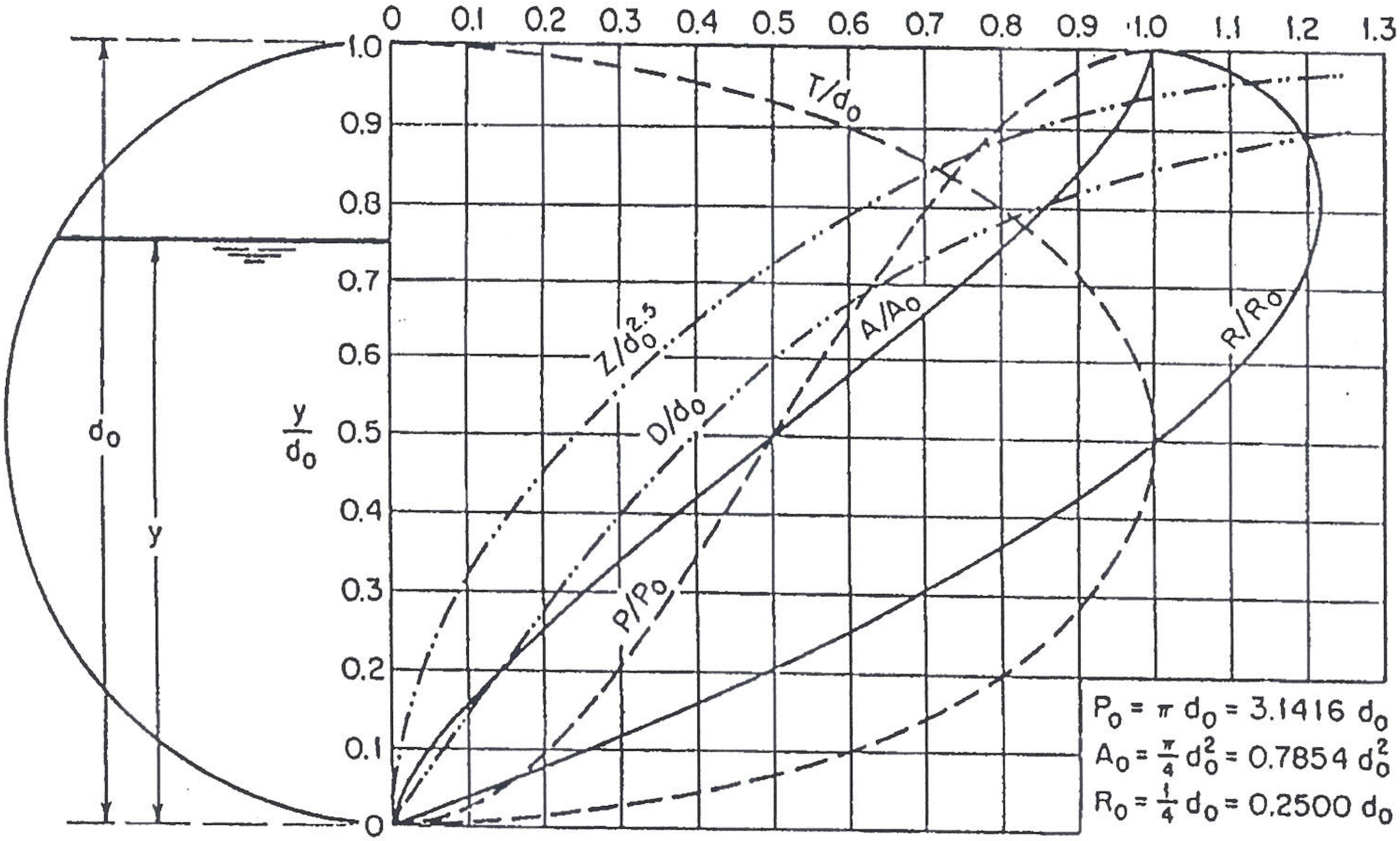
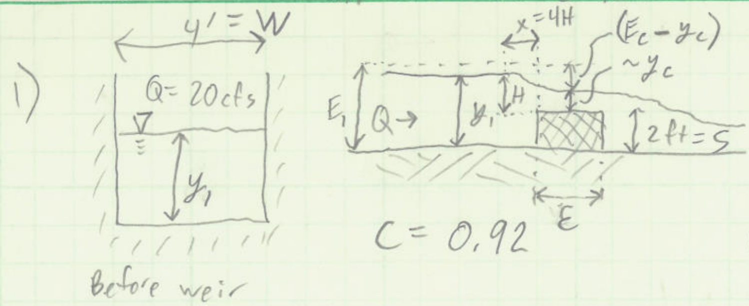


FIG. 2-1. Geometric elements of a circular section. ven te. Chow (1959)

1)



$V_c = \sqrt{g y_c}$

$y_c = \sqrt[3]{\frac{\left(\frac{Q}{W}\right)^2}{g}}$

$Q = W_c \sqrt{g} \left\{ 2 \frac{E_c}{3} \right\}^{3/2}$ equ. 13.3

$Q = 0.385\, C\, L (2g)^{1/2} H^{3/2}$ equ. 13.4

(H measured @ x)

p. 386 equ. 12.62

$Q = A_c V_c = W y_c\, g^{1/2} y_c^{1/2} = W \sqrt{g}\, y_c^{3/2}$

$\rightarrow y_c = \sqrt[3]{\frac{Q^2}{W^2 g}} = \sqrt[3]{\frac{20^2}{4^2 (32.2)}} = \boxed{0.9191 \text{ ft}}$

$E_c = H = \frac{3}{2} y_c$ (equ. 12.63 p. 386) $= \frac{3}{2}(0.9191) = \boxed{1.63 \text{ ft}}$

$E_1 = E_c + S = 1.63 + 2 = \boxed{3.63} = y_1 + \frac{V^2}{2g} = y_1 + \frac{Q^2}{2gA^2} = y_1 + \frac{Q^2}{2g(y_1(W))^2}$

$3.63 = y_1 + \frac{20^2}{2(32.2) y_1^2 4^2} = y_1 + \frac{1}{y_1^2}(0.3882)$

$y_1^3 - 3.63 y^2 + 0.3882 = \emptyset \qquad y_1 = \begin{Bmatrix} -0.3137 \\ +0.3437 \\ +3.6 \end{Bmatrix}$

choose $y_1 = \boxed{3.6 \text{ ft}}$ (length) 10

using equ 13.4 10

$H = \left(\frac{Q = 20}{0.385 (0.92)(4) \sqrt{2(32.2)}} \right)^{2/3} = \underline{1.457 \text{ ft}}$ 8

$y_1 = H + S = \boxed{3.457 \text{ ft}}$ 8

∴ equ 13.4 produced a slightly smaller y_1 than equ 13.3 under these conditions.

2) 12" Parshall flume ; $H_a = 32" = 2.67$ ft

8

$$Q = 4 W_c H_a^{1.522 w^{0.026}} = 4(1)(2.67 \text{ ft})^{1.522(1)^{0.026}} = 10.8 \text{ cfs}$$

✓ 17.8 ✗

---

3) sharp-crested weir, $L = 2.5$ ft, crest = 2 ft from bed. (S)
channel = 4 ft wide, $Q = 20$ cfs.

$$K = 0.4 + 0.05 \frac{H}{S} \qquad K_c = K\left(1 - 0.2 \frac{H}{L}\right)$$

$$Q = K_c L \sqrt{2g}\, H^{3/2} = \left[0.4 + 0.5 \frac{H}{2}\left(1 - 0.2 \frac{H}{2.5}\right)\right](8.025)\left(H^{3/2}\right) = 20$$

$$0.4 + 2.006 H^{5/2}\left(1 - \frac{H}{0.08}\right) = 20 = 0.4 + 2.006 H^{5/2} - 25.08 H^{7/2}$$

$$629 H^7 - 4.024 H^5 + 384.16 = \emptyset$$

www.wolframalpha.com

$$H = 0.207603 + 0.907663i$$
$$= \boxed{1.115 \text{ ft}}$$ ✗ 1.90'

$$y_1 = E_1 = H + S = 1.115 + 4 \; (2) = \boxed{5.115 \text{ ft}}$$

8

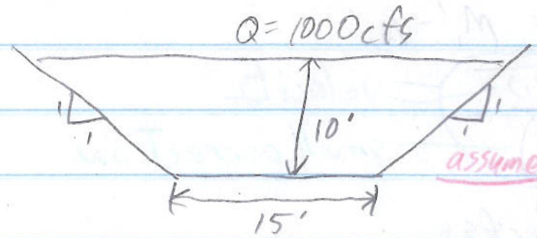
Hydro. Oct. 12 ①

Review for Test 1

$R_n = \frac{RV}{\nu}$ $\frac{\text{Hydro. radius Vel.}}{\text{Kin. viscosity}}$ $= 2.3 \times 10^6$ Turbulent

$N_F = \frac{V}{\sqrt{gD}}$ $= 0.26$ subcritical

Ex



Laminar or turbulent

Subcritical, supercritical, or critical

assume: $\nu = 10^{-5}\ ft^2/s$

Continuity

Energy

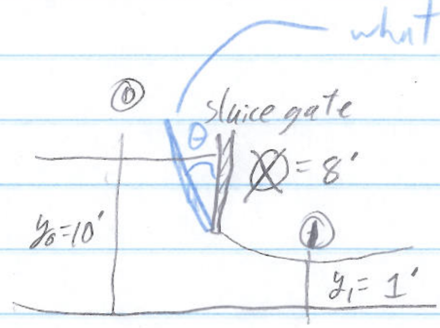
Momentum

$A = 250 ft^2 \therefore V = 4\ ft/s$

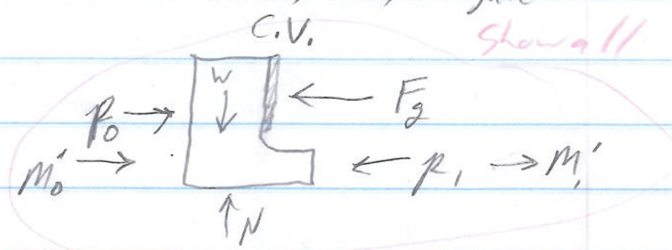
$B = b + 2zy$ Top width $B = 35$ wetted Perimeter = 43.3

$P = b + 2y\sqrt{1+z^2}$ mean depth $D = 7.14$ Hydraulic Radius = 5.77

Ex



Find V_1, Q, & F_{gate}



Continuity

$Q = y_1 w V_1 = y_0 w V_0$

$V_1 = \frac{Q}{y_1 w}$; $V_0 = \frac{Q}{y_0 w}$

$\Sigma F_x = P_0 - F_g - P_1 = \Delta M' = \rho Q (V_1 - V_0)$

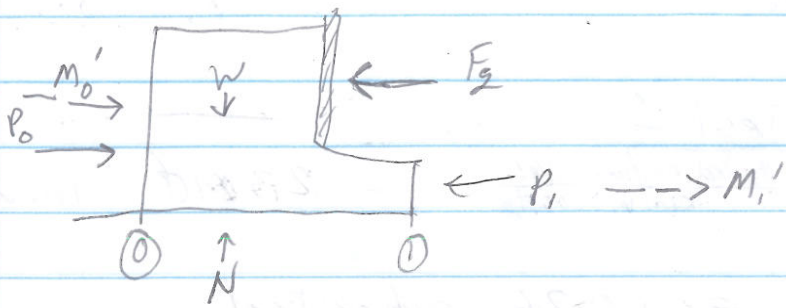
$P = \frac{1}{2}\gamma y^2 w$

Energy

$H_{T_1} = H_{T_2} + h_L$ (→ 0)

$y_0 + \frac{V_0^2}{2g} = y_1 + \frac{V_1^2}{2g}$

OVER



$$\Sigma F_x = P_0 - P_1 - F_g = M_1' - M_0'$$

$$Q = A_1 \sqrt{\frac{2g(y_0-y_1)}{\left(1-\left(\frac{y}{y_0}\right)^2\right)}}$$

velocity

small correction

$= 8(24.2) = \underline{193.6\ cfs}$

$V_1 = 24.2\ ft/s \qquad V_0 = 2.42$

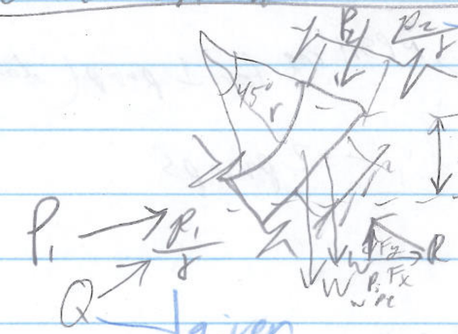
$F_g = 16.5^K$

Pump problem
- parallel & series

Hydro, Oct. 12 (2)

Three reservoir problem

Force on an elbow



$h_L \sim \phi$

if cross sect. const.
$V_1 = V_2$

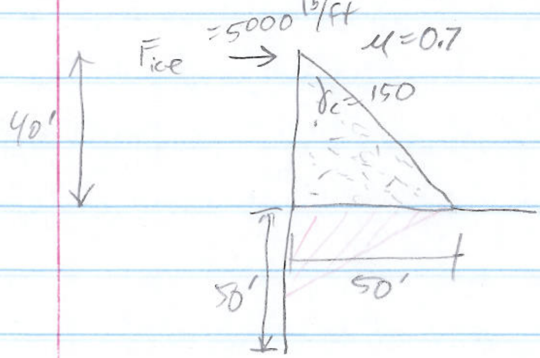
$Ɐ = A_p r \theta^{rad}$ $\quad W = \gamma Ɐ$

$P_1 = p_1 A_p$ $\quad P_2 = p_2 A_p$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + \Delta Z$$

$$V_1 = V_2 = \frac{Q}{A}$$

Hydrostatics (Force on gate & stability of Dams)



Find F_{os} for sliding (= 1.65 (safe)) & overturn (= 3.2 (safe))

is there tension ← $X_N = 26.6 > \frac{50}{3}$ (safe)

no tail water

Set up table

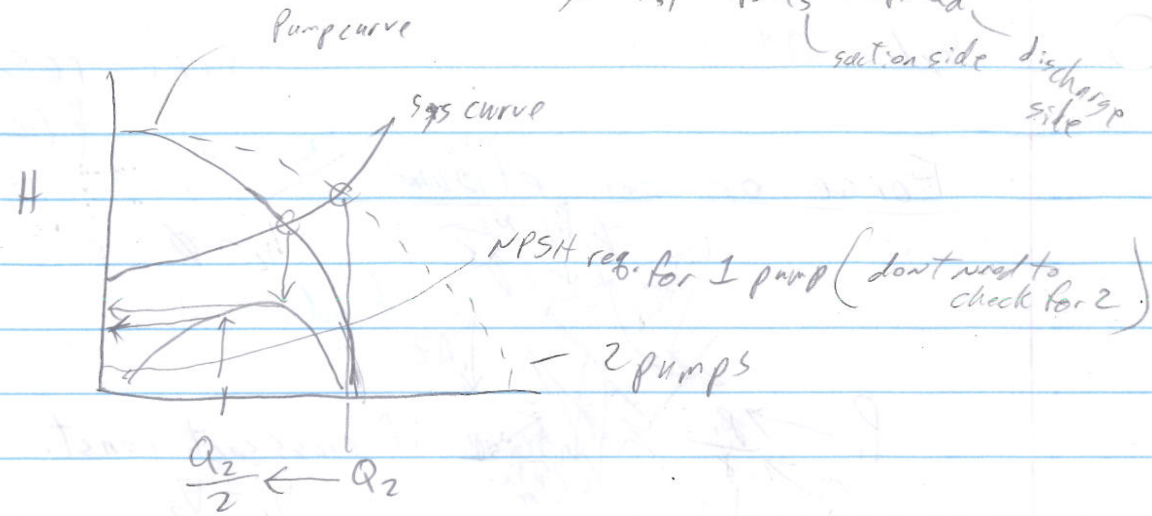
F	F_x	F_y	M / arm	↺+	−↻

$\phi = \phi_1 + \frac{S}{S_B} \Delta\phi$

OVER

$H_{sys} = H_{st} + K_{ps} Q_s^2 + K_{ps} Q_d^2$

static lift (for H_{st}); suction side (for $K_{ps} Q_s^2$); discharge side (for $K_{ps} Q_d^2$)

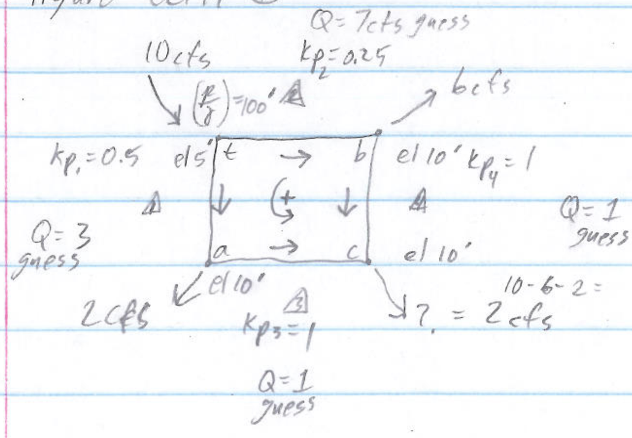


$$NPSH \Rightarrow \quad < \frac{P_{atm}}{\gamma} - \frac{P_{vapor}}{\gamma} - lift - K_{ps} Q_s^2$$

(34) above $\frac{P_{atm}}{\gamma}$; (1) above $\frac{P_{vapor}}{\gamma}$

lift: wet well to pump

Hydro oct 14 ①



$$\Delta Q = \frac{-\left[0.5(3)^2 + (1(1)^2) - 1(1)^2 - 0.25(7)^2\right]}{2(0.5)(3) + 2(1)(1) + 2(1)(1) + 2(0.25)(7)} \quad +0.73$$

New: $Q_1 = 3.73$, $Q_3 = 1.73$, $Q_2 = 6.27$, $Q_4 = 0.27$

@ terminal point

$\frac{p_t}{\gamma} = 100'$, $EL = 5'$

(flow is from t to a so h_L = pos)

$H_t = \frac{p_t}{\gamma} + EL_t = 105'$

$H_a = \frac{p_a}{\gamma} + EL_a = H_t - h_{t-a} = 105' - (0.5(3.73)^2) = 98'$

$\rightarrow \frac{p_a}{\gamma} = H_a - EL_a = 98 - 10 = 88'$

* Do the same to find p @ other points

$H_b = H_t - k_{p_2} Q_2^2 = \frac{p_b}{\gamma} + EL_b \rightarrow \frac{p_b}{\gamma} = 85.2'$

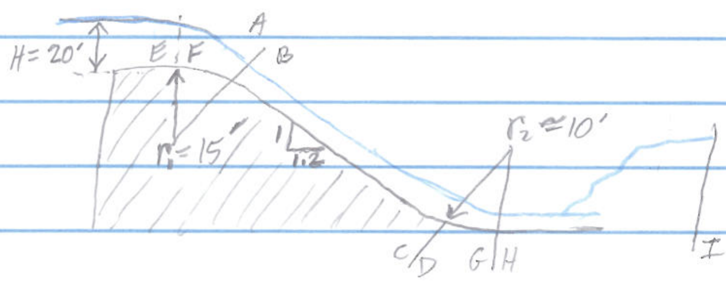
$H_c = H_a - k_{p_3} Q_3^2 = 98 - (1)(1.73)^2 = 95'$

$\frac{p_c}{\gamma} = H_c - EL_c = 95 - 10 = 85'$

Lecture 15
p.103

4) Est. press. head @ channel bed along spillway.
Find values @ A, B, H & I. Assume $\alpha = \beta = 1$; $d_B = d_A$;
$d_C = d_D$; $d_F = d_E$; $d_G = d_H$; No head loss A→H.

C = 0.5



Oct. 26

Lecture 15
Uniform Flow

Uniform Flow: is flow in a prismatic channel where the depth and velocity are constant along the channel. The depth in uniform flow is called the normal depth (y_n)

Vertical Distribution of Velocity in an Open Channel

Assuming Uniform Flow the form of the velocity distribution is approximated by Eq. 15.1 as shown in Figure 15.1.

$Q = Q_c = V_c A_c = \sqrt{g y_c}\, y_c W$

$y_c = \sqrt[3]{\frac{(Q/w)^2}{g}}$

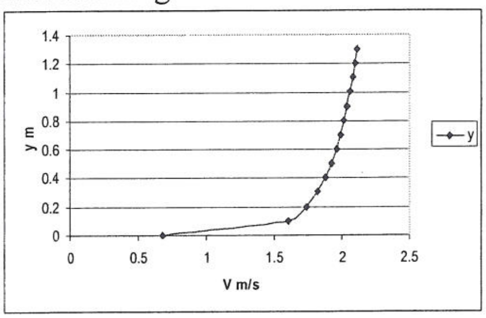
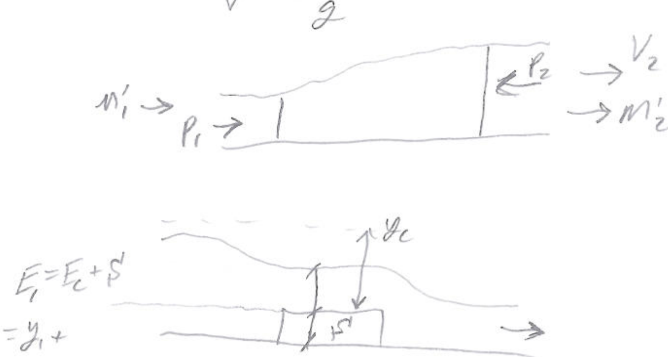


Fig 15.1



Vertical distribution

$$u = u^*/\kappa \{\ln y/y_o\} \qquad 15.1$$

where u^* = shear velocity = $(\tau_o / \rho)^{1/2}$; $\kappa = 0.4$; bed shear = $\tau_o = \gamma \boldsymbol{R} \boldsymbol{S_o}$

$u^* = \sqrt{g R S_o}$?

for smooth bed y_o = 1/9 (ν/u^*)

For rough bed $y_o = k_s/30$ where k_s = bed roughness~ D65

In real channels the actual velocity distribution is affected by the secondary flow due to channel curvature and the presence of the side walls. For narrow channels the maximum velocity occurs below the water surface.

Secondary currents

Secondary can be caused by channel curvature as shown below. In this flow the water surface is tilted to create a pressure force to compensate for the centrifugal force. The super-elevation of the water surface on the outside of the curve is given approximately by

$$\Delta z \sim \tfrac{1}{2}\ w V^2/(g\, r_c) \qquad 15.2$$

OVER

Momentum General equ (prismatic channel)

$$P_1 - P_2 - F_f + W\sin\theta = M_2' - M_1'$$

↳ Ø if channel ~ flat

↳ usually ~ Ø

$$P_1 - P_2 = M_2' - M_1' \text{ (separate by knows)}$$

$$\rightarrow P_1 + M_1' = P_2 + M_2'$$

Assume $\beta \sim 1$

$$\gamma A_1 y_{\text{center of gravity of } A_1} + \rho Q V_1 = \gamma A_2 y_{cog_{A_2}} + \rho Q V_2$$

$Q = V_1 A_1 = V_2 A_2$

$$\gamma A_1 y_{cog_1} + \rho \frac{Q^2}{A_1} = \gamma A_2 y_{cog_{A_2}} + \rho \frac{Q^2}{A_2}$$

÷ thru by γ

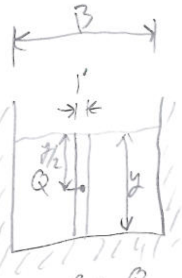
$$A_1 y_{cg_1} + \frac{Q^2}{gA_1} = A_2 y_{cg_2} + \frac{Q^2}{gA_2}$$

This term is called specific force & in Volumetric units

$$\mathcal{F} = \text{specific Force} = A y_{cg} + \frac{Q^2}{gA}$$

$$\mathcal{F}_1 = \mathcal{F}_2$$

if Momentum is constant, $\theta \sim Ø$, $\beta = 1$, $F_f \sim Ø$, prismatic channel (keeps same shape)



$q = \frac{Q}{B}$

Area × y_{cg}

$$\frac{y_1^2}{2} + \frac{q^2}{gy_1} = \frac{y_2^2}{2} + \frac{q^2}{gy_1}$$

use goal seek

To solve anylitically

create non unit variable so it will apply to all situations if you know it:

$$\phi = \left(\frac{y_2}{y_1}\right) \rightarrow \frac{2y_2}{y_1^3}$$

$$\phi^3 - (1 + 2N_{F_1}^2)\phi + 2N_{F_1}^2 = Ø$$

$$\left(N_{F_1}^2 = \frac{q^2}{gy_1^3}\right)\left(N_{F_1} = \sqrt{\frac{q^2}{gy_1^3}}\right)$$

÷ by $(\phi - 1)$ to turn it into a second degree instead of 3rd degree

$$\phi^2 + \phi - 2N_{F_1}^2 = Ø$$

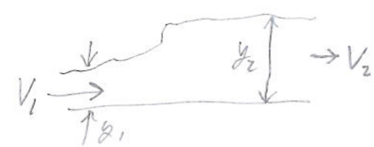
$$\phi = \frac{y_2}{y_1} = \left(\sqrt{1 + 8N_F^2} - 1\right)\frac{1}{2}$$ ☆

Hydraulic Jump

y_1 = inital depth, y_2 = sequent depth

together they are the conjugate depth

if y_1 is super critical then y_2 will be subcritical



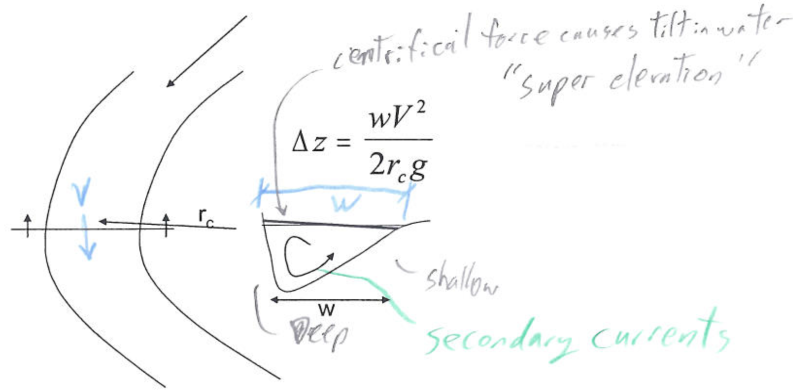


Fig 15.2 Secondary Flow

Uniform Flow Friction Equations

Uniform flow occurs in a prismatic channel when the friction and gravity forces are in equilibrium. This requires that $x > x_{est}$ or the turbulent boundary layer = $\delta_t = y$. Figure 15.3 shows the force balance for uniform flow.

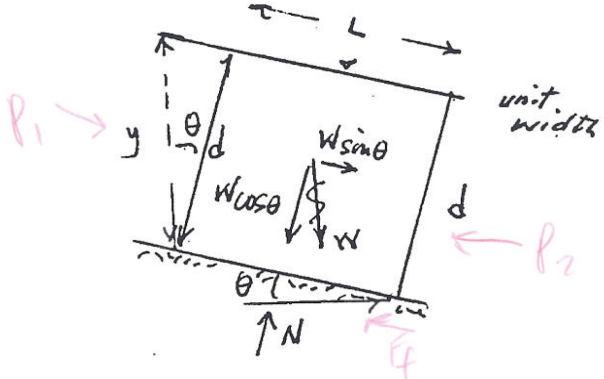


Figure 5.3 Uniform Flow in a Prismatic Channel

The force balance in the x-direction leads to

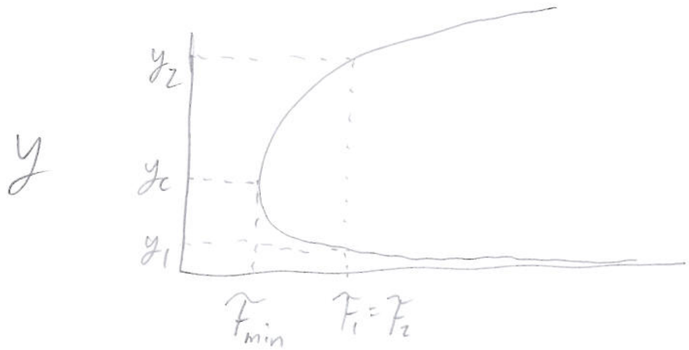
$$W\sin\theta = \tau_o PL \qquad 15.3$$

where τ_o = the average shear stress on the boundary P.

Equation 5.1 and the turbulent shear relationship

$$\tau_o = K\rho V^2 = \rho (gRS_o) \qquad 15.4$$

Not same as E curve b/c does not go off on a 45°



$$\mathcal{F} = \frac{y}{2} + \frac{q^2}{gy}$$

$$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8N_{F_1}^{\,2}} - 1\right)$$

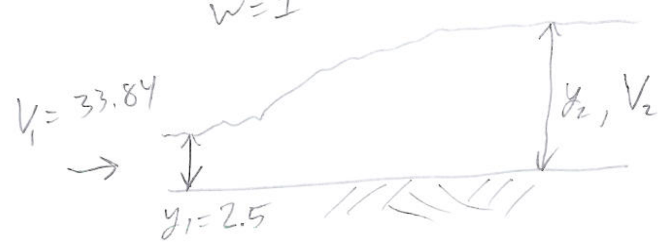
for assumptions $\beta = 1$, $F_f = 0$, 0 slope bed, prismatic

---

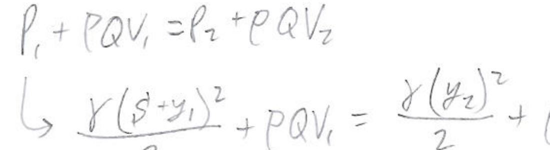
Example

Find E_1, E_2, $h_L = E_1 - E_2$, y_2, V_2

$w = 1$



Example "Hydro Jump @ a step"



(Bed no longer flat)

$$P_1 + \rho Q V_1 = P_2 + \rho Q V_2$$

$$\hookrightarrow \frac{\gamma (S + y_1)^2}{2} + \rho Q V_1 = \frac{\gamma (y_2)^2}{2} + \rho Q V_2$$

gives the Chezy friction equation

$$V = C\,(RS_o)^{1/2} \qquad 15.5$$

where $C = (g/K)^{1/2}$ 15.6

The C is a form of hydraulic conductivity and is inversely related to the degree or friction.

A commonly used alternative to the Chezy equation is the **Manning equation**; Manning's equation is based on field and laboratory data and has the form

$$V = (c'/n)\,R^{2/3}\,S_o^{1/2} \qquad 15.7$$

where n = Manning friction factor; c' = 1 for SI and 1.486 for US units. The value of n varies with the bed roughness, Reynolds Number, Froude Number, sediment transport, vegetation and channel shape (plan and section).
An simple estimate of n can be made using the Strickler Equation:

$$n = 0.034\,(D50\ \mathrm{ft})^{1/6} \qquad 15.8$$

where D50 is the median grain size on the bed.

__Note on Normal Depth:__

Uniform is defined as flow with a constant depth and constant velocity in the direction of the flow in a prismatic channel. The constant depth is called the normal depth and is denoted as y_n.

For uniform flow we use the Manning Equation with $S_f = S_o$ to solve for y_n.

For example for a trapezoidal channel we have:

$$Q = (c'/n)A\,R^{2/3}\,S_o^{1/2} = fcn(y_n) \qquad 15.9$$

where $A = y_n\,(b + z\,y_n)$

and $R = A/P$; $P = b + 2\,y_n\,(1 + z^2)^{1/2}$

Rearranging Eq. 15.9

$$C_Q = \frac{nQ}{c'\,S_o^{1/2}} = AR^{2/3} = f(y_n)$$

y_n = normal depth (const depth needed for uniform flow)

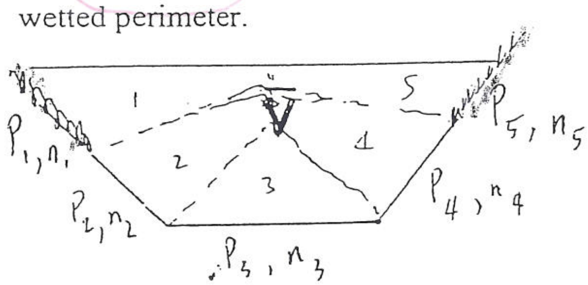
$F_f = W \sin\theta$ (friction force = gravity force for uniform flow)

Finding n in Composite Channels

Simple Channels

These are channels in which the mean velocity can be assumed to apply to the entire wetted perimeter. For example, circular, trapezoidal, triangular and parabolic. A single α and β are assumed to apply to the entire simple section.

The Manning's n for simple channels can be obtained from the following formula based on wetted perimeter.



Assumptions: $V_1 = V_2 = V_3 = \dots$

$S_1 = S_2 = S_3 = \dots$

logarithmic velocity distribution

$u \propto \ln y$

$$u = \frac{u_*}{k} \ln\left(\frac{y}{y_0}\right)$$

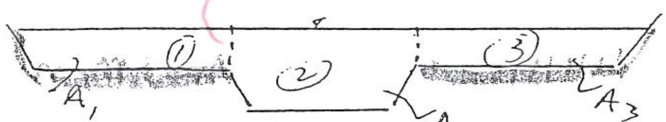
$$n = \left(\frac{\sum P_i n_i^{3/2}}{\sum P_i} \right)^{2/3}$$

Compound Channels

These are channels in which the mean velocity cannot be assumed to apply to the entire wetted perimeter. These channels can be considered to be made up of more than one simple channels. For example, rivers with flood planes are sometimes approximated by three trapezoidal sections.

The Manning's n for compound channels can be obtained from the following formula based on separating the total area into sub-areas.

Not included in P_1, P_2



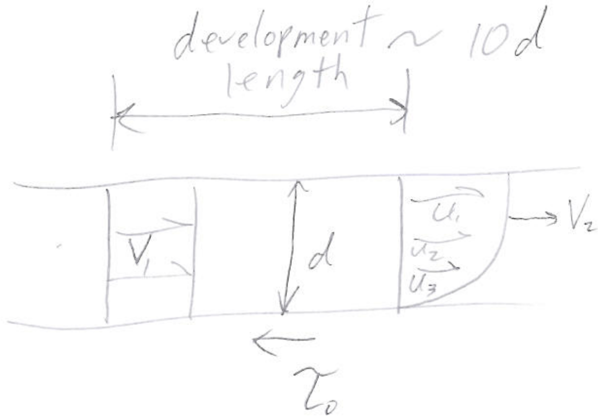
$$Q = \frac{c'}{n_T} A_T R_T^{2/3} S_0^{1/2} = \frac{c'}{n_1} A_1 R_1^{2/3} S_0^{1/2} + \frac{c'}{n_2} A_2 R_2^{2/3} S_0^{1/2} + \dots$$

$$\therefore \quad n_T = \frac{A_T R_T^{2/3}}{\sum (A_i R_i^{2/3} / n_i)}$$

The α and β for the compound section can be found from the basic definitions, i.e.

$$\alpha_T = \frac{\sum \alpha_i V_i^3 A_i}{\bar{V}^3 A_T} \; ; \quad \beta_T = \frac{\sum \beta_i V_i^2 A_i}{\bar{V}^2 A_T}$$

over


development length ~ 10d
V1
d
u1
u2
u3
V2
τ0

Lecture 16

Summary of Properties of Critical Flow

- **Froude Number = $V/(gD)^{1/2} = 1$**
- **$c = V_c = (gD)^{1/2}$**
- **E is minimum at critical depth for Q = constant**
- **For E = constant, Q is maximum at the critical depth**
- **Specific Force is minimum at Critical depth for Q = constant**
- **$E_c = E_{min} = y_c + D_c/2$** **********
- **$Q_c = V_c A_c = A_c (D)^{1/2}(g)^{1/2}$** *********
- **Critical depth is a *control* for upstream subcritical flow; downstream (supercritical region) disturbances can not propagate upstream of the critical depth section.**

Summary of Properties of Uniform Flow

- **Only possible in a prismatic channel**
- **Gravity force = Friction force**
- **Depth, y = normal depth = y_n, velocity, V, and Q are constant with x**
- **$S_o = S_f = S_e$; no eddy loss**
- **Typical computation Formulae are:**
  - **Mannings Eq.**

    $$Q = \frac{c' A}{n} (R)^{2/3} (S_f)^{1/2}$$ *********

  - **Chezy Eq.**

$$Q = C A (R)^{1/2} (S_f)^{1/2}$$ *********

$$Z = A\sqrt{D} = A\sqrt{\frac{A}{T}} \tag{2-3}$$

The *section factor for uniform-flow computation* $AR^{2/3}$ is the product of the water area and the two-thirds power of the hydraulic radius.

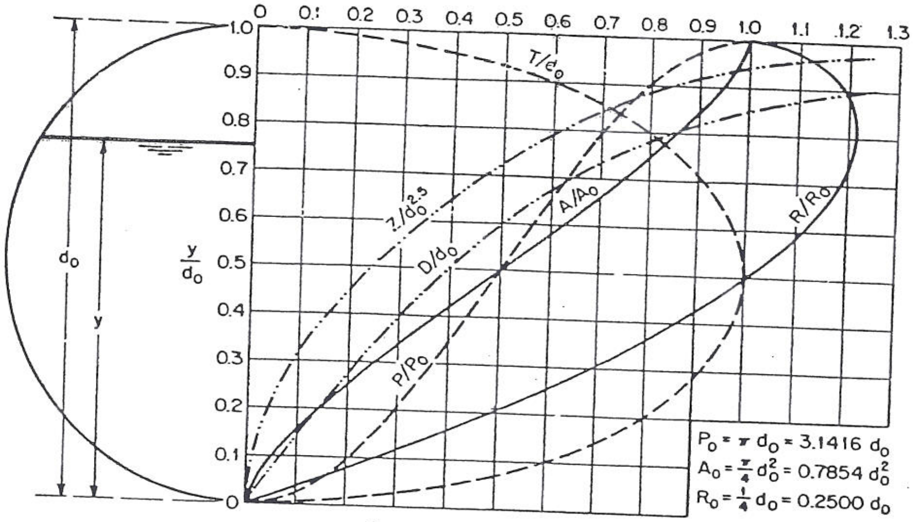


FIG. 2-1. Geometric elements of a circular section.

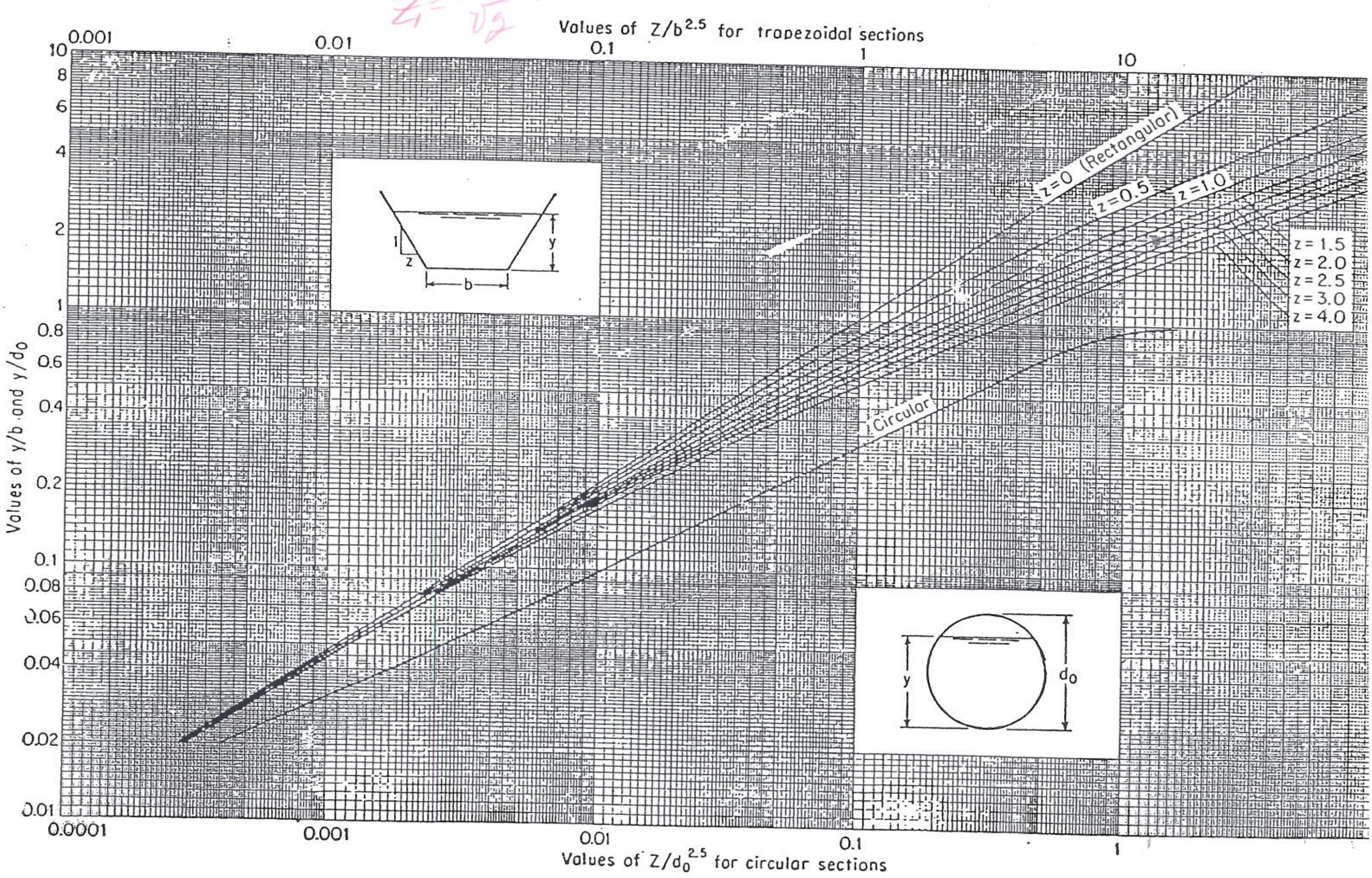
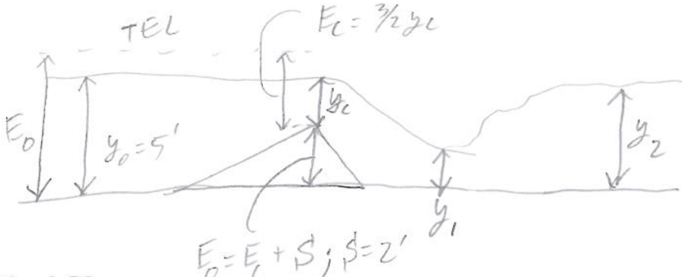


FIG. 4-1. Curves for determining the critical depth.



$y_c = \sqrt[3]{\frac{(Q/W)^2}{g}}$

Goal seek to get Q

$E_0 = y_0 + \frac{Q^2}{2g(y_0 W)^2} = E_c + S$

Then get $E_1 = y_1 + \frac{Q^2}{2g(W y_1)^2} = E_c + S$

Solve y_1, V_1, N_{F_1}

Now get

$y_2 = \frac{y_1}{2}\left(\sqrt{1+8N_F^2} - 1\right)$

Critical Slope

This is the slope required to maintain critical uniform flow;

- $S_o = S_f = S_c$;
- $y_c = y_n =$ **constant** ;
- $S_c = \{n\, Q_c/(c' A_c R_c^{2/3})\}^2$

Review Problems:

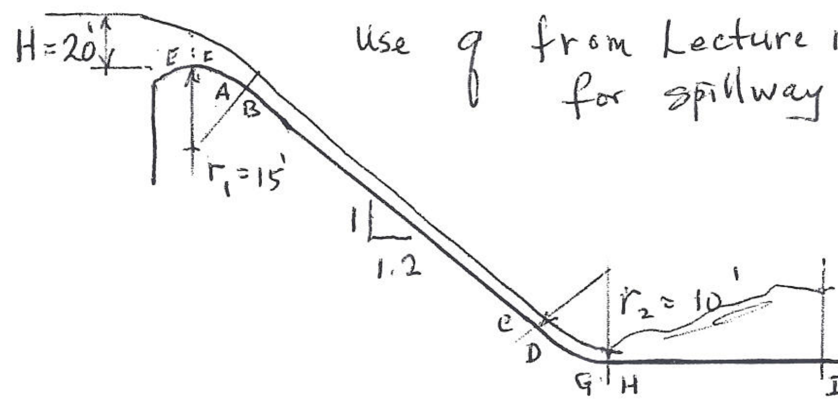
1.. Find the maximum flow through a 6 ft diameter (d_o) culvert with a specific energy of 5 ft. Assume no entrance loss and $\alpha = 1.0$. What is the critical slope if the Mannings n is 0.024?

2. Estimate the normal and critical depths in a triangular channel with side slopes of 2H:1V. Given Q = 100 cfs; n = 0.03; $S_o = 0.0009$.

3. Derive the sequent depth equation for the Classical Hydraulic Jump.

Assignment

4. Estimate the pressure head at the channel bed along the of the spillway shown below [determine values at locations A, B, H and I) .Assume: $\alpha = \beta = 1.0$; $d_B = d_A$.; $d_C = d_D$; $d_F = d_E$; $d_G = d_H$; no head loss between A and H.



Use q from Lecture 13 for spillway with c = 0.5.

6. A Parshall flume with a 12 inch throat has a discharge of 8 cfs. Find the theoretical and actual depths and specific energies at A. What is the theoretical discharge? Why is the theoretical discharge significantly less than the actual?

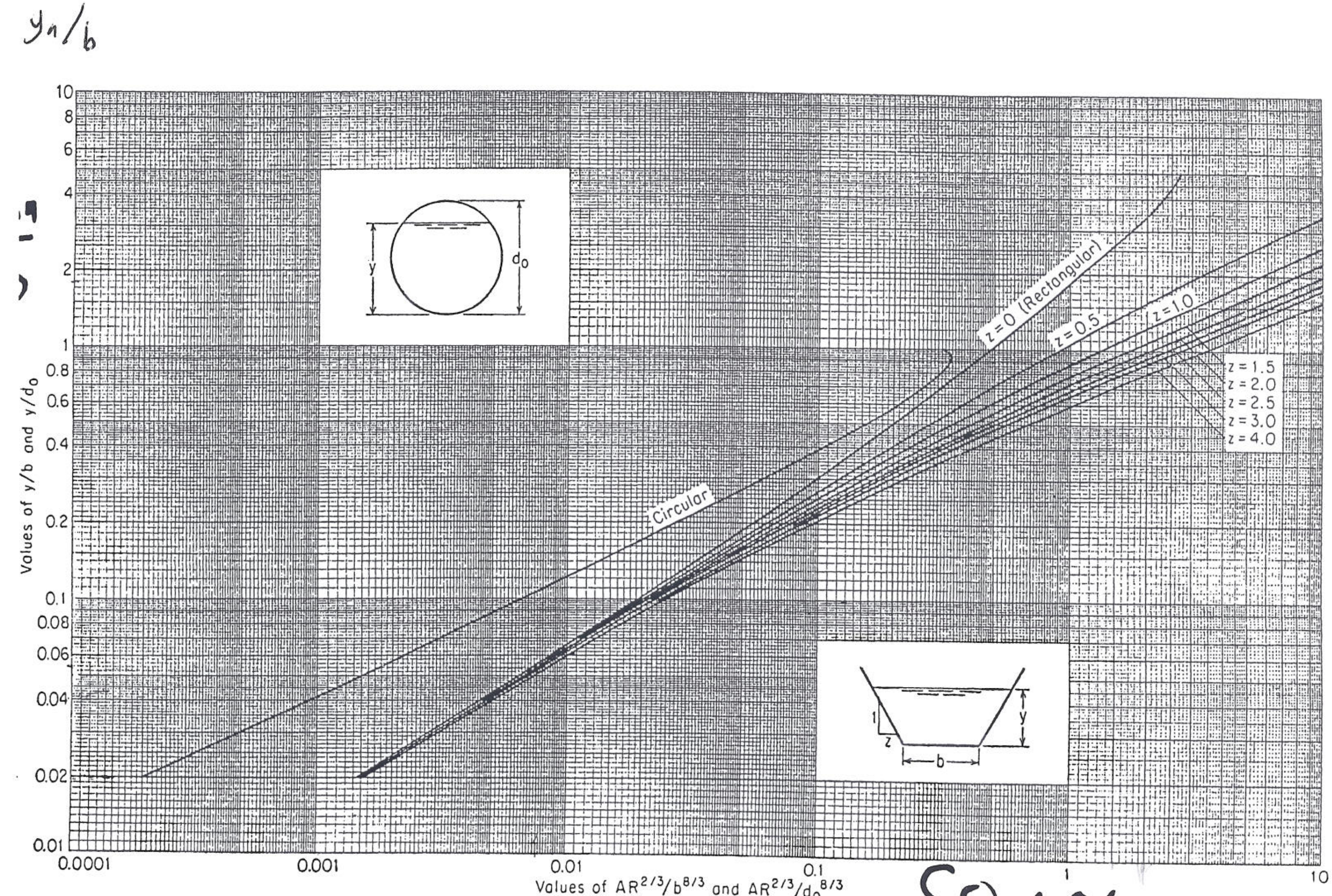


FIG. 6-1. Curves for determining the normal depth.

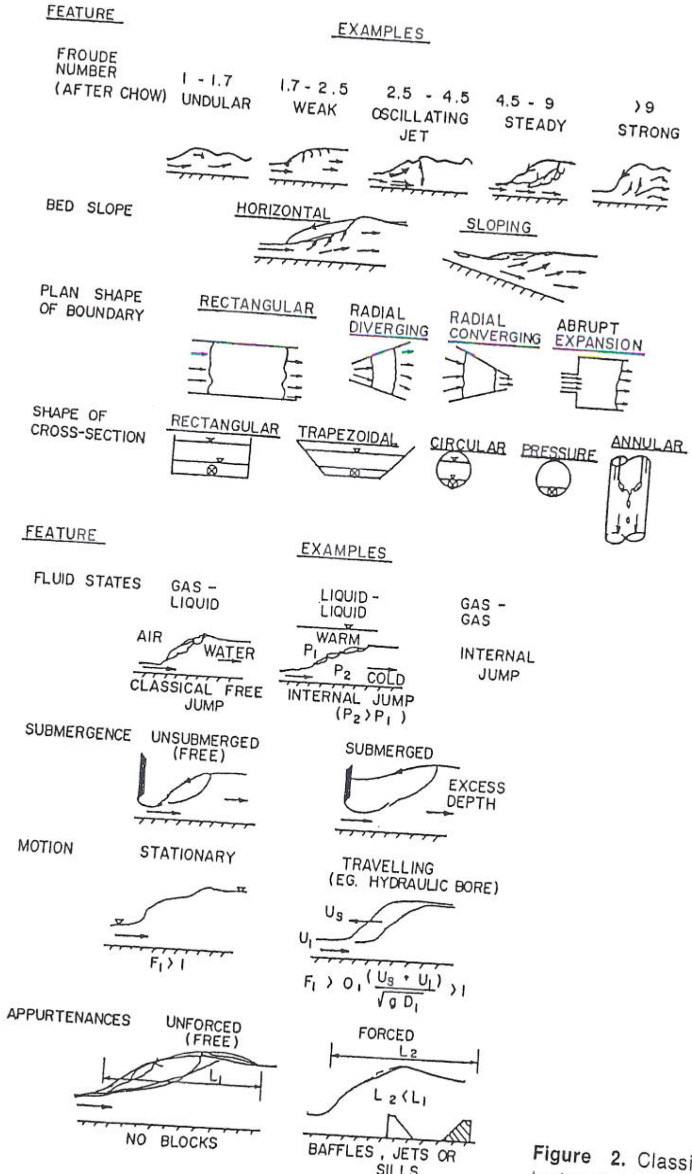


Figure 2. Classification of the hydraulic jump.

Hydraulic jumps are also classified according to the type of channel in which they occur (e.g.) prismatic channels (rectangular, trapezoidal, circular), nonprismatic (radially diverging, radially converging and curvilinear aprons) and horizontal and sloping beds).

The hydraulic jump is encountered in many engineering problems [17–24]. The most common application is the hydraulic jump stilling basin [17]. Other applications include.

1. Mixing of chemicals.
2. Entrainment of air.
3. Increasing pressure on channel aprons to reduce

Internal hydraulic jumps have been identified in wastewater clarifiers [18–19], waste heat discharges, mixing of fresh and salt water, and sediment-laden flow into lakes or reservoirs.

The abrupt surges in open channels such as the hydraulic bore [16] are in fact traveling hydraulic jumps. Traveling jumps are also believed to be an important mechanism in the transition of gravity to pressure flow in closed conduits [21, 24].

Three approaches have been used in studying the hydraulic jump phenomenon:

1. Experimental studies using physical models.
2. The momentum and mass conservation principles applied on a macroscopic basis.
3. Solution of the differential equations for continuity and momentum using numerical integration techniques.

This chapter treats the subject of hydraulic jumps in two parts, namely, macroscopic aspects of the jumps (i.e. sequent depth ratios and length characteristics) and internal flow (i.e. velocity distributions and turbulence characteristics).

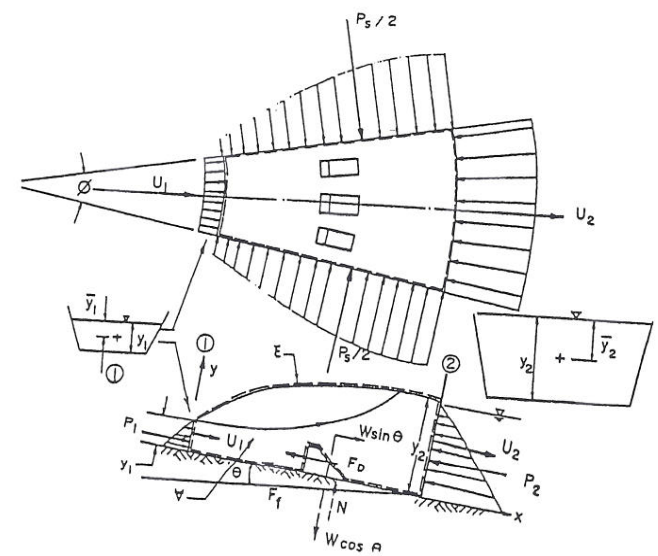
THE MACROSCOPIC APPROACH

General

Since the hydraulic jump takes place over a relatively short distance, of the order of five sequent depths, the transition is dominated by the initial momentum flux and pressure force due to the sequent depth. Boundary shear forces are secondary. Figure 3 shows an unsubmerged forced hydraulic jump in a radially diverging sloping channel. This will be used to illustrate the macroscopic approach.

Rouse, Siao, and Nagaratnam [12] have applied the Reynolds equation for turbulent flow in the form of

$$\frac{\partial(\bar{u}_i\bar{u}_j)}{\partial x_j} + \frac{\partial(\overline{u_i'u_j'})}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_j} + X_i + \frac{\mu}{\rho}\frac{\partial^2 \bar{u}_i}{\partial x_j\,\partial x_i} \qquad (2)$$



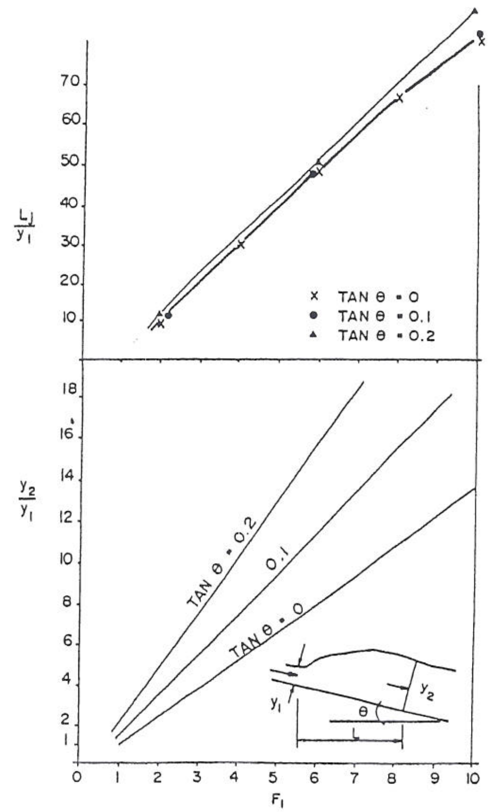


Figure 4. Jump length and sequent depth ratios for sloping rectangular channels [16].

$$\frac{L_j}{y_1} = a(N_F - 1) \quad a \sim 8 \text{ to } 10$$

$$\frac{L_j}{y_1} \approx 8N_F - 8$$

where $K_o = (KL_j/y_1) \tan \theta = f(\theta, F_1)$

An approximate solution to Equation 12 was given by [9].

$$y_o = \frac{1}{2\cos\theta}\left(\sqrt{1 + 8G_1^2} - 1\right) \tag{13}$$

where $G_1 = F_1\Gamma_1$ (14)
$\Gamma_1 = 10^{1.55\theta \text{rad}}$

Figure 4 compares the experimentally derived lengths and sequent depth ratios for horizontal and sloping hydraulic jumps. It is evident that the sequent depth increases sharply with increasing bed slope; however, the jump length in terms of y_1 is not greatly affected by θ.

A stilling basin with a sloping bed is sometimes used to accommodate uncertain or variable tailwater rating curves. In such cases, the stilling basin is designed to prevent sweep-out under the lowest tailwater levels. If higher tailwater levels are encountered, the start of the hydraulic jump will move upstream on the sloping apron. This arrangement is thought to give a more rapid reduction in the maximum velocity than would occur with a submerged hydraulic jump.

The Forced Hydraulic Jump in a Rectangular Channel

Another important case of Equation 4 is that involving appurtenances on the floor of the stilling basin. With the appropriate simplifications for a horizontal bed, the combination of Equations 4 and 6 gives

$$y_o^3 + \left[G\left(\frac{u_B}{u_1}\right)^2 F_1^2 - 2F_1^2 - 1\right] y_o + 2F_1^2 = 0 \tag{15}$$

where $G = S_B \frac{h_B^*}{y_1} C_D$ (16)

$S_B = \frac{w_B}{s_B + w_B}$ = blockage ratio (17)

$h_B^* = y_B$ or h_B whichever is less (18)

C_D = drag coefficient

s_B = baffle spacing

w_B = baffle width

h_B = baffle height

u_B = jet velocity at baffle

y_B = jet depth at baffle

The jet velocity at the baffle varies from u_1 to u_2 from the beginning to the end of the jump. McCorquodale and Regts [26] applied the momentum and continuity equations to estimate the expansion of the initial jet under the adverse pressure gradient of the forced hydraulic jump; the jet depth is

$$y_B = q/u_B \tag{19}$$

where $$\frac{u_B}{u_1} = 1 - \frac{y_o(x_B/y_1)}{\left(\frac{x_B}{y_1} + y_o\right)F_1^2}\left\{1 + \frac{1}{2}\left[\frac{y_o}{\frac{x_B}{y_1} + y_o}\right]\frac{x_B}{y_1}\right\} \tag{20}$$

x_B = distance from the initial section to the baffle

The determination of the drag coefficient on baffle blocks and sills in hydraulic jumps has been studied by Rajaratnam [6, 27], Harleman [28], Rand [29], Weide [30], Pillai and Unny [31], McCorquodale and Giratella [32], Narayanan [33], Tyagi et al. [34], and Karki [35]. Rajaratnam [9] represented the drag coefficient on a sill in a hydraulic jump as a function of the position of the wall from the start of the jump. He represented the drag force as

$$F_D = \tfrac{1}{2} C_d \rho u_1^2 h_B \tag{21}$$

where h_B = baffle height
$C_d = f(x/L_j)$

He found that C_d varied from about 0.6 at the start of the jump to about 0 at $x/L_j \simeq 0.8$; C_d then increased to about 0.12 for $x/L_j \geq 1.3$.

McCorquodale et al. [26, 32] attempted to define the drag coefficient, C_D, in terms of the baffle geometry. Thus, the drag force was defined, as in Equation 15, by

$$F_D = \tfrac{1}{2} C_D \rho u_B^2 A_B^* \tag{22}$$

where A_B^* = (area jet) ∩ (area baffle) (23)

Lecture 16 (cont'd)
Estimation Manning's n for Natural Channels.
Reference: Chapter 4 & part 10 plus handouts

Cowan Formula:

$n = (n_o + n_1 + n_2 + n_3 + n_4)\, m_5$

n_o material see Table 5.5
gravel, stone $n \simeq 0.034 (D_{50}\ ft)^{1/6}$

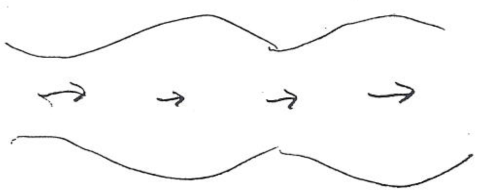
n_1 Bed form or bed irregularity

$V_o < 1\ ft/sec$ $V_1 > V_o$ ripples $V_2 > V_1$ dune $N_F < 1$ $V_3 > V_2$ $N_F \gtrsim 1$ $V_4 > V_3$

See Fig 2.

n_2

changes in cross-section affects $h_L = \frac{K(V_2 - V_1)}{2g}$



X-section changes.

Table 5.5.

n_3 Obstructions hard to predict Table 5.5

n_4 Vegetation Fig A-1

m_5 Alignment

"curvature effect"

straight
slightly meandering $1.5 < S_m \leq 2$
strongly " $S_m > 2$

Sinuosity

S_m = Sinuosity $= \frac{L_r}{L_V} < 1.5$

(L_r: flood length; L_V: Vally length)

$m \approx \sqrt[3]{S_m}$

$m_5 \approx \left(\frac{L_r}{L_V}\right)^{1/3}$

...wn wit[illegible] some weeds and brush, none of the vegetation in foliage, where [illegible] aulic radius is greater than 2 ft, and (c) growing season—bushy willows about 1 year old intergrown with some weeds in full foliage along side slopes, no significant vegetation along channel bottom, where hydraulic radius is greater than 2 ft.

(4) *Very high* for conditions comparable to the following: (*a*) turf grasses where the average depth of flow is less than one-half the height of vegetation, (*b*) growing season—bushy willows about 1 year old, intergrown with weeds in full foliage along side slopes, or dense growth of cattails along channel bottom, with any value of hydraulic radius up to 10 or 15 ft, and (*c*) growing season—trees intergrown with weeds and brush, all in full foliage, with any value of hydraulic radius up to 10 or 15 ft.

In selecting the value of m_5, the degree of meandering depends on the ratio of the meander length to the straight length of the channel reach. The meandering is considered *minor* for ratios of 1.0 to 1.2, *appreciable* for ratios of 1.2 to 1.5, and *severe* for ratios of 1.5 and greater.

In applying the above method for determining the n value, several things should be noted. The method does not consider the effect of suspended and bed loads. The values given in Table 5-5 were developed from a study of some 40 to 50 cases of small and moderate channels. Therefore, the method is questionable when applied to large channels whose hydraulic radii exceed, say, 15 ft. The method applies only to unlined natural streams, floodways, and drainage channels and shows a minimum value of 0.02 for the n value of such channels. The minimum value of n in general, however, may be as low as 0.012 in lined channels and as 0.008 in artificial laboratory flumes.

5-9. The Table of Manning's Roughness Coefficient. Table 5-6 gives a list of n values for channels of various kinds.[1] For each kind of channel the minimum, normal, and maximum values of n are shown. The normal values for artificial channels given in the table are recommended only for channels with good maintenance. The boldface figures are values generally recommended in design. For the case in which poor maintenance is expected in the future, values should be increased according to the situation expected. Table 5-6 will be found very useful as a guide to the quick selection of the n value to be used in a given problem. A popular table of this type was prepared by Horton [34] from an examination of the best available experiments at his time.[2] Table 5-6 is compiled

[1] The minimum value for Lucite was observed in the Hydraulic Engineering Laboratory at the University of Illinois [33]. Such a low n value may perhaps be obtained also for smooth brass and glass, but no observations have yet been reported.

[2] A table showing n values and other elements from 260 observations made on many existing artificial channels is also given by King [35].

TABLE 5-5. VALUES FOR THE COMPUTATION OF THE ROUGHNESS COEFFICIENT BY EQ. (5-12)

Channel conditions			Values
Material involved	Earth	n_0	0.020
	Rock cut		0.025
	Fine gravel		0.024
	Coarse gravel		0.028
Degree of irregularity	Smooth	n_1	0.000
	Minor		0.005
	Moderate		0.010
	Severe		0.020
Variations of channel cross section	Gradual	n_2	0.000
	Alternating occasionally		0.005
	Alternating frequently		0.010–0.015
Relative effect of obstructions	Negligible	n_3	0.000
	Minor		0.010–0.015
	Appreciable		0.020–0.030
	Severe		0.040–0.060
Vegetation	Low	n_4	0.005–0.010
	Medium		0.010–0.025
	High		0.025–0.050
	Very high		0.050–0.100
Degree of meandering	Minor	m_5	1.000
	Appreciable		1.150
	Severe		1.300

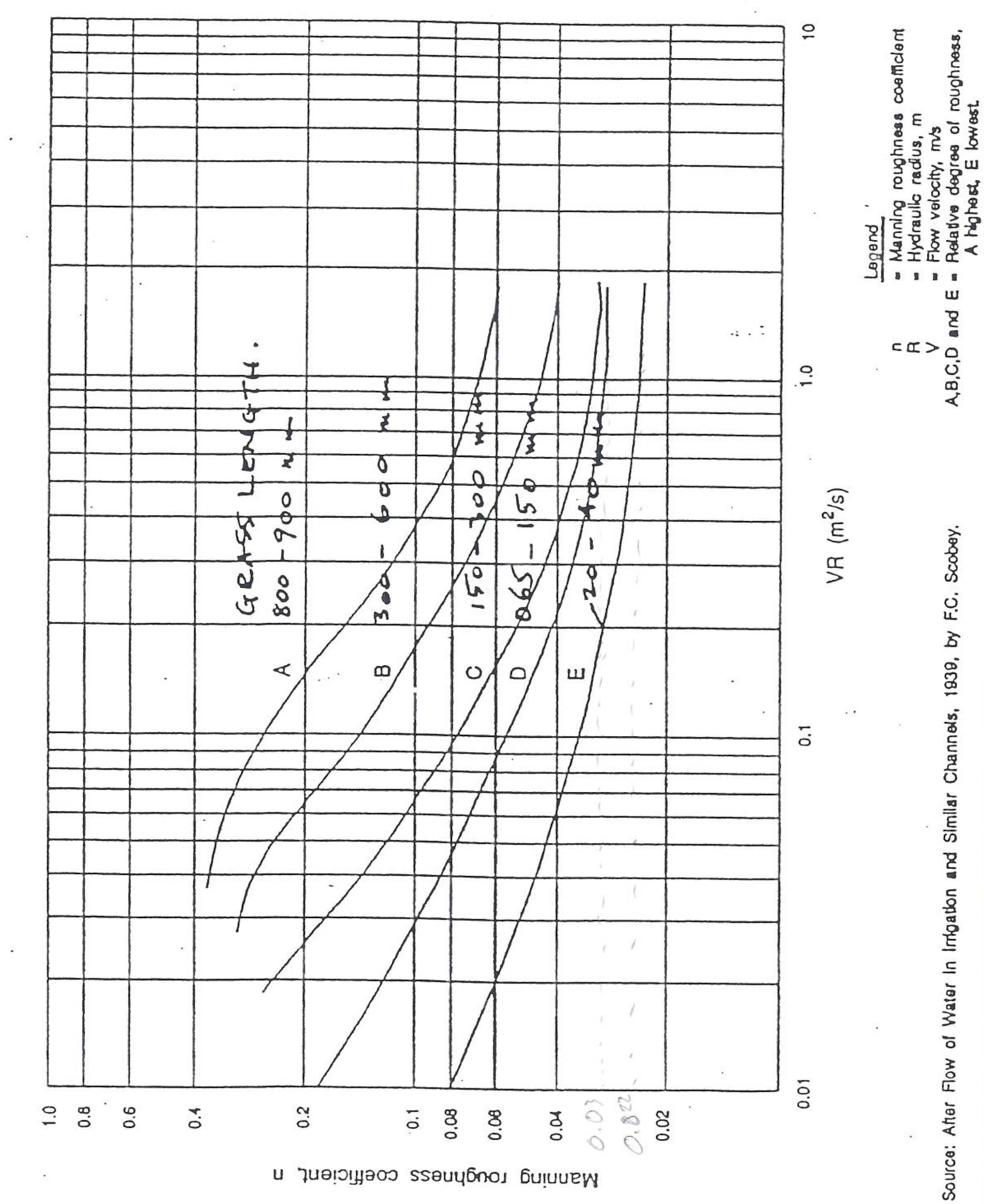
$$n = (n_0 + n_1 + n_2 + n_3 + n_4)\, m_5$$

Cowan Eq.

FLOW REGIME	BED FORM	BED MATERIAL CONCENTRATION	MODE OF TRANSPORT	WATER SURFACE BED PHASE RELATION	RESISTANCE TO FLOW
LOWER	Ripples	by weight 0 – 200 ppm	Discrete Steps	Out of Phase	Form roughness predominates - spacing and amplitude of roughness elements vary with the fall diameter of bed - material. n varies from 0.018 to 0.040
	Dunes	100-1200 ppm			
TRANSITION	Washed-out Dunes	1,000 – 1,200 ppm			Variable n = 0.012 – 0.02
UPPER	Plane	1,800 – 2,000 ppm	Continuous	In Phase	Grain roughness predominates – for Plane bed n varies from 0.012 to 0.018
	Antidunes	1,800 – 6,000 ppm	Continuous		
	Breaking Antidunes	1,800 – 100,000 ppm or more	Discontinuous		

Figure 2 The characteristics of flow in sand - bed channels

Figure A-1 **Vegetal Retardance Curves**



Source: After Flow of Water in Irrigation and Similar Channels, 1939, by F.C. Scobey.

Lecture 16 (cont.'d)
Estimation Manning's n for Natural Channels.
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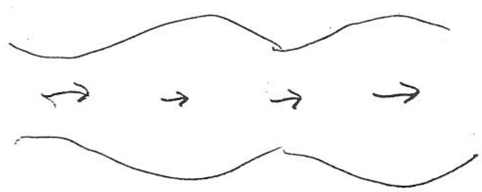
See Fig 2.

n_2



X-section changes.

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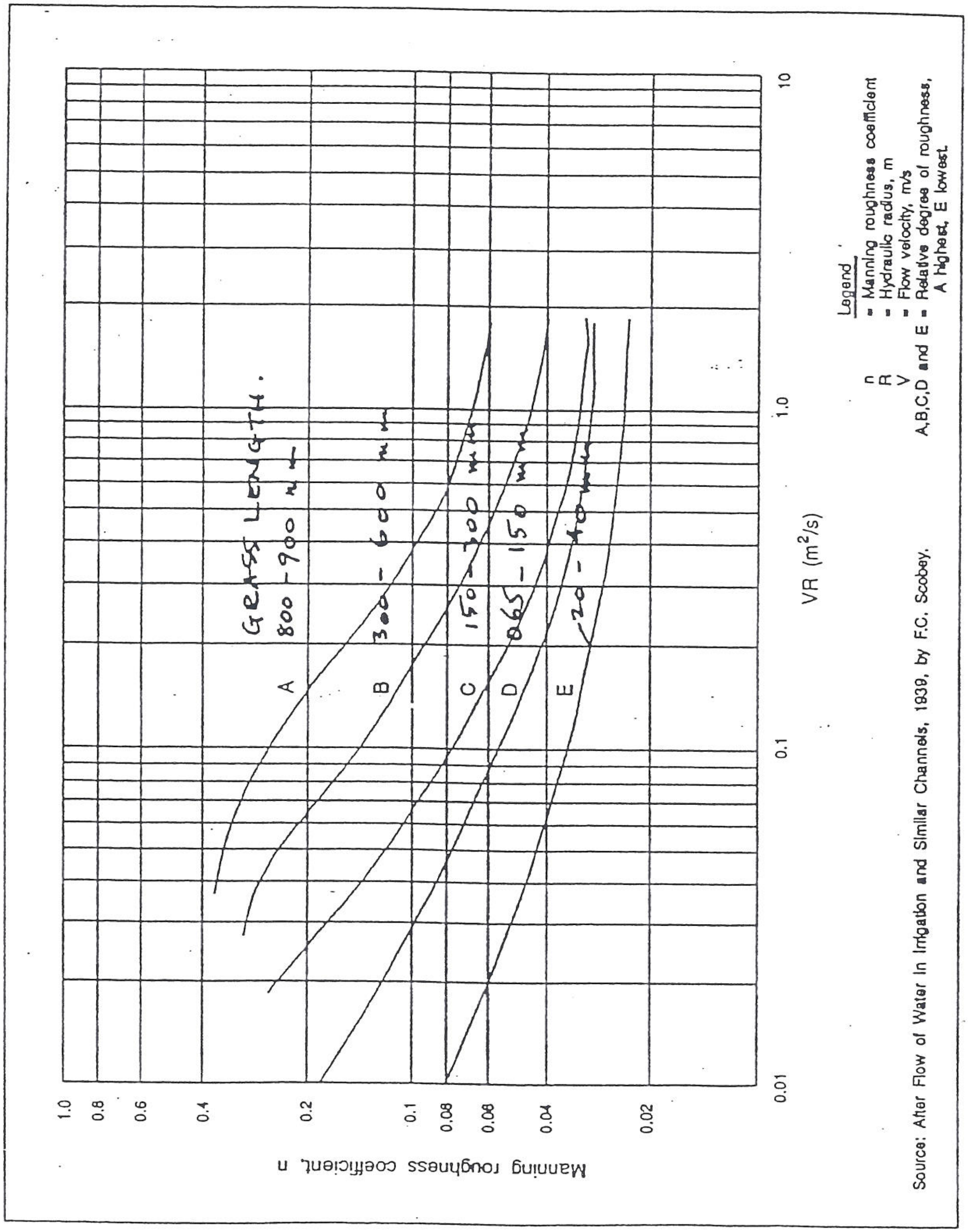


great graphics! 20/20

Lecture 17 Donald Serolleman Channel Design

1) Design High & Low Flows

$Q_{High} = 20000$ cfs ; $Q_{Low} = 5$ cfs

2) Side Slope : $Z = 2$

3) Bed Slope : $S_0 = 0.0005$

4) Lining → Concrete : $n = 0.013$

5) Max & Min Vel. : $V_{max} = 20$ ft/s ; $V_{min} = 2$ ft/s

6) Optimum b/y : equ. 17.3 → $\frac{b}{y} = 2(\sqrt{1+z^2} - z) = 0.472$

7) Depth : (design eq.) $y_n = C_Q^{3/8} \frac{(b/y + 2\sqrt{1+z^2})^{1/4}}{(b/y + z)^{5/8}}$; $C_Q = \frac{nQ}{c' S_0^{1/2}}$

$C_Q = 7824.7$

→ $y_n =$ **24.43 ft**

8) Find b : $b = \left(\frac{b}{y}\right) y_n = 0.472(24.43) =$ **11.5 ft**

9) Check V_{max}, V_{min}, N_F

$A = y(b + zy) = 24.43(11.5 + 2(24.43)) =$ **1475 ft²**

$V_{max} = Q_{max}/A =$ **13.56 ft/s** < 20 ∴ OKAY

$N_F = \frac{V}{\sqrt{gD}}$; $D = \frac{A}{B} =$ (For trapezoid) $\frac{1475}{2(2)(24.43 + 11.5)} = 13.5$

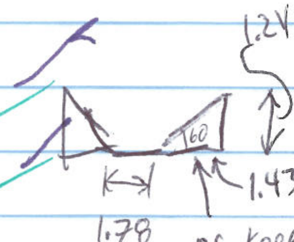
→ $N_F =$ **0.037** < 0.8 ∴ OKAY

$y_c = 20.8$ / 10?

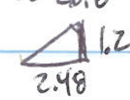
calc

10) Freeboard : $FB1 = 0.439 \ln Q_{[cfs]} - 1.5 =$ **2.85 ft**

$FB2 = 0.476 \ln Q_{[cfs]} - 0.2 =$ **4.5 ft**



or keep @ 2:1 ~ 26.6°



11) Design sub channel For Low Flow

$Q = 5\text{cfs} = \frac{c'}{n}\sqrt{3}\, y^2 \left(\frac{y}{2}\right)^{2/3} S_0^{1/2}$ → **$y_{min} = 1.24$ ft**

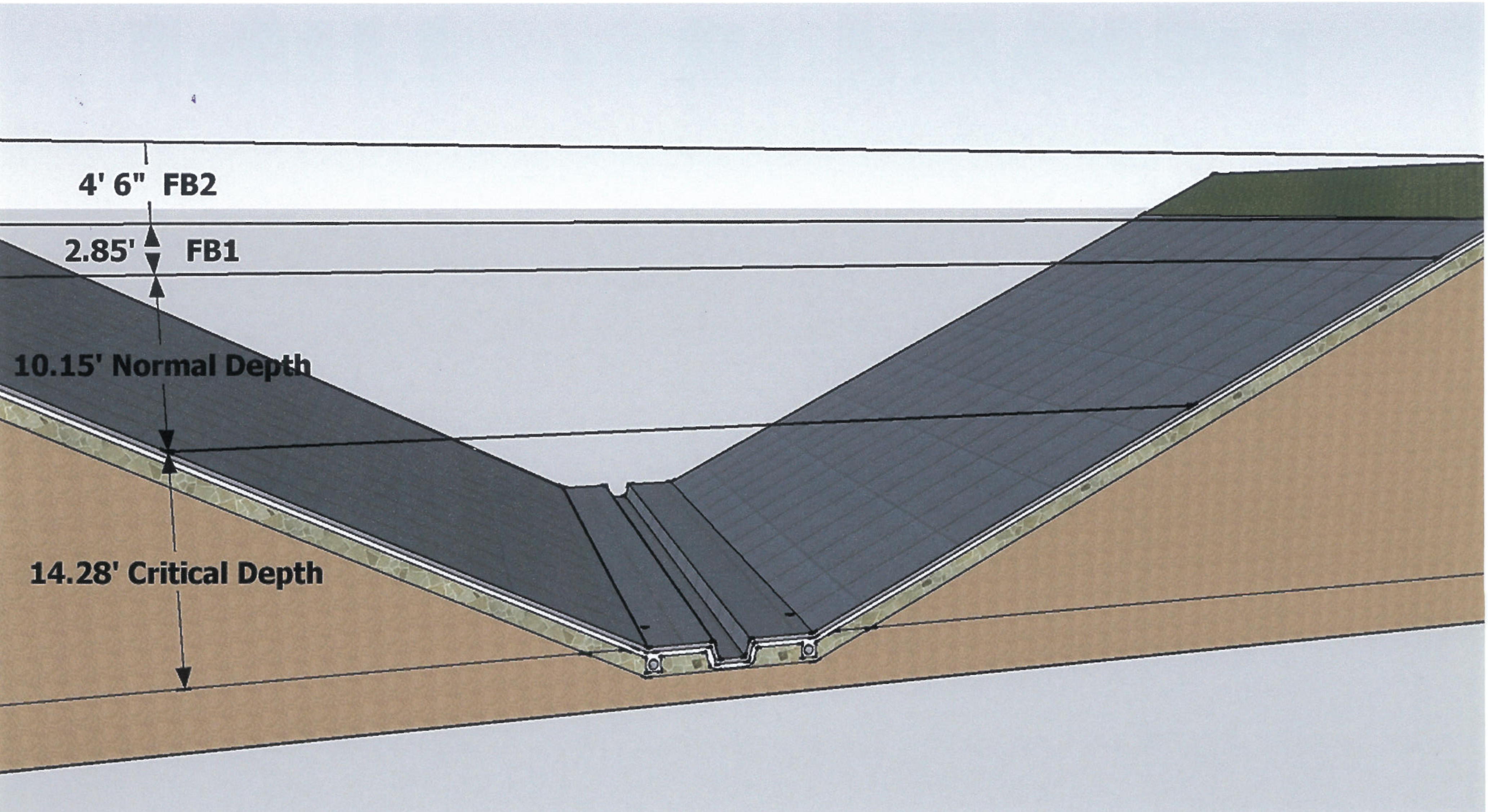
$b^* = \frac{2}{\sqrt{3}} y =$ **1.78 ft**

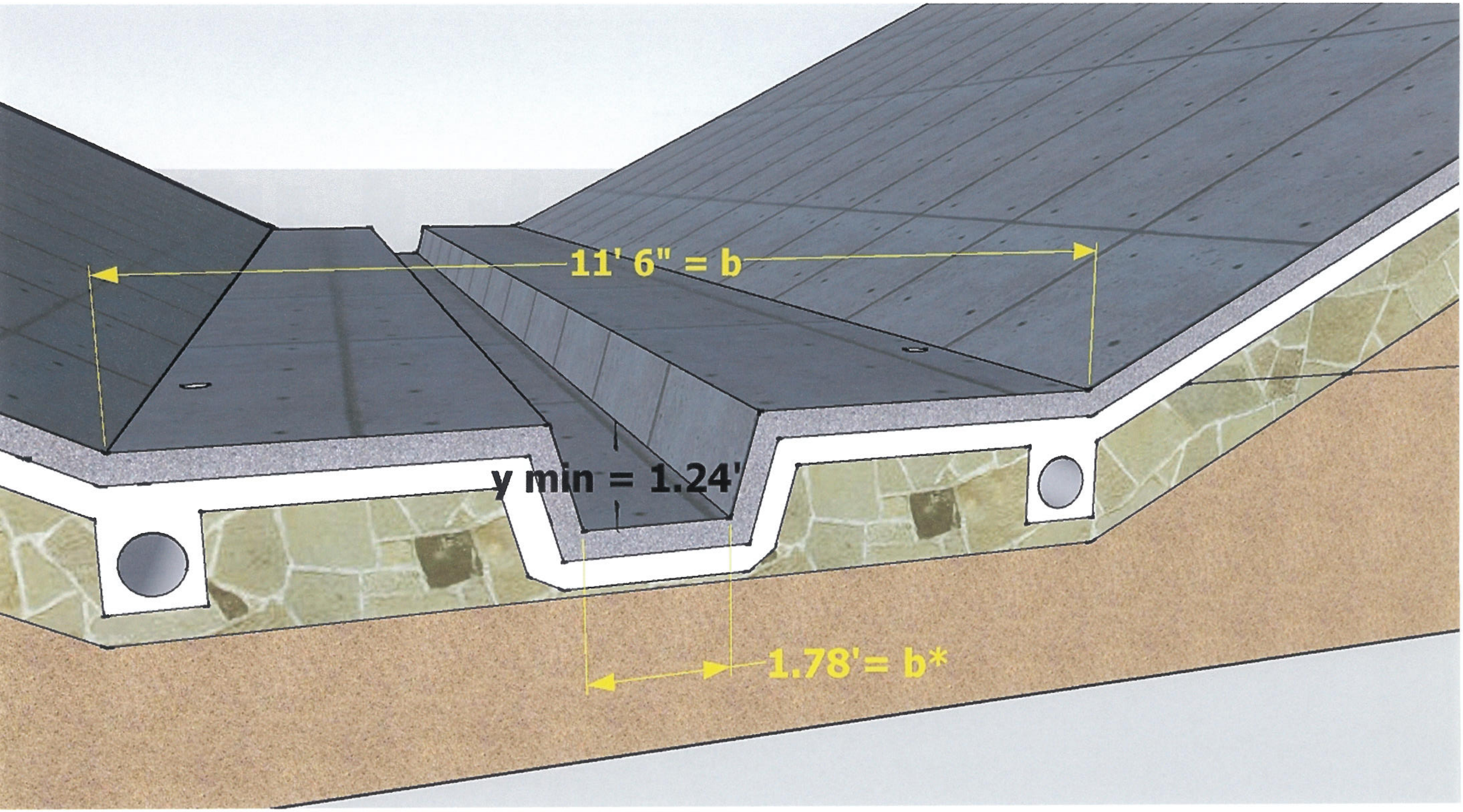
$A^* = \sqrt{3}\, y =$ **2.66 ft²** $A = \frac{b+B}{2}(y) \rightarrow B = 2.51$ ft

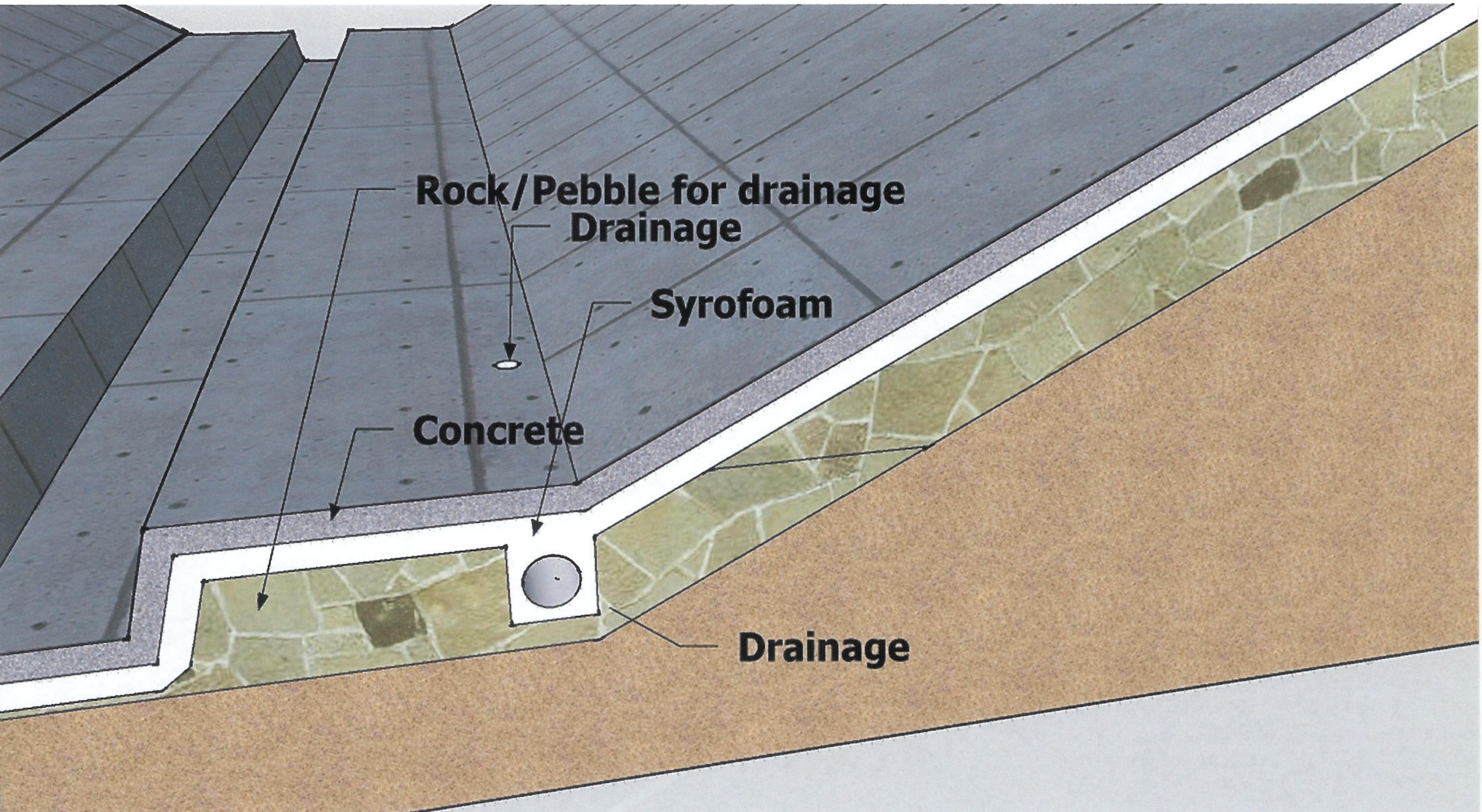
$V_{min} = Q_{min}/A_{min} =$ **1.88 ft/s** < 2 ∴ must allow for maintenance

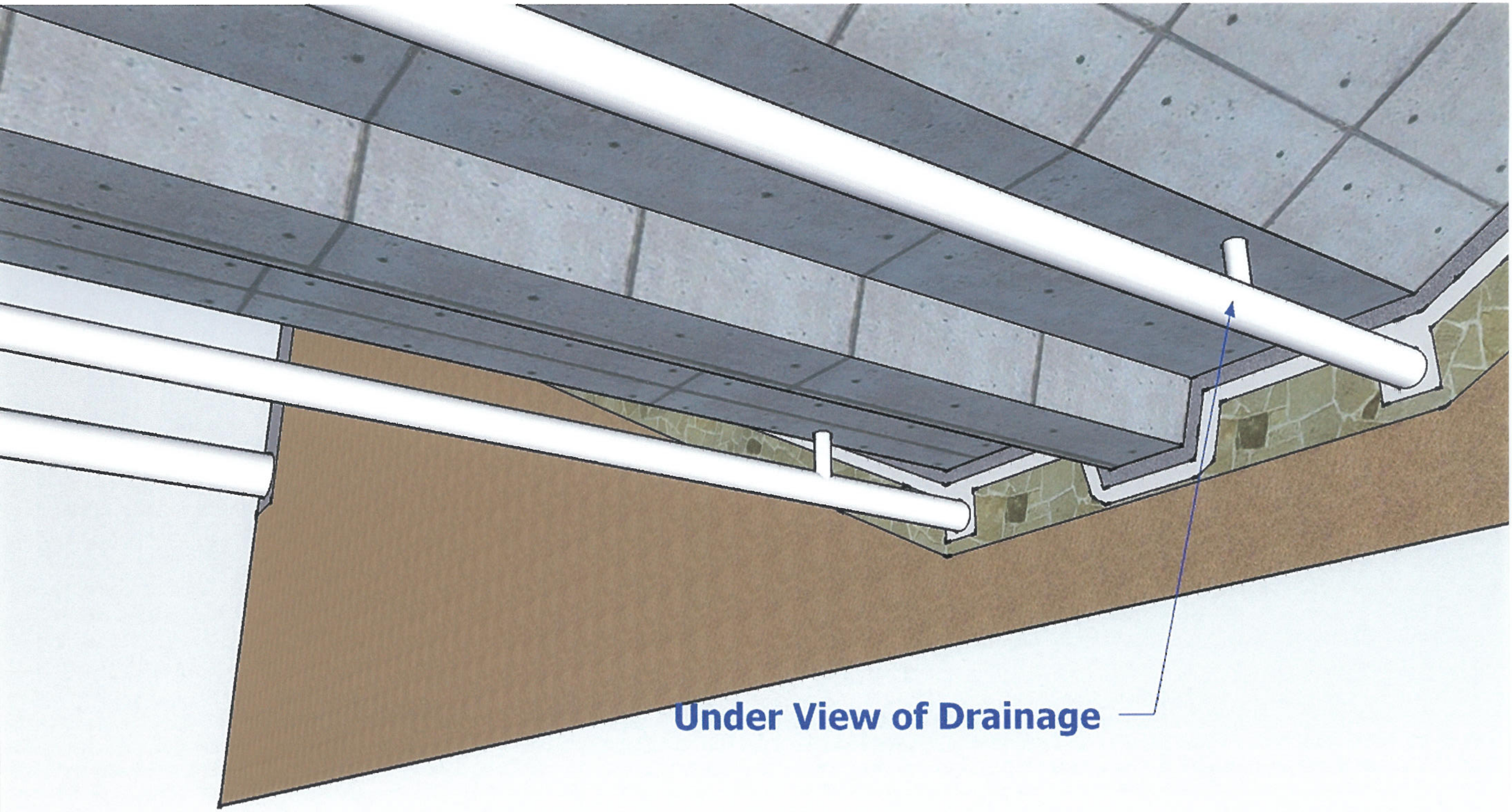
12) Critical Depth : Assume $Q_c = 20{,}000$ cfs

$Q_c = \sqrt{g D_c}\, A_c = \left[32.2 \frac{y(48 + 2y)}{2(2y) + 48}\right]^{1/2} y(48 + 2y) \rightarrow y_c =$ 10


4' 6" FB2
2.85' FB1
10.15' Normal Depth
14.28' Critical Depth


11' 6" = b
y min = 1.24'
1.78'= b*


Rock/Pebble for drainage
Drainage
Syrofoam
Concrete
Drainage


Under View of Drainage

Hydro. Nov 2

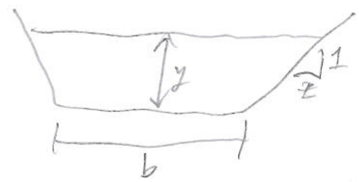
Lecture 17
Design of Lined Channels.

Channel Design Using the Manning Equation

Design variables:

- Design flows (maximum and minimum flows).
- channel shape ← usually
  - e.g. trapezoidal,
  - Most economical section (Best Hydraulic Section)
- bed slope, (S_o)
- side slope, ($\frac{1}{z}$)
- lining material, (n)
- bottom width, (b)
- freeboard.
- Q high, low

Q high, Q low



Design Equations:

(1) Continuity Q = VA

(2) Manning $Q = (c'/n) A R^{2/3} S_o^{1/2}$ (friction equ) → $C_Q = \frac{nQ}{c' S_o^{1/2}} = AR^{2/3} = \left[y_n(b+zy_n)\right]\left[\frac{y_n(b+zy_n)}{b+2y_n\sqrt{1+z^2}}\right]^{2/3}$ 17.1 17.2

n = usually concrete; constants

Eq 17.2 can be use to find the normal (y_n) or uniform depth for uniform flow with a given Q, slope, b, n and z .

Design Criteria:

- Design Flows: 1) Based on demand, e.g. aqueduct or irrigation canal
  2) Based on flood frequency analysis/hydrology
  3) Low flows

- Selection of lining material, *n* (usually concrete)

- Best hydraulic section: (use for min. cost ∴ less concrete)
  $b/y = 2\{(1 + z^2)^{1/2} - z\}$ for best trapezoidal channel 17.3
  $z = \cot(\varphi)$ where φ = Side slope angle.
  (See example problem)

  $Q = \left(\frac{c'}{n} S_o^{1/2}\right)\frac{A^{5/3}}{P^{2/3}}$

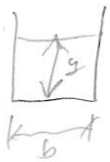
  Q_{max} due to P_{min} see *

- *Side slope is determine by slope stability -> z*

Material	*z*
Rock	*0*
Stiff clay	*0.5 to 1*
Firm clay	*1.5 to 2*
Soft clay	*3*
Sand	*2 to 3*

- *Bed slope must meet project goals but is influenced by cut/fill requirements, S_o*

* Design for Q_{max}



$P = b + 2y \rightarrow \dfrac{\partial P}{\partial y}$ set to $\emptyset \rightarrow \dfrac{\partial P}{\partial y} = -\dfrac{A}{y^2} + 2 = \emptyset \rightarrow A = 2y^2$

or $by = 2y^2$

$A = by$

$b = \dfrac{A}{y}$ $\rightarrow P = b + 2y = \dfrac{A}{y} + 2y$

$\rightarrow \dfrac{b}{y} = 2$ (Best Hydraulic section for a rectangle)

---

cont

$$\frac{b}{y} = 2\left(\sqrt{1+z^2} - z\right)$$

optimum $z = \cot 60°$

best trapezoid

best rectangle

Lecture 17

Assignment **Due Date : One week from this lecture.**

1.Design a trapezoidal channel with concrete lining with Q = 20,000 cfs; b/y = best hydraulic section; n = 0.013; z = 2 and S_o = 0.0005. Low Flow is 5 cfs. What is the critical depth in this channel? Show design features

OKAY FOR concrete

2. Repeat Problem No. 1 based on a maximum velocity of 5 ft/sec with a Mannings n of 0.0225.

1) Optimal trapezoid $\frac{b}{y} = 2\left(\sqrt{1+z^2} - z\right)$, where $z = \cot 60° = 2$

$\frac{b}{y} = 0.472$; side slope $= \frac{1}{z} = 0.5$, Bed Slope $= S_0 = 0.0005$,

$Q_{high} = 20{,}000\ ft^3/s$, $Q_{LOW} = 5\ ft^3/s$, $n = 0.013$ $\quad C_{a_{High}} = \frac{n Q_{High}}{c' S_0^{1/2}} = \frac{0.013(20000\ \frac{ft^3}{s})}{(1.486)\sqrt{0.0005}} = 7824.7$

Fig: 4-1 $\left(z=2, \frac{y}{b} = 2.119\right) \rightarrow \frac{Z}{b^{2.5}} = \frac{Q}{\sqrt{g}\, b^{2.5}} \sim 1.3$

Preferable $\lesssim 0.8$

- $N_F < 1$ To prevent wavy flow; < 1 causes shock waves (can be greater than depth)

- *$V \geq Vmin$ to prevent siltation and vegetation (approximately range 2 to 2.5 ft/sec)*

- *$V \leq Vmax$ to prevent abrasion (for concrete channels 15 to 20 ft/sec)*

- *Solve for y and b from Eqs 17.2 and 17.3*

- *Freeboard = fcn(F, Q)* (adds on to calculated depth, AKA S.F.)

Details

- Lateral subsurface drains
- Uplift relief drains
- Frost protection
- Maintenance considerations

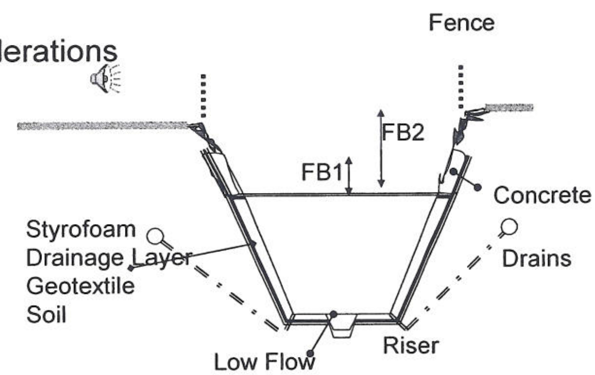


Figure 17.1 Lined Trapezoidal Channel Design Considerations

Valik, Yelena, [illegible], Mara, Opal

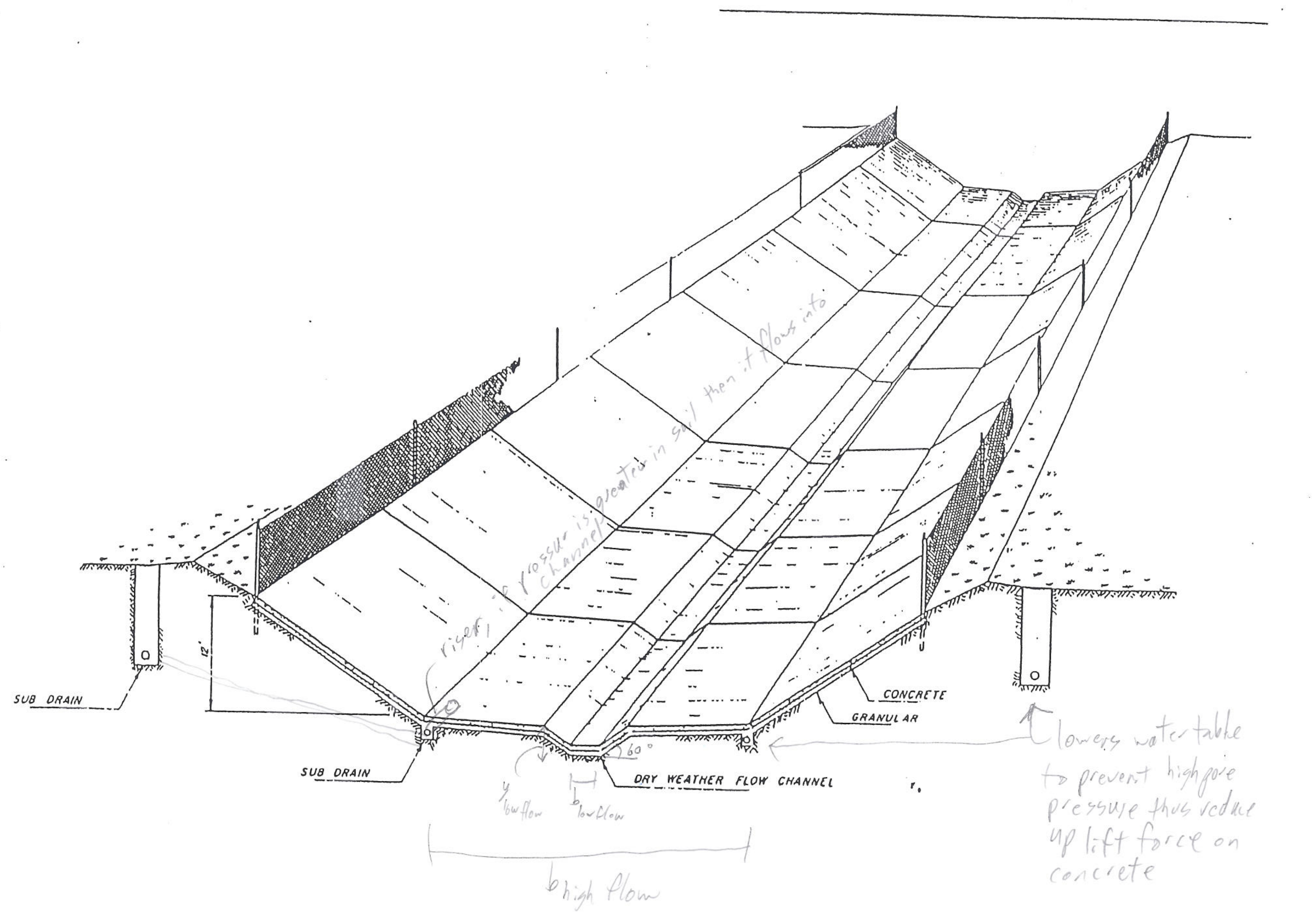
Leif

married

brother
sister

marries

sister



SUB DRAIN
12'
SUB DRAIN
DRY WEATHER FLOW CHANNEL
60°
CONCRETE
GRANULAR
riser, if pressure is greater in soil then it flows into channel
$y_{low flow}$
$b_{low flow}$
$b_{high flow}$
lowers water table to prevent high pore pressure thus reduce up lift force on concrete

Lecture 18
Assignment
P.122

1) Design a straight canal w/ $Q = 10{,}000$ cfs & $S_0 = 0.0009$ w/ bed & bank material as follows: $D_{25} >= 1"$; $D_{50} = 0.75"$ slightly rounded, Non-colloidal.

Compare solutions for max permissible velocity & max permissible unit tractive force (shear).

Donald Jerolleman

6/10

Lecture 18 Assignment P. 119

Given $Q = 2000$ cfs ; $D_{50} = 0.02''$; Find P, R S_o

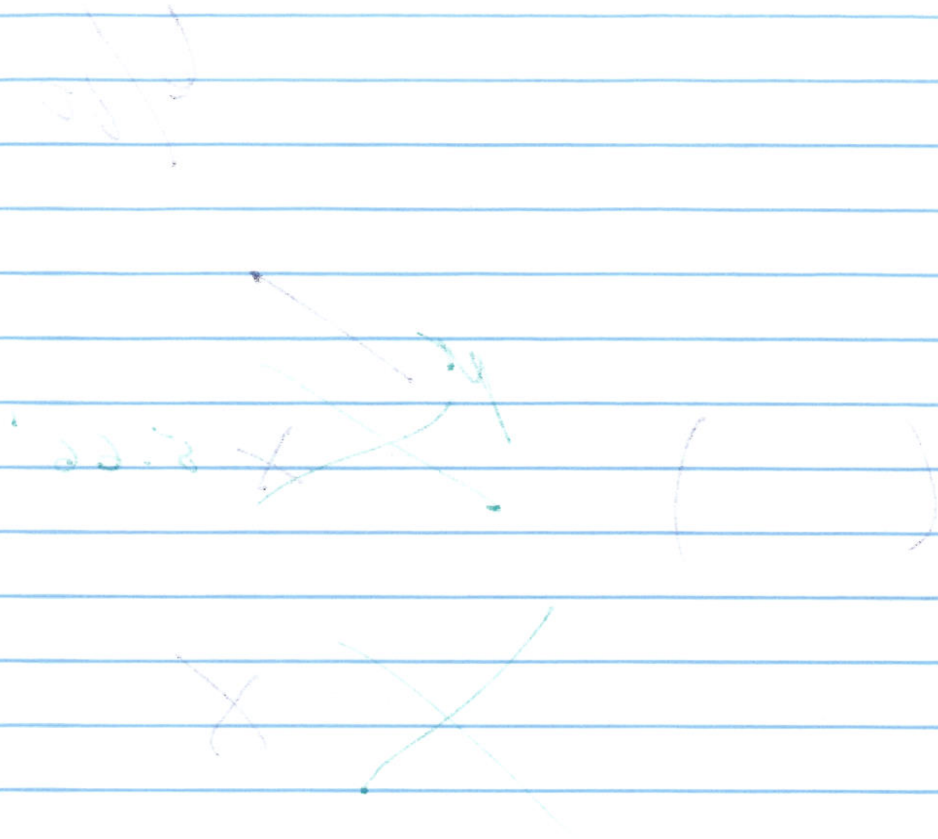
$f_s = 8(D_{50}[in])^{1/2} = 1.1314$

$P = 2.67\sqrt{Q} = \boxed{119.4}$ ✓

$R = \left(\frac{Q}{1.17P\sqrt{f_s}}\right)^{2/3} = \boxed{71.3}$ ✗ 5.66'

$V = \frac{Q}{PR} = 0.235$ ft/s

$S_o = \left(\frac{V}{16R^{2/3}}\right)^3 = \boxed{6.23\times10^{-10}}$ ✗



Lecture 18
Design of Erodible Channels.
Reference: Handouts

Design Procedures for Unlined Channels

Methods for Unlined Channels

- Maximum Permissible Velocity Method
- Maximum Permissible Shear Method
- Regime Method
- Sediment Transport Method

Regime Method

- *The Regime Concept is "In nature there is a unique set of stable dimensions (depth, width and slope) for a given flow and silt load."*

Lacey was one of the early engineers to formulate this approach. He based his Regime equations on his observations of irrigation canals. He identified the non-silting non-scouring canals and developed the following equations to describe their stable dimensions:

- **$P = 2.67\,Q^{1/2}$ ft with Q in cfs**
- **$f_s = 8.(d50 \text{ inches})^{1/2}$**
- **$V = 1.17[f_s R]^{1/2}$**
- **$V = 16\,R^{2/3} S_o^{1/3}$**
- **$Q = AV = PR\,V$**

Or

- **$V = 1.17[f_s R]^{1/2} = Q/(PR)$**

Therefore $R = \{Q/[1.17P\, f_s^{1/2}]\}^{2/3}$

&

- **$So = \{V/(16\,R^{2/3})\}^3$**
- **$V = Q/A = Q/(PR)$**

Assignment: Given: Q= 2000 cfs: D50=0.02 inches. Find: P, R, So

Dominant flow

Q_{Dom} = avg. annual flood

$T_r \sim 2.33$ yrs

method 2

Core of Engr. modification of Lacy's equations

Table 5.9 Simons and Albertson (1963) Modified Regime Equations

clay

	Sand Bed and Sand Banks	Sand Bed and Cohesive Banks	Cohesive Bed and Cohesive Banks
$P = C_1\, Q^{0.512}$	3.3	2.51	2.12
$R = C_2\, Q^{0.361}$	0.37	0.43	0.51
$A = C_3\, Q^{0.873}$	1.22	1.08	1.08
$V = C_4\, (R^2\, S)^{1/3}$	13.9	16.1	16.0
$W/D = C_5\, Q^{0.151}$	6.5	4.3	3.0

Simons and Albertson (1963) explain the limitations of the Indian and their own regime equations. Simons and Albertson (1963) also provide guidance for designing with their equations:

1. Canals that are formed in coarse non-cohesive material of the type studied by the USBR (sediment transport < 500 ppm).

2. Canals that are formed in sandy material with sand beds and banks (sediment transport < 500 ppm).

3. Canals that are formed in sand beds and slightly cohesive to cohesive banks (good results when sediment transport < 500 ppm, qualitative results when sediment transport > 500 ppm).

4. Canals having cohesive beds and banks (sediment transport < 500 ppm).

The USACE (1994) provides guidance on channel design. Their recommendation is to use locally or regionally developed equations for channel design. However, when this is not possible, Figures 5.34, 5.35, and 5.36 can be used to provide rough estimates for top width, depth, and slope of a channel given the channel-forming discharge and bed material. Limitations associated with the charts are provided in the following paragraphs.

USACE Regime Chart Limitations

1. Where possible, reach-averaged data for existing channels should be plotted and compared with the indications of the charts, using bankfull discharge as the channel-forming. If bankfull discharge is not determinable, a 2-year recurrence discharge can be used as the channel forming. This comparison can indicate how compatible the stream system is with the assumptions of the charts. The trends of the charts can then be used to estimate changes appropriate for modifications due to increased in-channel flows.

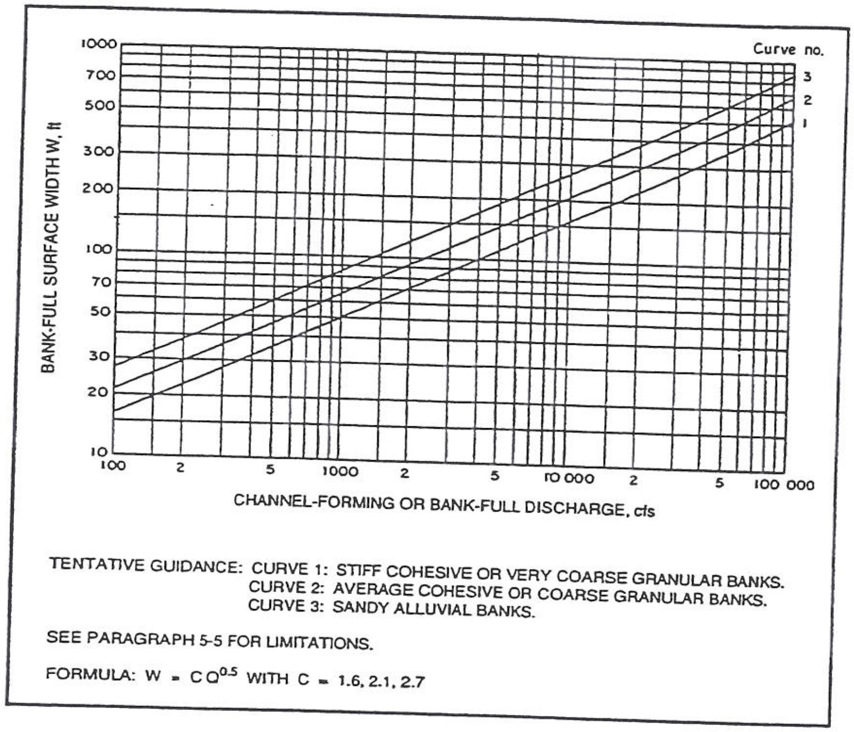


Figure 5.34 Top Width as Function of Discharge (USACE, 1994)

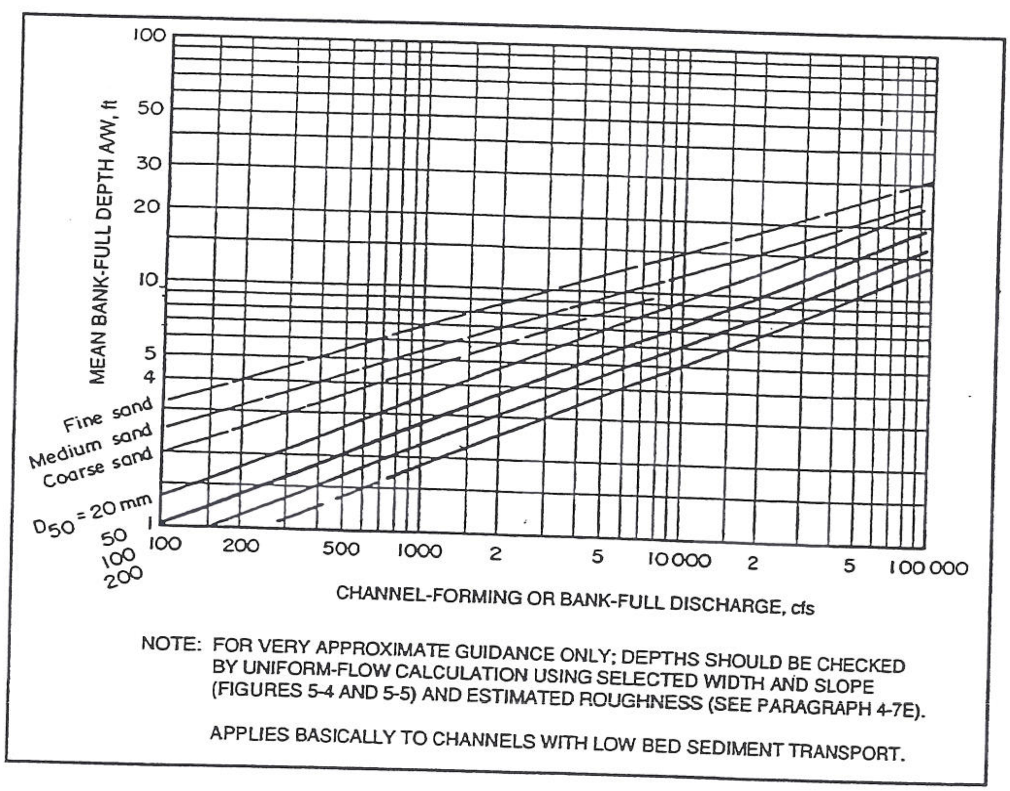


Figure 5.35 Depth as Function of Discharge (from USACE, 1994)

$A_{trapazoid} = y(b + zy)$

Design Procedure

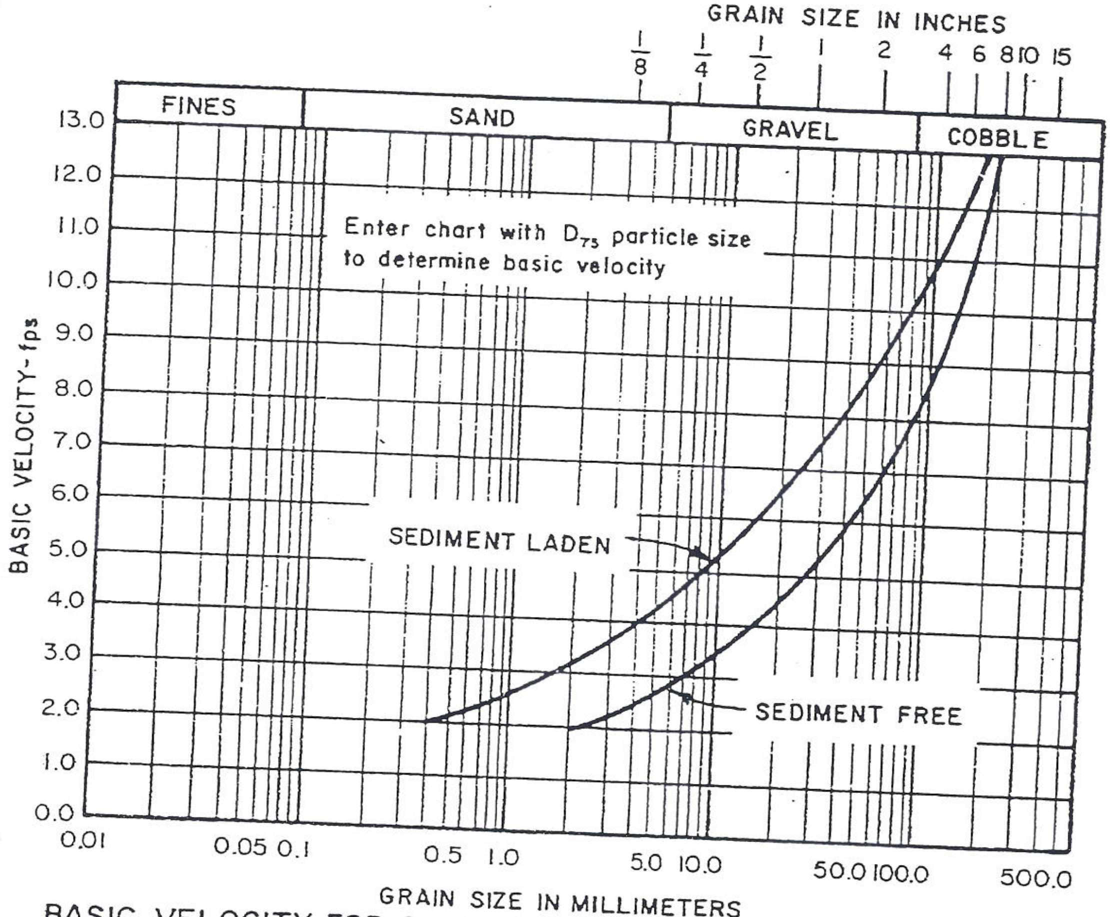
- **1. Determine the design flow, Q.**
- **2. Determine the design bed slope, So.**
  ↳ from Regime or Terain
- **3. Use soil properties estimate the side slope, z.**
- **4. Based on the bed material, estimate Mannings n.**
  ↳ Table 7-3
- **5. Based on the bed material and sediment load, estimate *Vmax***
  ↳ 7-3 or p. 158
- **6. Calculate the flow area $A = Q/V_{max} = y\,(b + zy)$**
  if trapazoid
- **7. Calculate the hydraulic radius from Mannings Eq.**
  $$R = \{V_{max}\, n/(c'\, S_o^{1/2})\}^{3/2}$$
  Kind of a measure of depth
- **8. Now $P = A/R = b + 2\,y\,(1 + z^2)^{1/2}$**
  trapazoid
- **9. Solving for y from Eq 6 and 8 gives:**
- $$\{P - 2\,y\,(1 + z^2)^{1/2}]\}\,y + z\,y^2 - A = 0$$
- $$[z - 2\,(1 + z^2)^{1/2}]\,y^2 + P\,y - A = 0$$
- **or** $y = [P \pm Disc^{1/2}]/\{2\,[2\,(1 + z^2)^{1/2} - z\,]\}$
- **here** $Disc = \{P^2 + 4\,A\,[z - 2\,(1 + z^2)^{1/2}]\}$
- **10. Then** $b = P - 2\,y\,(1 + z^2)^{1/2}$
  ↳ put into (6)
- **11. Add FB2.**
  Free board
  ↳ chart in last lecture

Assignment. Due in 1 weeks.

1. Design a straight canal with Q = 10,000 cfs and So = 0.0009 with bed and bank material as follows: $D_{25>}$ =1"; D_{50} = 0.75"; slightly rounded, non-colloidal.
Compare the solutions for maximum permissible velocity and maximum permissible unit tractive force (shear).

================

From Corps of Engineers manual



BASIC VELOCITY FOR DISCRETE PARTICLES OF EARTH MATERIALS (v_b)

ALLOWABLE VELOCITIES FOR UNPROTECTED EARTH CHANNELS

CHANNEL BOUNDARY MATERIALS	ALLOWABLE VELOCITY
DISCRETE PARTICLES	
Sediment Laden Flow	
$D_{75} > 0.4$ mm	Basic velocity chart value x D x A x B
$D_{75} < 0.4$ mm	2.0 fps
Sediment Free Flow	
$D_{75} > 2.0$ mm	Basic velocity chart value x D x A x B
$D_{75} < 2.0$ mm	2.0 fps
COHERENT EARTH MATERIALS	
PI > 10	Basic velocity chart value x D x A x F x C_e
PI < 10	2.0 fps

Figure 5.28a Allowable Velocities for Unprotected Earth Channels (from USDA, 1977)

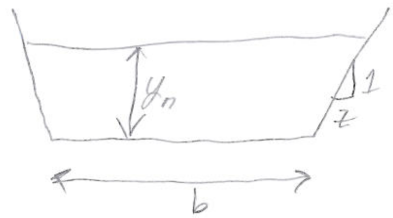
Donald Serolleman

Assignment: Due Date : Next lecture.

1.Find the normal and critical depths for the following:
A trapezoidal channel with Q = 1000 cfs; b = 10 ft; n = 0.03; z = 2 and S_o = 0.0005.

2. Find the bottom width for the following:
A trapezoidal channel with Q = 1000 cfs; y_n = 8 ft; n = 0.03; z = 2 and S_o = 0.0005.

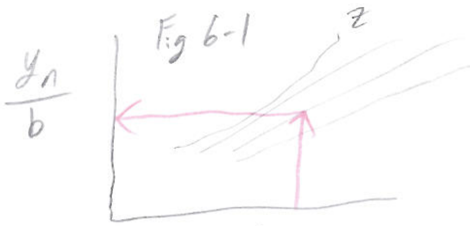
1) $A = y_n(b + zy_n)$ $\quad P = b + 2y_n\sqrt{1+z^2}$



$Q = \frac{c'}{n}\frac{A^{5/3}}{P^{2/3}}S_o^{1/2}$ Known (n = cst.)

Put know into one constant: $C_Q = \frac{nQ}{c' S_o^{1/2}} = \frac{A^{5/3}}{P^{2/3}} = \frac{[y_n(b+zy_n)]^{5/3}}{(b+2y_n\sqrt{1+z^2})^{2/3}}$

use trial & error; goal seek; Fig 6-1 (attached) use to get started



$C_Q = \frac{0.03(1000 \frac{ft^3}{s})}{1.486(0.0005)^{1/2}} = 902.85$

good starting point

$\frac{C_Q}{b^{8/3}} = \frac{902.85}{(10)^{8/3}} = 1.945 \longrightarrow \frac{y_n}{b} = 1 \rightarrow y_n = 10$

9.85' approx

$902.85 = \frac{[y_n(10+2y_n)]^{5/3}}{(10+2y_n\sqrt{1+z^2})^{2/3}}$

$y_n = 10 \rightarrow C_Q = 932.7$

$y_n = 11 \rightarrow C_Q = 1155$

$y_n = 9.75 \rightarrow C_Q = 881$

$y_n = 9.9 \rightarrow C_Q = 912$

$y_n = 9.89 \rightarrow C_Q = 910$

$y_n = 9.85 \rightarrow C_Q = 901.8$

$\boxed{y_n = 9.855 \rightarrow C_Q = 902.84}$

EQ 12.18 $\frac{Q}{\sqrt{g}} = (b+ty_c)y_c\sqrt{\frac{(b+ty_c)y_c}{b+2ty_c}}$

$\rightarrow \frac{1000}{\sqrt{32.2}} = (10+2y_c)y_c\sqrt{\frac{(10+2y_c)y_c}{10+2(2)y_c}}$

(wolframalpha.com)

$\rightarrow \boxed{y_c = 4.91\ ft}$ ✓

10/10

over

2) $Q = 1000$ cfs, $y_n = 8$ ft, $n = 0.03$, $Z = 2$, $S_0 = 0.0005$

$$C_Q = \frac{nQ}{c' S_0^{1/2}} = \frac{(0.03)(1000)}{(1.486)(0.0005)^{1/2}} = \underline{902.85} = AR^{2/3} = A\left(\frac{A}{P}\right)^{2/3}$$

$$= \frac{A^{5/3}}{P^{2/3}} = \frac{[y_n(b + z y_n)]^{5/3}}{[b + 2 y_n \sqrt{1+z^2}]^{2/3}} = \frac{[8(b+16)]^{5/3}}{[b + 16\sqrt{5}]^{2/3}}$$

$$\rightarrow 902.85(b + 16\sqrt{5})^{2/3} = (8b + 128)^{5/3}$$

wolframalpha.com → $\boxed{b = 21.4 \text{ ft}}$

9/10

1541

Donald Jerolleman   Assignment Lecture 20   Hydraulics F2010

$y_2 = 5.5'$ $y_1 = 6'$ $x_1 = \emptyset$ $Q = 1000\ cfs$ $n = 0.02$ $S_0 = 0.001$ $B = 40' = b$

y	dy	A	P	$AR^{2/3}$	$A\sqrt{D}$	f	$\bar{f}$	$\Delta x = \Delta y \bar{f}$	X
6	0	240	52	665.3	587.9	2224	2087	0	0
5.5	0.5	220	51	582.99	515.95	1891	2087	1028.8	1028.8

Signs

$q = \frac{Q}{b} = \frac{1000}{40} = 25\ cfs/ft$; $C_Q = \frac{nQ}{c' S_0^{1/2}} = \frac{0.2(1000)}{1.486(0.001)^{1/2}} = 425.61$

$$f_6 = \left(1 - \left(\frac{1000}{587.9\sqrt{32.2}}\right)^2\right) \Big/ 0.001\left(1 - \left(\frac{425.6}{665.3}\right)^2\right) = 2224$$

$$f_{5.5} = \left(1 - \left(\frac{1000}{516\sqrt{32.2}}\right)^2\right) \Big/ 0.001\left(1 - \left(\frac{425.6}{585}\right)^2\right) = 1891$$

$\bar{f} = 2057.6$

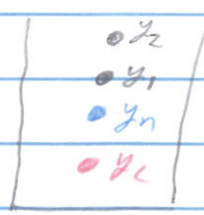
$$y_n \Rightarrow Q = \frac{c'}{n}(b\,y_n)\left(\frac{b\,y_n}{b\,4 2 y_n}\right)^{2/3} S_0^{1/2} = \frac{1.486}{0.02}(40\,y_n)\left(\frac{40 y_n}{40 + 2 y_n}\right)^{2/3} S_0^{1/2}$$

$\rightarrow y_n = \boxed{4.48\ ft}$

$$y_c \Rightarrow Q_c = Q = \sqrt{g}\sqrt{D_c}\,A_c = \sqrt{g}\sqrt{y_c}\,(y_c)(b)$$

$$1000\ cfs = \sqrt{32.2}\ y_c^{3/2}(40')$$

$\rightarrow y_c = \boxed{2.69\ ft}$



M_1 Curve

Hydro Nov 11

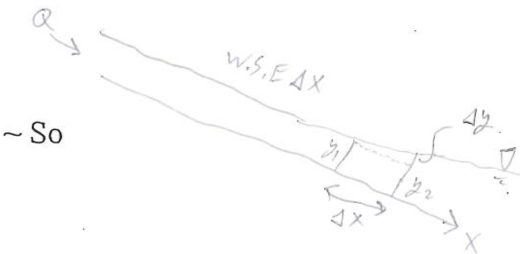
Lecture 20

Gradually Varied Flow *(Steady Flow)*

Chapters 14 & 15 ,Handouts and HEC RAS Manual

In gradually varied flow the change in depth with distance along the channel is small.

Common Simplifying Assumptions:



1. $|(dy/dx)| < 1/20$ (slope of line parallel to bed)
2. Q = constant;
3. Bed slope is small; cos0 ~ 1; sinθ ~ tanθ ~ So
4. Hydrostatic pressure; c = 0;
5. n is constant;
6. α ~ 1 and β are constant;
6. Eddy loss is small;
7. Prismatic channel.

Equation for water surface slope:

The total mechanical energy head at a section in an open channel is:

$$H_T = h_z + y + V^2/(2g) \qquad \text{.........................} \; 20.1$$

$S_f = \left(\frac{nQ}{c'AR^{2/3}}\right)^2$

The Continuity Eq. requires $V = Q/A$ at all sections.

$S_o \sim \sin\theta \sim \tan$

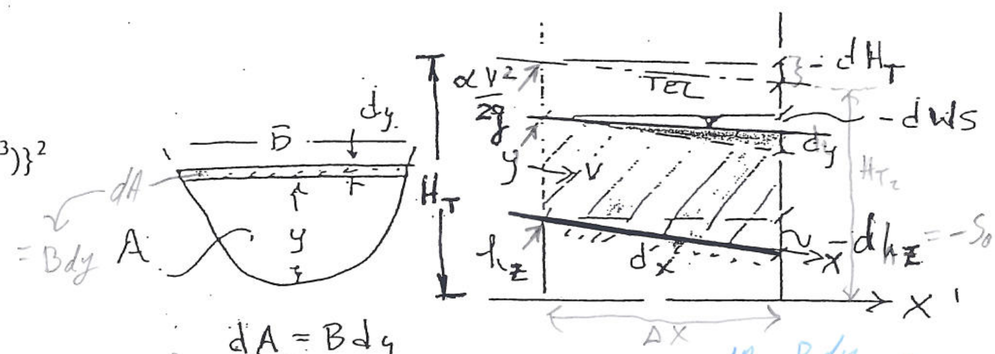
$$\frac{dH_T}{dx} = \frac{dh_z}{dx} + \frac{dy}{dx} + \frac{d[V^2/(2g)]}{dx} \qquad \text{.........................} \; 20.2$$

$-S_f\,\Delta X = \Delta H_T$

Note: $\frac{dH_T}{dx} = -S_f$

$S_o\,\Delta X = dh_z \rightarrow \quad \frac{dh_z}{dx} = -S_o$

where $S_f = \{n\,Q/(c'\,A\,R^{2/3})\}^2$



$dA = B\,dy$

$\frac{dA}{dy} = \frac{B\,dy}{dy} = B$

Defining Diagram

Expanding $\frac{d[V^2/(2g)]}{dx} = \frac{d}{dx}\left(\frac{Q^2}{A^2 2g}\right) = \frac{2Q^2}{2g}\frac{-1}{A^3}\frac{dA}{dx}\cdot\frac{dy}{dy} = -\frac{Q^2}{gA^3}\frac{dA}{dy}\cdot\frac{dy}{dx}$ 20.3

$\frac{Q^2}{2g}\left(\frac{-2}{A^3}\frac{dA}{dx}\right)$

$$= \frac{-Q^2}{gA^2D}\frac{dy}{dx}$$

$$\therefore \; -S_f = -S_o + \frac{dy}{dx} - \frac{Q^2}{gA^2D}\frac{dy}{dx}$$

Substituting Equation 20.3 into 20.2 gives:

$$\frac{dy}{dx} = \frac{(S_0 - S_f)}{1 - \frac{Q^2}{gA^2D}} = f(y) \text{ "non-linear in y \& really can't be solved"} \qquad \text{.................. 20.4}$$

$$\frac{dy}{dx} = \frac{S_0 - \left(\frac{nQ}{c'AR^{2/3}}\right)^2}{1-\left(\frac{Q^2}{gA^2D}\right)} = \frac{S_0\left(1-\left(\frac{nQ}{c'AR^{2/3}S_0^{1/2}}\right)^2\right)}{1-\left(\frac{Q^2}{gA^2D}\right)} = \frac{S_0\left(1-\left(\frac{A_nR_n^{2/3}}{AR^{2/3}}\right)^2\right)}{1-\left(\frac{A_c\sqrt{D_c}}{A\sqrt{D}}\right)^2}$$

$= C_Q = A_nR^{2/3}$

b/c $\frac{Q}{\sqrt{g}} = A_c\sqrt{D_c}$ → $\frac{Q^2}{g} = A_c^2D_c$

For a **wide rectangular channel**, A ~ y; R ~ y; D ~ y, therefore $A_n = y_n$, $R_n = y_n$, $D_c = y_c$, $A_c = y_c$

w/ unit width

$$\frac{dy}{dx} = \frac{S_0\left(1-\left(\frac{y_n}{y}\right)^{10/3}\right)}{1-\left(\frac{y_c}{y}\right)^3} \qquad \text{.................. 20.5}$$

{still can't integrate, use spreadsheet}

General case for trapezoidal channel:

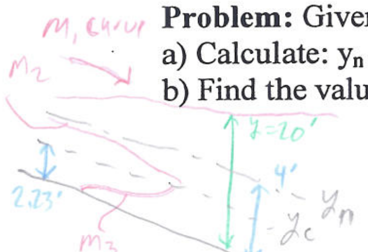
$$\frac{dy}{dx} = \frac{S_0\left(1-\left(\frac{y_n}{y}\right)^N\right)}{1-\left(\frac{y_c}{y}\right)^M} \qquad \text{.................. 20.6}$$

get from charts, Fig 6.2 & 4.2

Better of not using this & going back to Q, R, ... methods

Problem: Given a very wide channel with q = 20 cfs/ft; So = 0.0016; n = 0.03.
a) Calculate: y_n and y_c.
b) Find the value of dy/dx for i) y = 20 ft;
ii) y = 3 ft;
iii) y = 1 ft.

{Boundary conditions "initial cond."}



i) $q = \frac{c'}{n} y_n y_n^{2/3} S^{1/2}$

$C_q = \left(\frac{nq}{c'S_0^{1/2}}\right) \simeq 10.09 = y_n^{5/3}$

$\rightarrow y_n = C_q^{3/5} =$ **4 ft**

$q = \sqrt{g}\sqrt{y_c}\,y_c = \sqrt{g}\,y_c^{3/2}$

$\rightarrow y_c =$ **2.32 ft** (Anything above is sub-crit. & bellow = supercritical)

$\frac{dy}{dx} = \frac{S_0\left(1-\left(\frac{y_n}{y}\right)^{10/3}\right)}{1-\left(\frac{y_c}{y}\right)^3}$ } will be pos ∴ depth will increase w/ x

$= S_0(0.999) \sim S_0$

If $\frac{dy}{dx} = S_0$ the water level slope is nearly horizontal "M_1 curve"

For y = 3 ft

$\frac{dy}{dx} = -0.0048$ "M_2 curve" decreasing

$\frac{dy}{dx}$ w/ y = 1 ft

=

M_1 = caused by Dam, waterway, etc

M_2 = " " weir, waterfall

M_3 = " " gate,

value of N decreases rapidly as the depth of flow approaches the top of the channel. Further mathematical analysis has revealed that the value of N will be equal to zero at $y/d_0 = 0.938$ and will then become negative at greater depths. The significance of this fact will be discussed later in this article and the next.

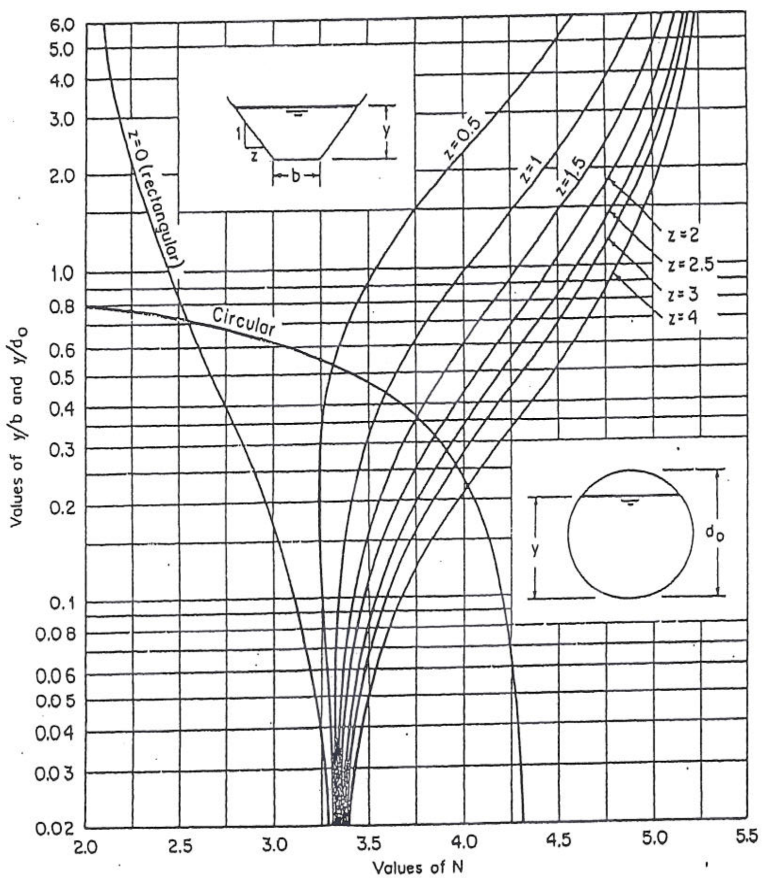


FIG. 6-2. Curves of N values.

For channel sections other than the rectangular, trapezoidal, and circular shapes, exact values of N may be computed directly by Eq. (6-14), provided that the derivative dP/dy can be evaluated. For most channels, except for channels with abrupt changes in cross-sectional form and for closed conduits with gradually closing top, a logarithmic plot of K as ordinate against the depth as abscissa (Fig. 6-3) will appear approximately as a straight line. This can also be seen from the dimensionless curves for , in Fig. 6-1, which are plotted similarly except that the ordinate

This equation indicates that the value of M for the trapezoidal section is a function of z and y/b. For values of z = 0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, and 4.0, a family of curves for M versus y/b are constructed (Fig. 4-2). These curves indicate that the value of M varies in a range from 3.0 to 5.0.

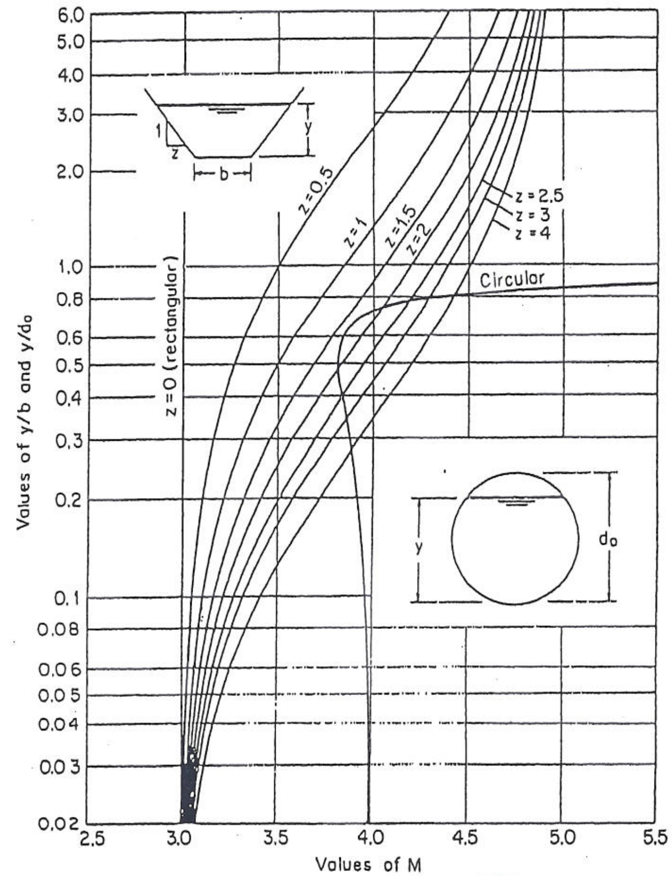


FIG. 4-2. Curves of M values.

A curve for a circular section with M plotted against y/d_0, where d_0 is the diameter, is also shown (Fig. 4-2). This curve was developed by a similar procedure but constructed from a much more complicated formula. The curve shows that the value of M varies within a rather narrow range for values of y/d_0 less than 0.7 or so, but increases rapidly as the value of y/d_0 becomes greater than 0.7. The significance of this

$z(y/b)$ may be constructed. It is obvious that this curve would be identical with the curve for $z = 1$ in Fig. 4-2. For convenience in application, however, a family of curves of M versus y/b are shown, using z as a parameter.

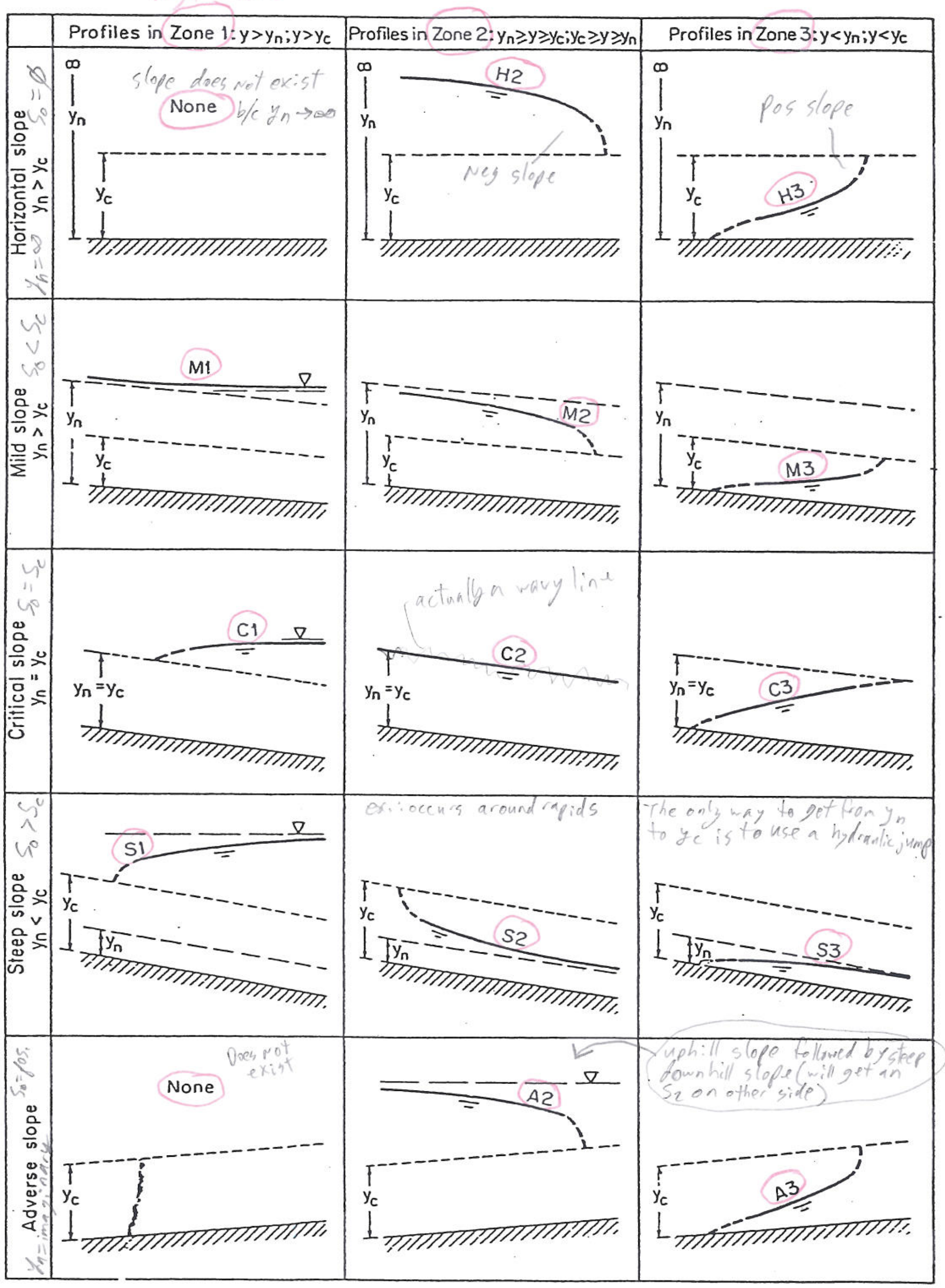


FIG. 9-2. Classification of flow profiles of gradually varied flow.

see ex. 9-7

Guide to Sketching/Calculating and Classifying Flow Profiles

1. Draw channel invert profile with an exaggerated vertical scale.

2. Compute and plot $\mathbf{y_n}$ **and** $\mathbf{y_c}$ on the channel profile.

3. Identify the slopes of all of the reaches by the slope classification (*H, M, C, S, A*) using the table (b) on page 1 of Lecture 21.

4. Identify the two or three possible (depth) zones in all of the reaches by the classification (*1, 2, 3*) using the table (b) on page 1 of Lecture 21.

5. Identify and mark all possible **control points [boundary conditions]**:
   - outlet stage at a lake or large river
   - critical depth control(s)
   - artificial controls levels e.g. sluice gates, weirs, spillways

When **two or more controls are in conflict**, use the one which gives **the highest energy level**

6. Identify and mark all possible **normal depth limits [asymptotic conditions]**:
   - Upstream end of a long mild slope
   - Downstream end of a long steep slope

7. Sketch profiles starting at possible control points using the following rules:
   - in a **subcritical** flow region always sketch (and compute) in the **upstream direction.**
   - in a **supercritical** flow region always sketch (and compute) in the **downstream direction.**
   - identify **hydraulic jumps (HJ/RVF)** where flow changes from supercritical to subcritical
   - identify **hydraulic drops (HD/RVF)** where flow changes suddenly from subcritical to supercritical such as at a weir or spillway - often when an Adverse slope precedes a Steep slope.
   - use the table of possible flow profiles on page 2 of this lecture to identify the shape of the curves for each reach. Mark all profiles with the classification based on bed slope (*H, M, C, S, A*) and depth zone (*1, 2, 3*), e.g. ***M1***.
   - Note that there can be one or more possible profile that can only be determined by complete analysis, e.g. a supercritical flow profile and a subcritical profile may appear to be feasible when you are sketching the curves; however, in the final analysis a hydraulic jump will occur and result in a transition from the supercritical profile to the subcritical profile. The location of the hydraulic can be found be determining the location where the Specific Force is the same for the supercritical and subcritical profiles. Except for hydraulic jumps the curves should be smooth.

Lecture 22
Gradually Varied Flow *(Steady Flow)*
Computation of Flow Profiles

Some of the possible methods of computing water surface profiles are:

Method	dependent variable	independent variable	Limitations	Comments
1. Graphical Integration	**distance x**	**depth y**	**Prismatic channel**	**Constant n Eddy loss ≃ 0**
2. Numerical Integration	**distance x**	**depth y**	**Prismatic channel**	**Constant n Eddy loss ≃ 0**
3. Direct Step	**distance x**	**depth y**	**Prismatic channel**	**Constant n Eddy loss ≃ 0**
4. Standard Step	**depth y**	**distance x**	**Prismatic & non-prismatic channel**	**Variable n. Eddy loss included. Basis of HEC2**

Graphical and Numerical Integration are similar. The Water Surface Profile slope equation (Eq. 20.3) for a prismatic channel can be inverted and written as:

$\frac{dx}{dy} =$ 22.1

= function(y) which can be graphed as

dx/dy versus y as shown at the left. We can write,

$\{x_2 - x_1\} =$ 22.2

1. Classify and sketch the flow profiles that correspond to a) y = 1 ft; b) y = 3 ft; and c) y = 10 ft. Given: A trapezoidal open channel with: b = 20 feet; Q = 440 cfs/ft; z = 1; n =0.03; $S_o = 0.0009$. Use a spreadsheet program to compute and plot the water surface profiles for each case. Use numerical integration.

Hydraulic Jump causes super critical to sub
Hydro Drop causes Sub to Super

Hydraulic Drop ~ $\frac{y_c}{2}$

2. Sketch and mark the possible flow profiles in the channel shown on the attached sketch.

Critical Control
* Internal Boundry Condition
* Any point that can cause critical dept
* y_c are possible when slope changes from (A, H, M, C) going to S, C
* A lake boundry condition "l" exists
* An Artificial B.C. exist "a" ie: a gate
* Asymtotic conditions (see curves)
* If flow is sub-crit. we calc. upstream super-crit. calc downstream

9-7. Sketch the possible flow profiles in the channels shown in Fig. 9-13.

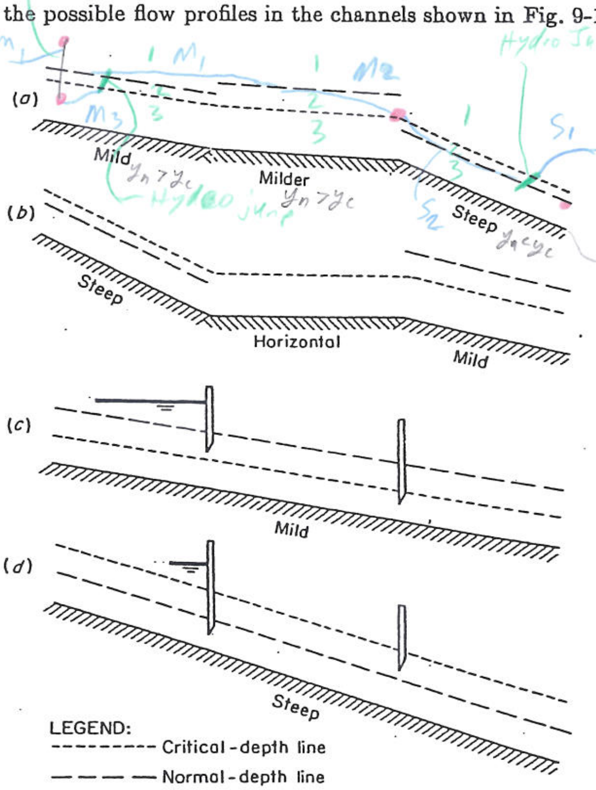


FIG. 9-13. Channels for Prob. 9-7. The vertical scale is exaggerated.

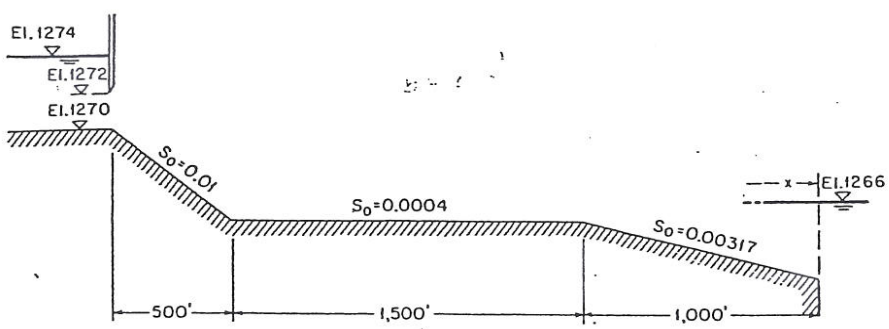


FIG. 9-14. A channel profile for Prob. 9-8.

9-8. A rectangular channel (Fig. 9-14), 20 ft wide, consists of three reaches of different slopes. The channel has a roughness coefficient $n = 0.015$ and carries a discharge of 500 cfs. Determine:

a. the normal and critical depths in each reach

$$\frac{dy}{dx} = \frac{S_o\left(1 - \left(\frac{A_n R_n^{2/3}}{A R^{2/3}}\right)^2\right)}{1 - \left(\frac{Q/\sqrt{g}}{A\sqrt{D}}\right)^2} = \frac{S_o\left(1 - \left(\frac{C_Q}{A R^{2/3}}\right)^2\right)}{1 - \left(\frac{Q/\sqrt{g}}{A\sqrt{D}}\right)^2}$$

Nov 16

Lecture 23 & 24

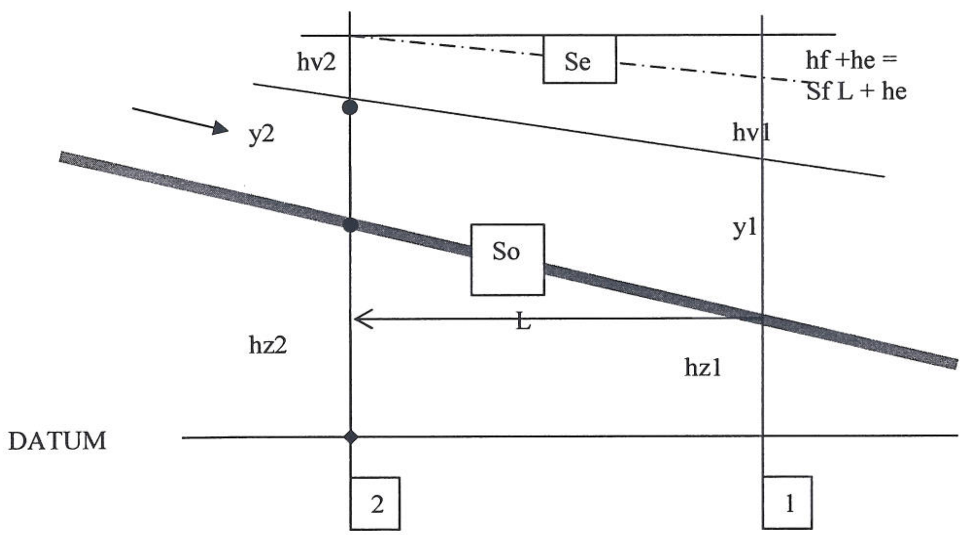
Standard Step Method for Non-Prismatic Channels

In Non-Prismatic channels, it is common to use x as the independent variable (known) and y as the dependent variable (unknown).

The Differential Equation must be modified to include the effect of change in geometry with x.

However, either the energy or the momentum equation is usually used to relate the flow conditions(x known and y, V unknown) at one section to those at another section (x, y, V known).

Se



Defining Sketch for Backwater Case

Applicable Equations

Assume subcritical flow/steady state; treat y at section 1 as known

By definition

$$H_{T1} = h_{z1} + y_1 + h_{v1}$$
$$H_{T2} = h_{z2} + y_2 + h_{v2} \qquad (A)$$

By the energy principle

$H_{T2} = H_{T1} + h_f + h_e$ ***(B)*** **[computation proceeding upstream]**

$h_f = ½ \{S_{f1} + S_{f2}\}$ & $h_e = K_{ce} |h_{v2} - h_{v2}|$

If the flow is subcritical the Eqs A and B are solved for the depth and velocity at section 2.

For this case all the variables are known at section 1.

The standard step method for solving these equations is as follows:

1. *Assume y_2 or WSEL2*

2. *Calculate: A_2, $V_2 = Q/A_2$, hv_2, S_{f2}, ave S_f, h_f, h_e and WSEL2*

3.Calculate: $H_{T2}^{(A)} = h_{z2} + y_2 + h_{v2}$

4. *Calculate: $H_{T2}^{(B)} = H_{T1} + h_f + h_e$*

5. *Compare: $H_{T2}^{(B)} = H_{T1} + h_f + h_e$: $H_{T2}^{(A)} = h_{z2} + y_2 + h_{v2}$*

IF $H_{T2}^{(B)} = H_{T2}^{(A)}$ GOTO NEXT SECTION AND REPEAT FROM STEP 1

ELSE GOTO STEP 1 AND REPEAT (FOR THE SAME SECTION).

Note: For supercritical flow the computation proceeds downstream. Eq B becomes:

$H_{T2}^{(B)} = H_{T1} - h_f - h_e$

Otherwise the procedure is the same.

Application of the U.S. Army Corps of Engineers HEC RAS (HEC2) Program for Computing Water Surface Profiles

Ref. Handouts from the HEC-RAS Manual

Assignment

1. Using the Standard Step Method estimate the depth of the flow at Section 2 in the channel shown below (set up a shreadsheet solution with at least the following columns:

x ft	Q cfs	Invert EL	WS EL	y	A	V	h_v	H_T^a	R	S_f	$S_{f\,ave}$	h_f	h_e	H_T^b

Note: Use "solve for " or " Goal Seek" to find the WSEL or y that gives $H_T^a = H_T^b$.

y = WSEL - Invert EL;

V = Q/A;

$h_v = \alpha V^2/2g$

$S_{f\,ave} = ½ \{S_{f\,n} + S_{f\,n+1}\}$;

$S_f = \{Q\, n/(c'AR^{2/3})\}^2$

$H_T^a{}_{n+1} = \{WSEL + h_v\}_{n+1} = H_T^b{}_{n+1} = H_T^a{}_n + \{h_f + h_f\}_{n \text{ to } n+1} = fcn(y_{n+1})$ [backwater]

$h_f = \Delta x\, S_{f\,ave}$;

$h_e = K_{ec}\,|h_{v\,n+1} - h_{v\,n}|$

n = downstream section; n+1 = upstream section in backwater calculation.

DATA:

Section 1: Trapezoidal; x = 0

Q = 1000 cfs; b = 20 ft; z = 2; n = 0.03; WSEL = 110.0; invert El. = 100.0 ft

Section 2: Rectangular; x = 300.0 ft

Q = 1000 cfs; b = 20 ft; z = 0; n = 0.03; invert El. = 100.18 ft.
Distance between Section 1 and 2 is 300 ft.

Section 3: Rectangular; x = 600.0 ft

Q = 1000 cfs; b = 20 ft; z = 0; n = 0.024; invert El. = 100.22 ft
Distance between Section 2 and 3 is 300 ft.

2. Prepare a HEC2 or HEC-RAS input file for the river sections in problem 1. Run the HEC2/HEC-RAS program with your data file and submit the standard summary table of your results along with a profile showing the invert, water surface elevation and energy level. Assume the channel is straight with no overbank flow.

Guide for HEC-RAS

Example:

River Name: Ence
Reach 1

Flow 2000 cfs
Downstream Stage 23.0 ft

Section 0 d/s

Station	0	35	45	50	60	70	100		
Elevation	23.6	21	11.5	10.5	11.25	20.8	24		

	LOB	Ch	ROB
Dx to d/s section	0	0	0
Manning n	0.04	0.025	0.045
Stations for Floodplains	35.0	70.0	
Kc	0.1		
Ke	0.4		

Section 300 mid

Station	0	30	36	42	53	60	85		
Elevation	24.0	21	12.0	10.8	11.5	21.5	25		
	LOB	Ch	ROB						
Dx to d/s section	300	300	300						
Manning n	0.04	0.025	0.045						
Stations for Floodplains	30.0	60.0							
Kc	0.1								
Ke	0.4								

Section 600 u/s

Station	0	10	20	25	30	40	65		
Elevation	25.0	22.0	12.0	11.0	12.5	21.0	26		
	LOB	Ch	ROB						
Dx to d/s section	300	300	300						
Manning n	0.04	0.025	0.045						
Stations for Floodplains	10.0	40.0							
Kc	0.1								
Ke	0.4								

Application Setup

The opening Window for HECRAS looks like this:

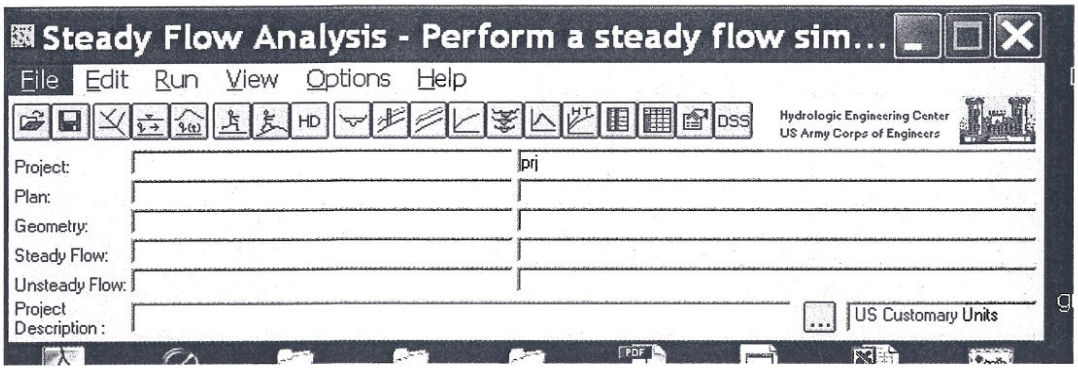


Fig .1 Opening Window

1. To start a new project click "file" and then "new project"
2. A new window will open as shown below. You will need to enter a name for the project in the first row column 1 (e.g. ENCE 6319 Example) and a file name in column 2 with the *.prj* extension as shown below (e.g. EN6319.prj). In addition you can create a new directory by clicking on "Create Directory", for "ENCE 6319 Test" on the Desktop. Finally click OK to accept this information.

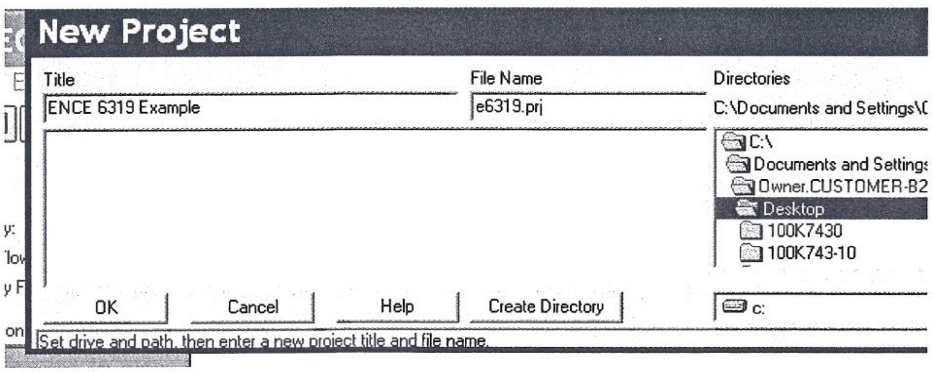


Fig. 2 Project name and directory

3. The next step is set the units (by default US units are set). This is done under options in Fig. 1.
4. Next we start the geometry file from the Window in Figure 1. First click on the 3rd icon from the left (tree branch) which will open the "geometry" window shown below.

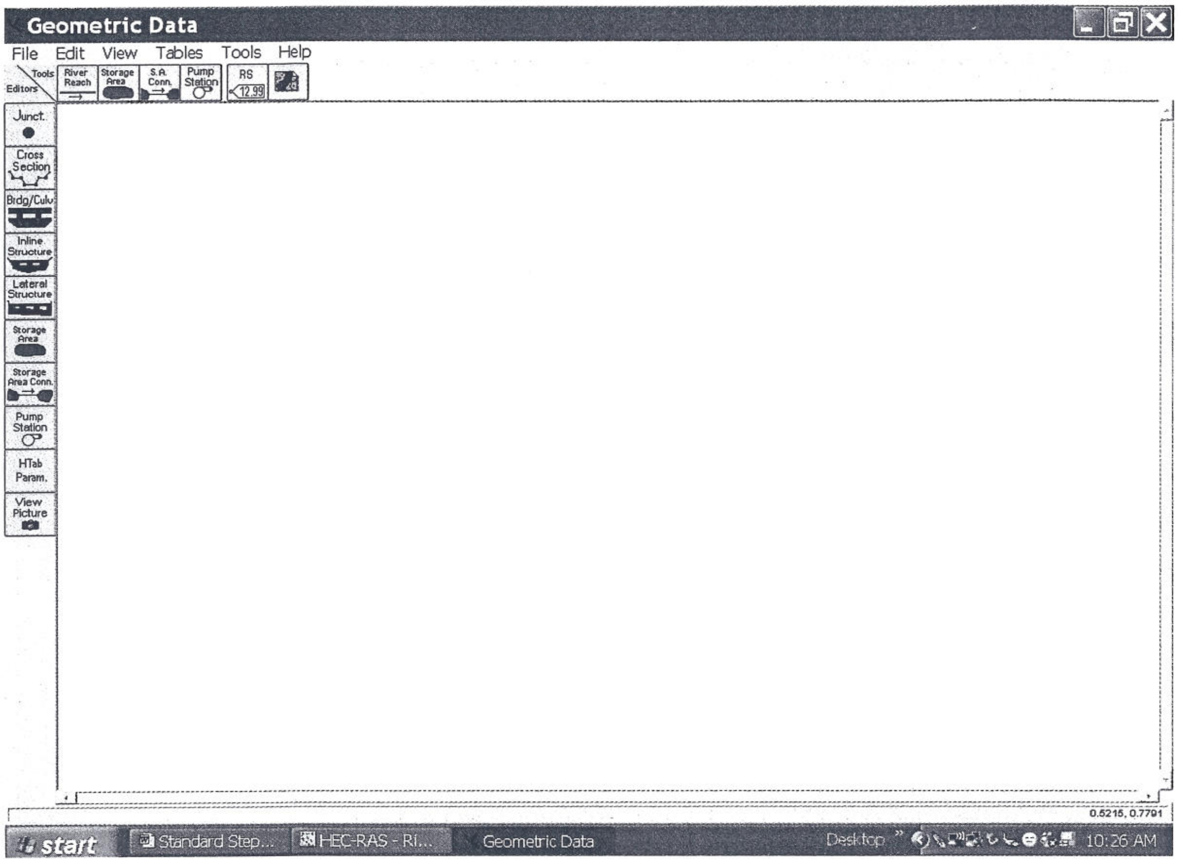


Figure 3a. Geometry Window

Now click on "River Reach" and a "pen" will appear. You use this to sketch the river plan (approximately) starting at the upstream end. This is shown in Figure 3b

A small window will open asking for the River name and the reach number which are illustrated on the Figure 3b.

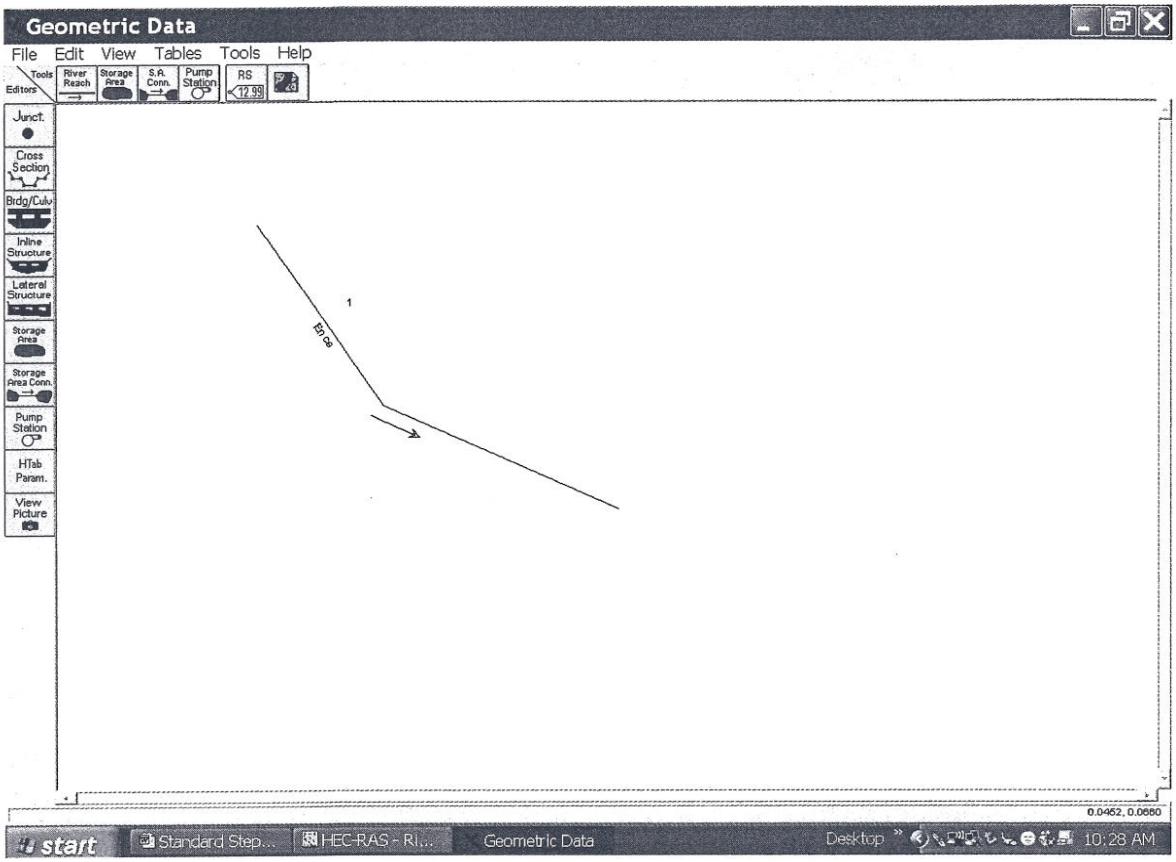


Figure 3b. River Plan with Name and Reach Number

5. Once the River plan is finished, we click on Cross-section to enter the cross-sectional data in the window shown below.

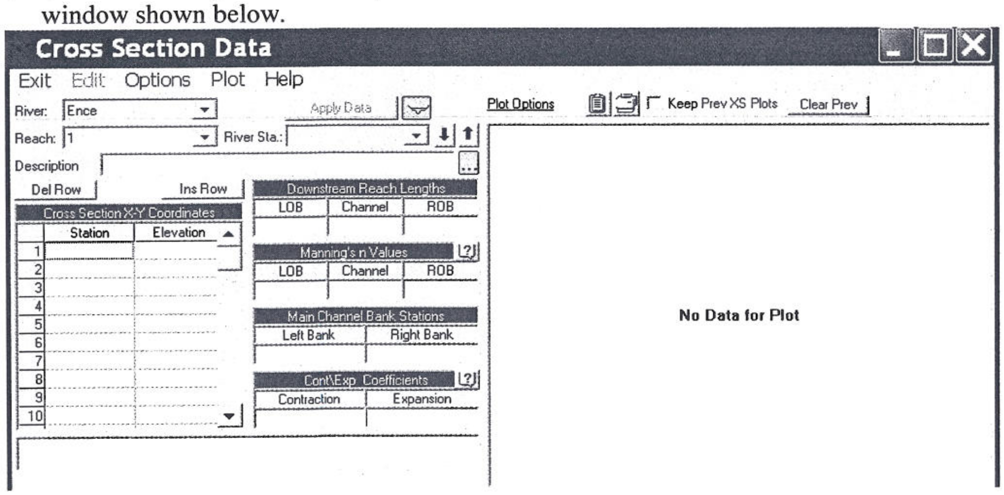


Figure 4a. Cross-section Entry Window

6. We start by clicking on "Options" and selecting "Add a New Cross-section" which will allow us to enter the coordinates for the cross-section from the right. Note: "station" means the lateral coordinate entered in increasing sequence (only). Then the elevation of each "station" is entered. You can have about 200 coordinates. When you finish the coordinates you can click "Apply Data" and the cross-section will be plotted to the right as shown below.

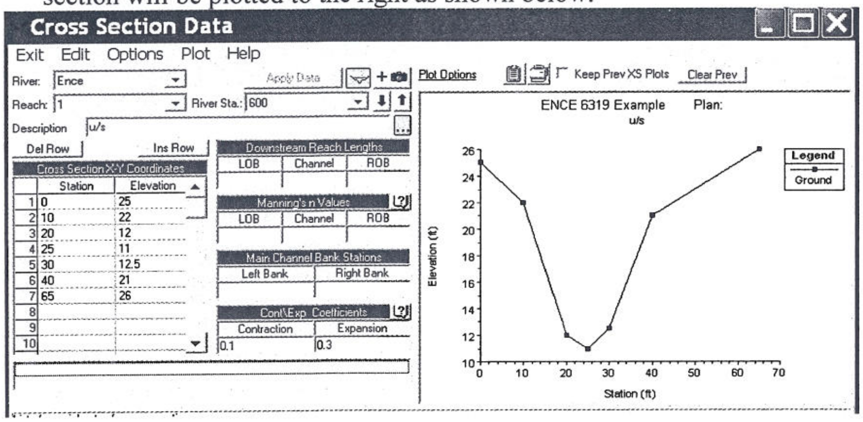


Figure 4b. Coordinates and Plot

7. Next we enter the distance to the next downstream station these are the Dx values and must be entered for the Left Floodplain (LOB), the main channel, and the right floodplain (ROB). Then we specify the Manning n across the section {left floodplain (LOB), main channel and right floodplain (ROB). This is followed by a "station" value corresponding to where the floodplain or "overbank" flow starts on each side of the main channel. The program uses looking downstream to set left and right but for the calculations it is not material. Finally we enter the contraction and expansion loss coefficients. See Figure 4c.

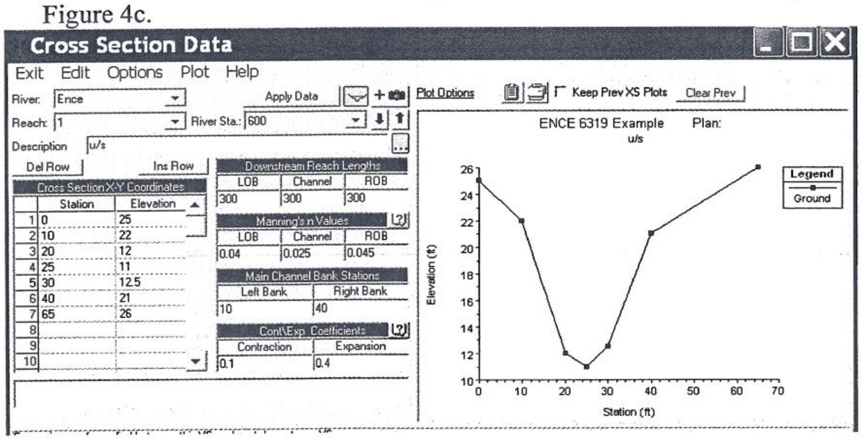


Figure 4c. Entry of Δx's, Manning's n, floodplain-channel stations and Kc & Ke

NOTE: After entering these data be sure to click "Apply data"

This process is repeated for each cross-section by going back to "options" and selecting " "Add a New Cross-section".

After all the sections are entered and "Apply Data" clicked, then go back to the Main Geometry window and to "file" and save the geometry with a descriptive name.

8. Now we must enter the Boundary Conditions by clicking on the 4th icon in Figure 1 (Opening Window). This will bring up the window shown in Figure 5a.

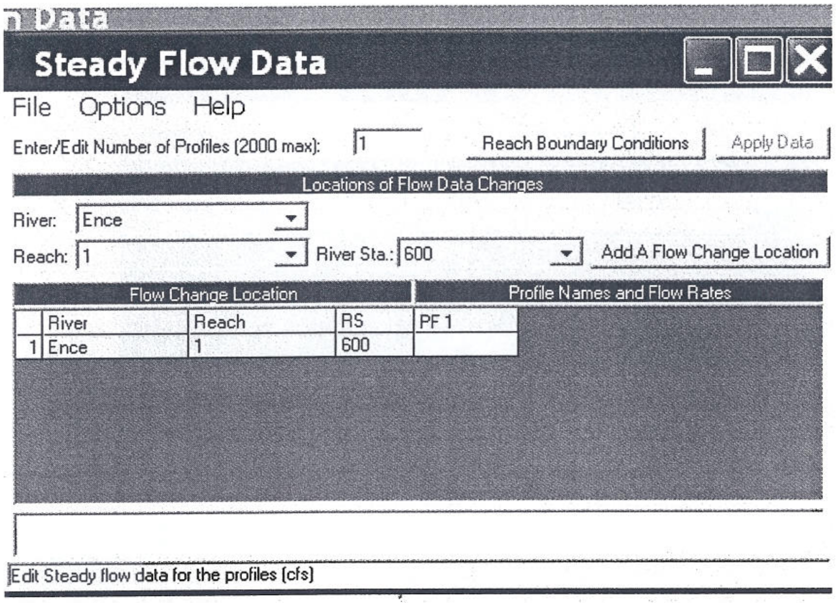


Figure 5a. Window for Steady Flow Boundary Conditions

9. Where it shows PF1 for RS 600, we enter the inflow for this section as shown below.

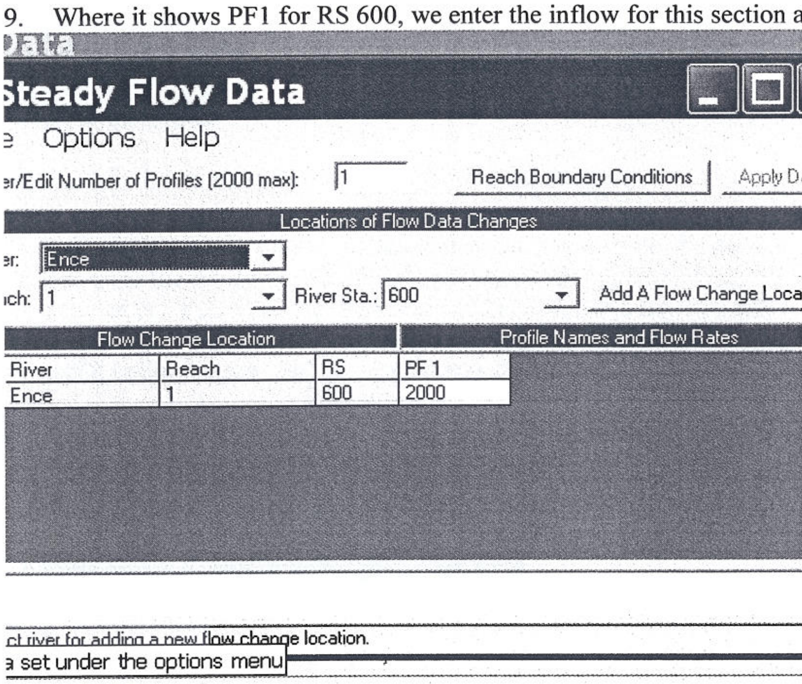


Figure 5b. Entering Flows

10. Now go to "Reach Boundary Conditions" to obtain the window below.
    You will see 4 choices: Known WL, Critical Flow, Normal Depth and Rating Curve.

    The example below selects known WL of El. 23.0 ft. Then click OK which sends you back to the window in Fig. 5b. You must click "Apply Data" and then go to "File" and save along with a descriptive name (not used in the cacluations).

Output

1. If there were no errors, then you can review the output in many ways, e.g. from the "Main Menu" select the icon showing profiles.

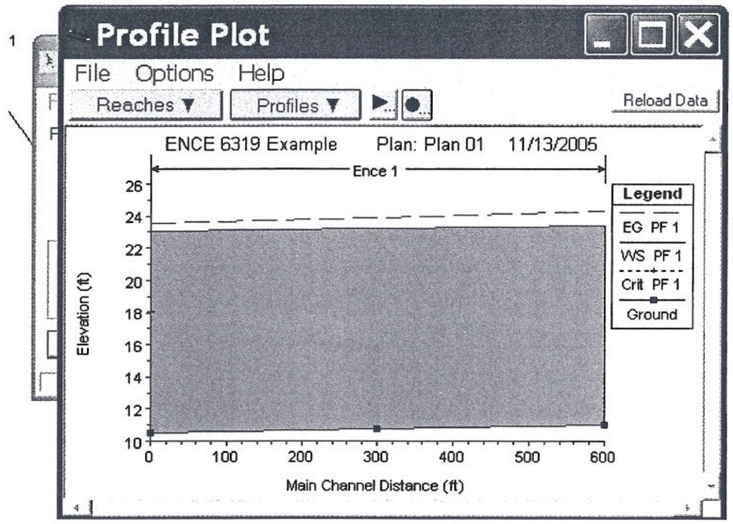


Figure 8. Profile Output

2. The output also can be obtained in Tables, e.g. second last icon give the Standard Table shown below.

Profile Output Table - Standard Table 1

File Options Std. Tables Locations Help

HEC-RAS Plan: N1 River: Ence Reach: 1 Profile: PF 1

Reach	River Sta	Profile	Q Total	Min Ch El	W.S. Elev	Crit W.S.	E.G. Elev	E.G. Slope	Vel Chnl	Flow Area	Top Width	Froude # Chl
			(cfs)	(ft)	(ft)	(ft)	(ft)	(ft/ft)	(ft/s)	(sq ft)	(ft)	
1	600	PF 1	2000.00	11.00	23.36		24.34	0.001475	8.01	263.91	46.31	0.49
1	300	PF 1	2000.00	10.80	23.18		23.88	0.000930	6.77	323.79	63.83	0.38
1	0	PF 1	2000.00	10.50	23.00	18.03	23.57	0.000730	6.13	368.11	82.55	0.36

Figure 9. Profile Output

	1	2 (1st Trial)	2 (2nd Trial)
Invert elv.	100	100.18	
depth y	10	10	
W.S.El. assumed	110	110.18	110.0943
A	trap. $y(b+zy)$ 400	Rec. $b \times w$ 200	
$V = \frac{Q}{A}$	2.5	5.0	
$V^2/2g$	0.097	0.388	
H_T [ft] $h_z + y + \frac{V^2}{2g}$	100+10+ 0.097= 110.097	100.18 + 10 0.380 = 110.568	

friction slope	1	2
S_f	2.246×10^{-4}	1.192×10^{-3}
$S_{f_{avg}}$		$\frac{\text{1} + \text{2}}{2} = 7.0817 \times 10^{-4}$
$h_{L\,obsv.} = H_{T_2} - H_{T_1}$		110.568 − 110.097 = 0.471
h_L calc.		$S_{f_{avg}}$ (L=300) 0.3
$H_{T\,calc.}$	110.097	$W.S.El_1 + V_1^2/2g + h_{L\,1-2}$ 110.0+ 0.097 + 0.3 = 110.397 ← stop if matches

If Not

	1	2
W.S.el. calc	110	$H_{T_2\,calc} - V_2^2/2g$ = 110.397 − 0.388 = 110.009
Avg W.S.el = (W.S.E.l. calc + W.S.el assumed)($\frac{1}{2}$)	110	$\frac{110.009 + 110.18}{2}$ = 110.0943

Final y_2 w/ goal seek:

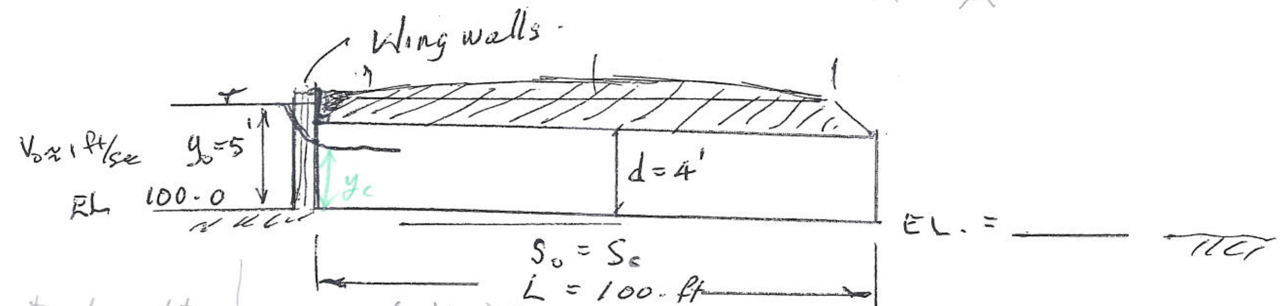
$y_2 = 9.826$ ft ;

$y_3 = 10.101$ ft

Assignment **NAME** Donald Serolleman

Due Date : In class assignment.

Estimate the maximum flow for a 4-ft by 8.ft wide concrete box culvert with a well designed entrance (low contraction) and an upstream depth of 5 ft. The upstream invert is 100. ft. Assume: n = 0.015; Ken=0.0; Kex=1.0   W = 8ft



Assume critical conditions → y_c @ Inlet (Type I)

$E_o = y_o + \frac{V_o^2}{2g} = 5.02'$

Assume $E_c = E_o = \frac{3}{2} y_c$

→ $y_c = 3.34'$

$\frac{y_o}{d} = \frac{5}{4} = 1.2 < 1.25$ so should not be submerged

$S_o = S_c = \left(\frac{n V_c}{c' R_c^{2/3}}\right)^2$

→ $V_c = \sqrt{g y_c} = 10.38$ ft/s

→ $A_c = y_c W = 3.34(8)$

→ $Q_c = V_c A_c = 278$ cfs

→ $R_c = \frac{A_c}{P_c} = \frac{3.34(8)}{2(3.34)+2(8)} = 1.18'$

∴ $S_o = S_c = 0.0088$

∴ downstream invert $= 100 - S_o(L)$

$= 99.12$

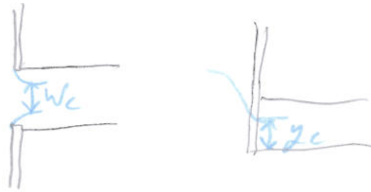
T.W.L $= 99.12 + y_c$

$= 99.12 + 3.34$

$= 102.46'$

What bed slope should be used to ensure the maximum flow?

What is the restriction on the downstream depth?



Type 1

2. Estimate the maximum flow for a 100 ft long 4-ft by 8-ft wide concrete box culvert with a flush entrance and an upstream stage of 105 ft and a downstream stage of 101 ft. The upstream invert is 100.0 ft and the So =Sc. Assume an approach velocity of 1 ft/sec and the tailwater velocity of 1 ft/sec. Contraction Coeff. $C_c = 0.62$ b/c of Flush conditions

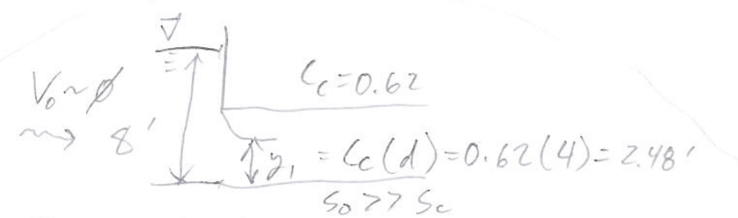
Assume Entrance is unsubmerged; n = 0.015; Kex=1.0.

$W_c = W(C_c) = 8(0.62) = 4.96'$

$E_c = 5.02' = \frac{3}{2} y_c \rightarrow y_c = 3.34'$

$V_c = 10.38$ ft/s

$Q_c = W_c y_c V_c = 172$ cfs



n = 0.015

3. Assuming orifice control, estimate the flow for a 100 ft long 4-ft by 8-ft wide concrete box culvert with a flush entrance and an upstream stage of 108 ft and a downstream stage of 100 ft. The upstream invert is 100. ft. Determine the downstream invert to ensure orifice control. The approach velocity and the tailwater velocity are very low (< 1 ft/sec). Assume Cc=0.62 at entrance. Set the bed slope to ensure free surface flow in the culvert, e.g. $y_n < 0.75d$.

$y_1 = 0.62(4) = 2.48'$

$V_1 = \sqrt{2g(y_0 - y_1)}$

$= \sqrt{2g(8 - 2.48)} = 18.88$ ft/s

$Q_1 = V_1 A_1 = 18.88(2.48)(8) = 375$ cfs

$y_n \geq 0.75d$

$S_o = \left(\frac{n V_n}{c' R_n^{2/3}}\right)^2$ (c' = 1.486)

$V_n = \frac{Q_1}{W y_n} = \frac{375}{8(4(\frac{3}{4}))} = 15.6$ ft/s

$A_n = 8(3) = 24$

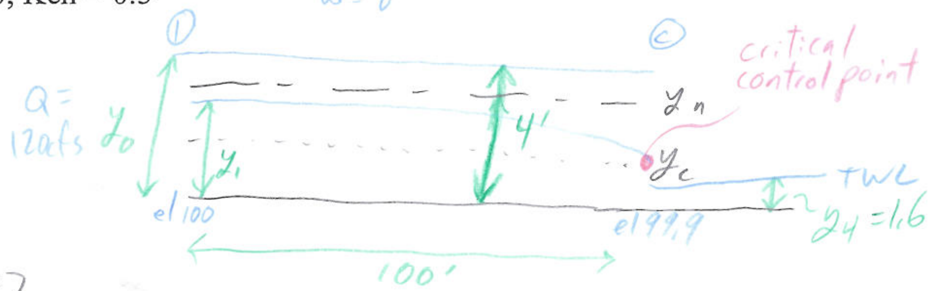
$R_n = \frac{A_n}{P_n} = \frac{24}{2(8) + 2(3)} = 1.09'$

$\therefore S_o = 0.022$

$= 100 - 0.022(100)$
$= 97.8'$

TW = 97.8 + 3 = 100.8' needed to maintain flow

5. Assuming outlet control and a flow of 120 cfs, estimate the upstream depth for a 100 ft long 4-ft by 8-ft wide concrete box culvert with a flush entrance and a downstream stage of 101.5 ft. The upstream invert is 100. ft and downstream invert is 99.9 ft. Neglect the approach velocity and the tailwater velocity in the stream. Assume: n = 0.015; Kex=1.0; Ken = 0.5



$$S_0 = \frac{100 - 99.9}{100} = 0.001$$

(el_1, el_2; $100 \leftarrow L$)

$$y_c = \sqrt[3]{\frac{\left(\frac{Q}{W}\right)^2}{g}} = 1.91' > y_4$$

$$S_c = \left(\frac{n V_c}{c' R_c^{2/3}}\right)^2 ; \quad V_c = \sqrt{g y_c} = 7.85 \text{ ft/s} \qquad R_c = \frac{A_c}{P_c} = \frac{y_c W}{2y_c + W} = 1.29$$

$S_c = 0.004445 > S_0 \therefore$ outlet control

$y_n = 3.25$ ft

$y_1 \sim \frac{1}{2}(y_c + y_n) = 2.55$ ft

$H_{T_c} = 99.9 + 1.91(1.5) = 102.77$ ft

Guess $y_1 \sim 2.55$ ft

(a)

$$H_{T_1} = h_{z_1} + y_1 + \frac{V_1^2}{2g} \quad \begin{matrix} \rightarrow A_1 \\ \rightarrow V_1 = 5.88 \text{ ft/s} \end{matrix}$$

$\therefore \hookrightarrow 103.09$ ft

(b)

$H_{T_1} = H_{T_c} + h_{L_{c-1}}$

$= 102.77 + L(S_{AV}) = 103.09'$

$S_{AV} = \frac{1}{2}(S_c + S_1) = 0.0032$

(a) + (b) agree $\therefore$ correct

$$S_1 = \left(\frac{n V_1}{c' R_1^{2/3}}\right)^2$$

$$H_{T_0} = H_{T_1} + h_{en} \; ; \; h_{en} = K_{en} \frac{V_1^2}{2g}$$

Hydro Dec 2

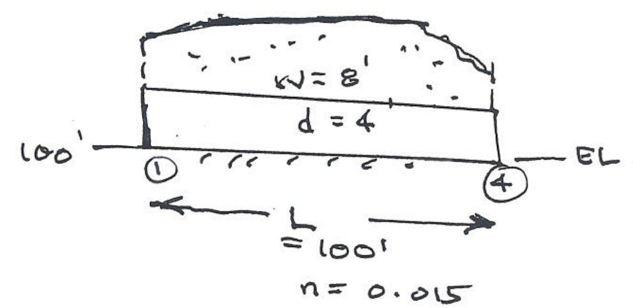
1			2				3				4			
w	8		w	8	Cc	0.62	w	8	Cc	0.62	w	8	Ken	0.5
d	4		d	4			d	4			d	4	Kex	1
n	0.015		n	0.015			n	0.015			n	0.015		
L	100		L	100			L	100			L	100		
INV u/s	100		INV u/s	100			INV u/s	100			INV u/s	100		
yo	5		yo	5			yo	8			yo	8		
Vo	1		Vo	1			Vo	1			Vo	0		
Eo	5.02	ft	Eo	5.016	ft		Eo	8.02	ft		Eo	8	ft	
Ec	5.02		Ec	5.016			y1	2.48			Hto	108		
yc	3.34		yc	3.344	Wc	4.96	V1	18.88	Wc	4.96	Ht4	105		
Vc	10.38		Vc	10.376							H	3		
Ac	26.75		Ac	16.585			Ac	19.84			Rp	1.33333		1.95
Qc	277.56	cfs	Qc	172.087	cfs		Qc	374.597	cfs		Ap	32		
Rc	1.82	ft	Rc	1.821	ft		yn~	3			V1	9.96	ft/sec	9.96
							Rn	1.71			Ap	32	ft2	
Sc	0.00493	~ 0.5%	Sc	0.00493	~ 0.5%		Vn	15.61			Q	319	cfs	
							Sc	0.0121	~ 1.2	%				

$S_o > 0$

Let $S_o = S_c = 0.005$

5	Q	120	cfs	So	0.001	~ 0.1 %			
w	8		y4	1.6	ft				
d	4		yc	1.91	ft	y4<yc	So<Sc	Outlet Control	
n	0.015		Vc	7.85	ft/sec	yn>yc	Slope =M		
L	100		Rc	1.29	ft				
INV u/s	100		Sc	0.00445					
yn	3.25	ft	HTc	102.77	ft				
An	26.0		hz1	100	ft				
Vn	4.62	ft	y1	2.55	ft				
Rn	1.79		A1	20.41	ft²				
CQ	38.30		V1	5.88	ft/sec				
AnRn23	38.30		Ht1 a	103.09	ft				
f	0.00		R1	1.56	ft				
			S1	0.00195					
			Sav	0.0032					
			hf	0.32	ft	negl.	hobby		
			Ht1 b	103.09	ft	-0.0005			
			Hen	0.27	ft				
			Hto	103.36	ft ✓	yo~	3.35602	ft	check

$y_o < d$



Lecture 26
Culvert Hydraulics

Reference *Modern Sewer Design (AISI) and Handouts.*

Combined Rapidly and Gradually Varied Steady Flow.

There are six commonly encountered types of flow in culverts. These are illustrated on the attached Figure.

Type 1 has ***Critical Flow Inlet Control*** which means that there is critical depth at the inlet and there is free surface flow in the culvert. To ensure critical depth, the bed slope must be equal or greater than the critical slope. The actual critical area may also be affected by the flow contractions due to poor entrance conditions, e.g. a pipe projecting into the upstream flow. If the tailwater is too high, it may submerge y_c and reduce the culvert capacity.

$Q = A_c V_c = A_c \sqrt{g D_c}$

Type 2 has ***Critical Flow Outlet Control*** which means that there is critical depth at the outlet. This may occur if the bed slope is less than the critical slope. There is a backwater profile between the downstream and the upstream end of the culvert. The tailwater is less or equal to y_c for this case. There may or may not be free surface flow in the culvert for this case.

less efficient than Type I

Type 3 has *Tailwater Outlet Control* which means that the tailwater depth exceeds the critical depth at the outlet and/or the critical depth at the entrance. There is a backwater profile between the downstream and the upstream end of the culvert. The tailwater is greater than the critical depth ($h_4 < y_c$) for this case. There may or may not be free surface flow in the culvert for this case.

Type 4 has *a Submerged Outlet Control* which means that the tailwater depth exceeds the pipe depth at the outlet ($h_4 > D$). There is pressure flow in the pipe. The energy principle is commonly used to analyze this case. Usually, the inlet is also submerged in this case.

$h_L = H_{T0} - H_{T4} = K_m \frac{V_p^2}{2g} + K_e \frac{V_p^2}{2g} + h_f$

Type 5 has ***a Submerged Inlet Control with Free Surface Flow*** which means that there is an orifice type control at the inlet and there is atmospheric pressure downstream of the entrance. Generally, the upstream depth is about 50% higher than the pipe depth. This also requires one or more of the following conditions: low tailwater depth ($h_4 < D$), a steep slope, a short pipe length, low friction and unsubmerged outlet. There is no pressure flow in the pipe. The orifice equation is commonly used to analyze this case.

← otherwise back pressure builds & raises water level before pipe

want "free surface flow" or orifice flow ← Do not completely fill pipe

Type 6 has ***a Pipe Friction Control with an Unsubmerged Outlet*** which means that there pressure flow downstream of the entrance but the tailwater is less than the outlet pipe depth ($h_4 < D$). Generally, the upstream depth is about 50% higher than the pipe depth This may occur under one or more of the following conditions: a mild slope, a long pipe length, high friction. There is pressure flow in the pipe. The energy principle is commonly used to analyze this case. Assumption $h_4 < 0.85D$ then $h_4 = 0.85D$.

The US Bureau of Public Roads has developed nomographs for several of these types of flows; examples of these nomographs are attached to this lecture. The nomographs included:

1) Various inlet controls (Types 1 and 5) and pressure flow (Types 4 and 6).
2) Circular and box culverts.
3) Corrugated-metal ($n = 0.024$) and concrete pipes ($n = 0.012$ Note: this may be low especially for older pipes).

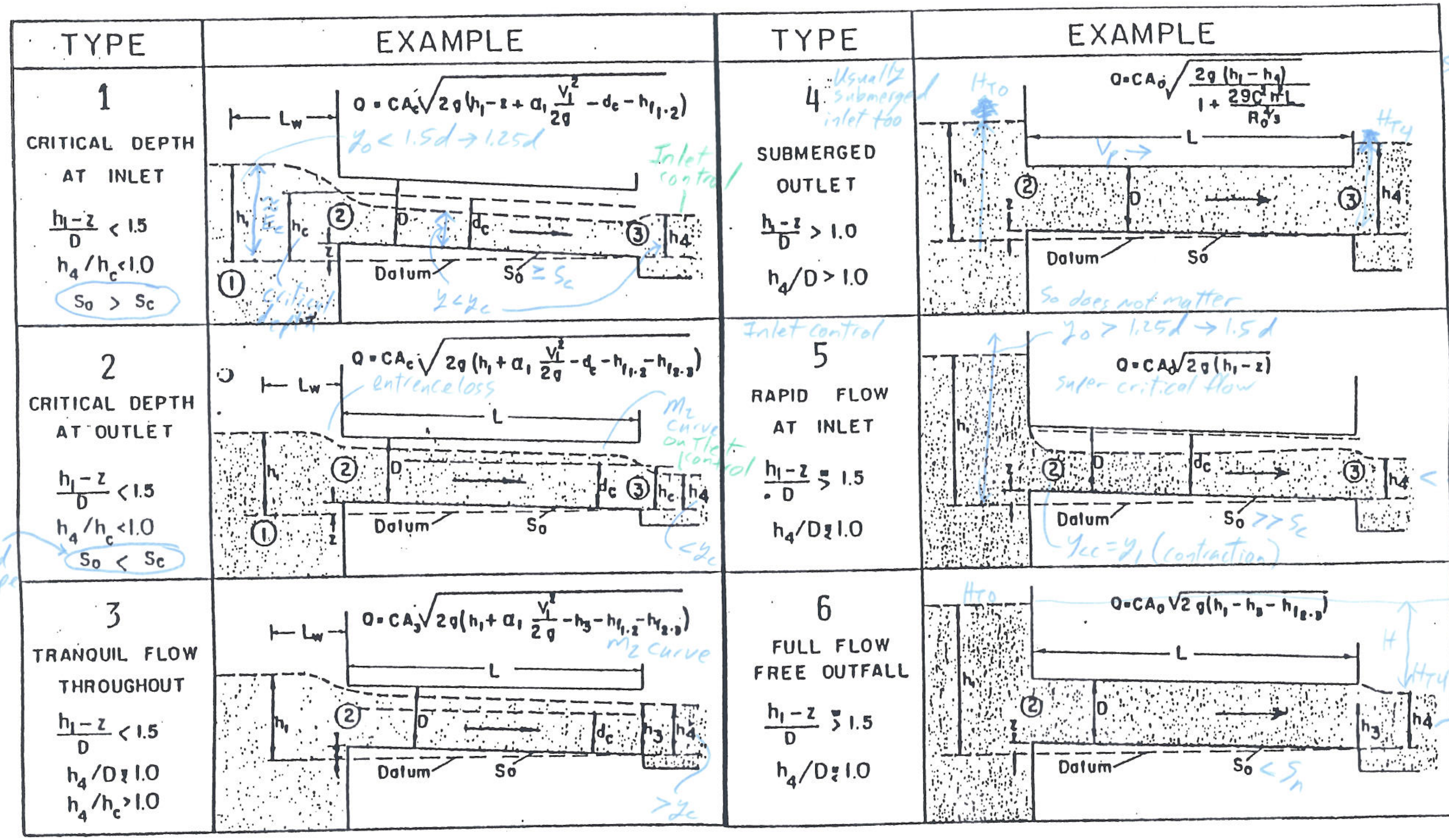


Fig. 2.2. U.S. Geological Survey Culvert Flow Classification (Bodhaine, 1968)

lically sh~~t and a hydraulically long culvert. Under suitable conditions, a hydra. ‍lly short culvert with submerged entrance may prime itself automatically and thus flow full. According to the laboratory investigations by Li and Patterson [28], this self-priming action is due to a rise of the water up to the top of the culvert caused in most cases by a hydraulic jump, the backwater effect of the outlet, or a standing surface wave developed inside the barrel.

For practical purposes, culvert flow may be classified into six types, as shown in Fig. 17-28. The identification of each type may be explained according to the following outline:

- A. Outlet submerged....................................Type 1
- B. Outlet unsubmerged
  - 1. Headwater greater than the critical value
    - *a.* Culvert hydraulically long........................Type 2
    - *b.* Culvert hydraulically short........................Type 3
  - 2. Headwater less than the critical value
    - *a.* Tailwater higher than the critical depth.............Type 4
    - *b.* Tailwater lower than the critical depth
      - i. Slope subcritical..............................Type 5
      - ii. Slope supercritical...........................Type 6

If the outlet is submerged, the culvert will flow full like a pipe, and the flow will be of type 1. If the outlet is not submerged, the headwater may be either greater or less than the critical value. When the headwater is greater than the critical value, the culvert may be either hydraulically short or long; these can be differentiated by means of the charts in Figs. 17-26 and 17-27. The flow is of type 2 if the culvert is hydraulically long and of type 3 if it is hydraulically short. When the headwater is less than the critical value, the tailwater may be either higher or lower than the critical depth of the flow at the culvert outlet. For higher tailwater, the flow is of type 4. For lower tailwater, the flow is of type 5 if the culvert slope is subcritical and of type 6 if the slope is supercritical.

In the above classification, there is an exception in that type 1 flow can occur with tailwater slightly higher than the critical depth or with tailwater higher than the top of the outlet if the bed slope is very steep. The first two types of flow are pipe flow, and the other types are open-channel flow. For type 3 flow, the culvert acts like an orifice. The coefficient of discharge varies approximately from 0.45 to 0.75. For type 4, 5, and 6 flows, the entrance is not sealed by water and it acts like a weir. The discharge coefficient varies approximately from 0.75 to 0.95, depending on the entrance geometry and headwater condition. As shown in Fig. 17-28, type 4 flow is subcritical throughout the barrel length. Type 5 flow is subcritical and, hence, the control section is at

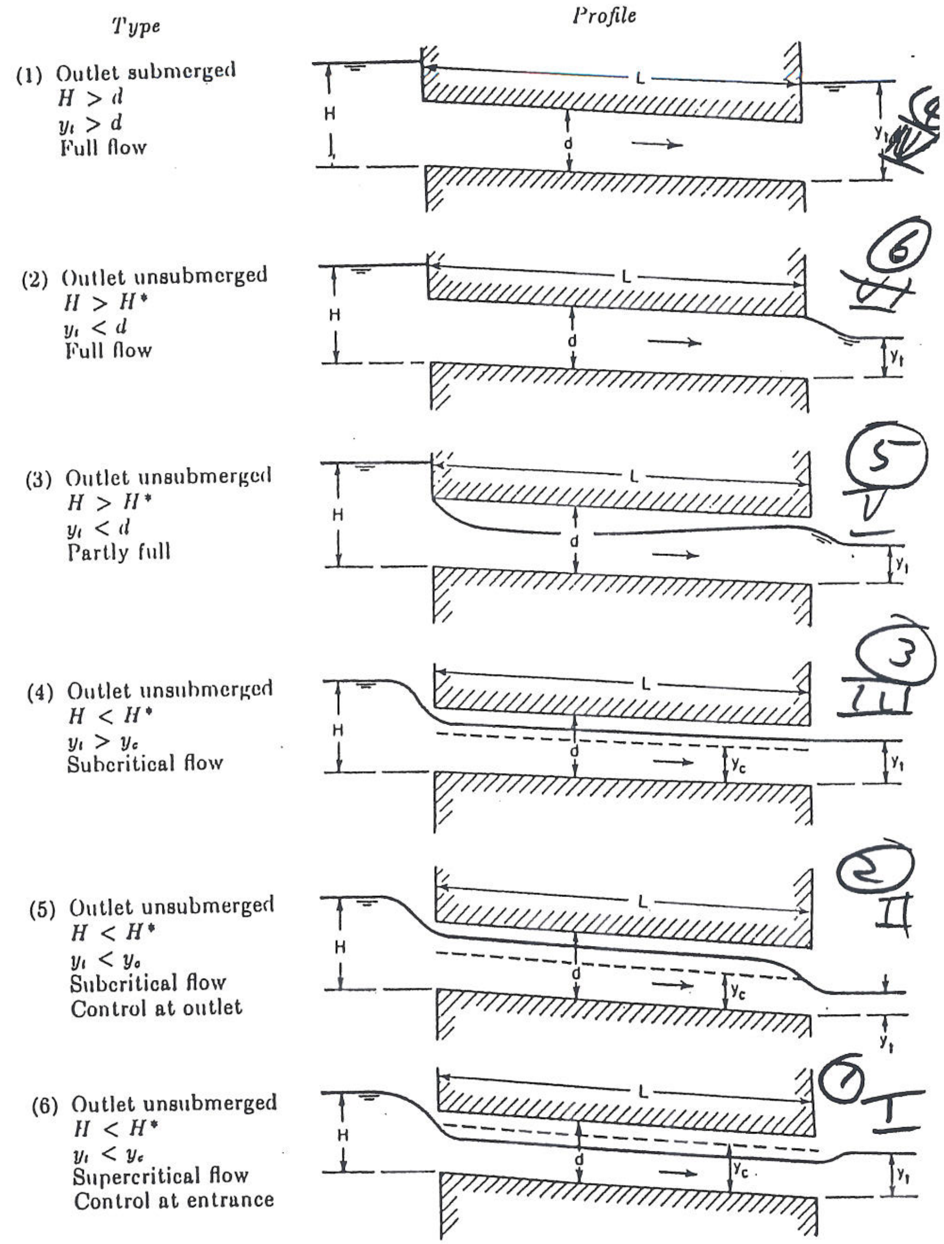
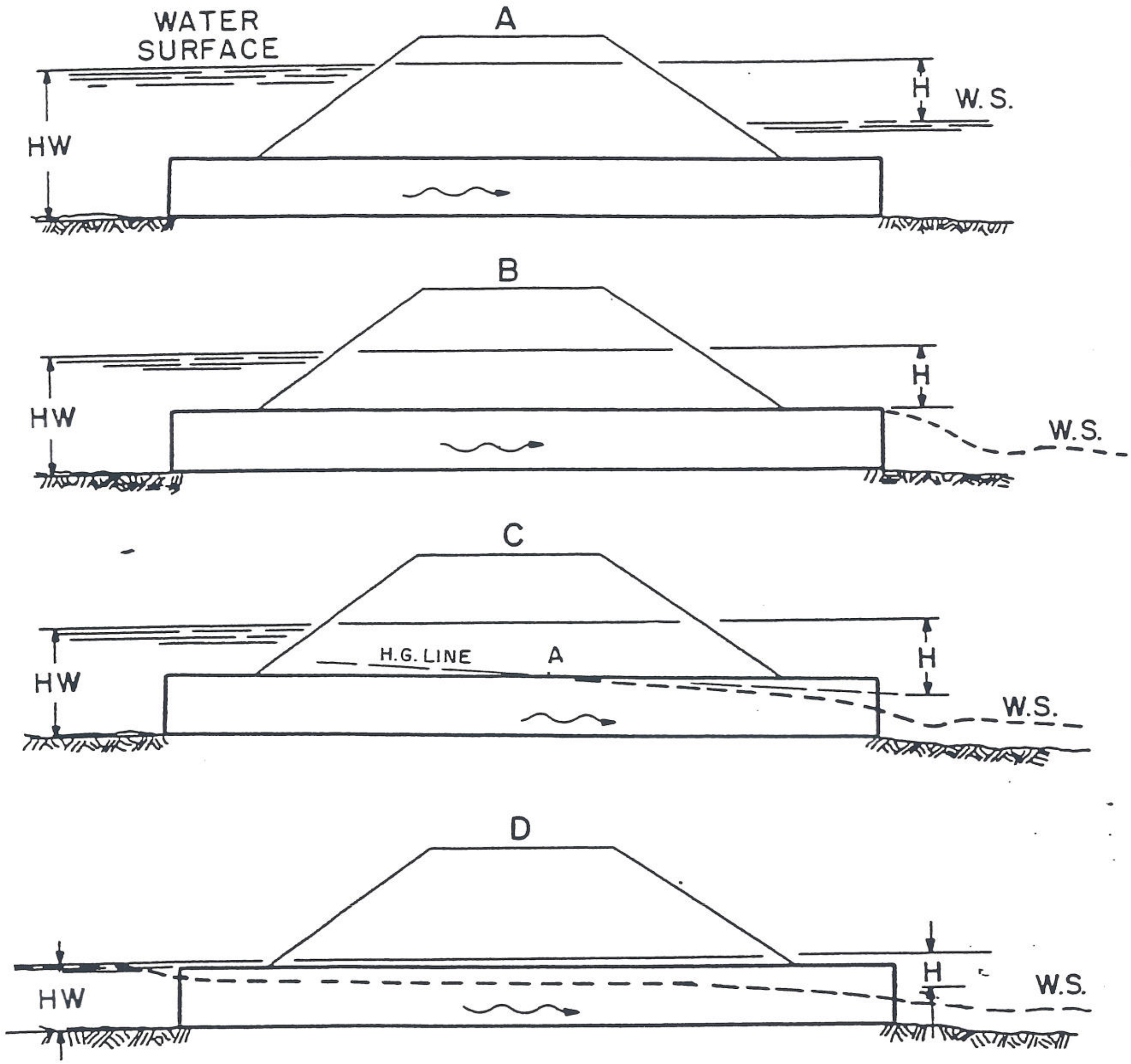
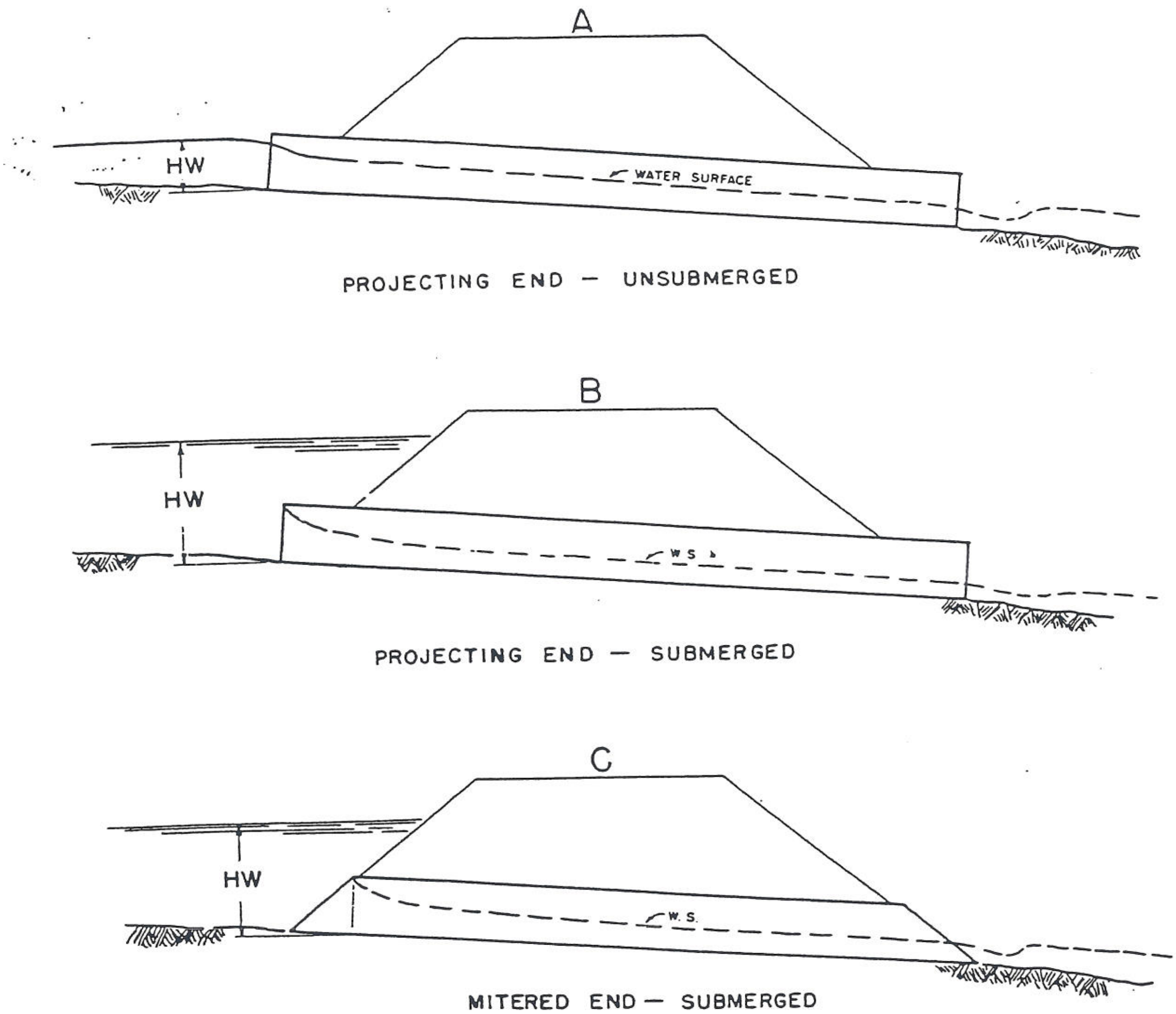


Fig. 17-28. Types of culvert flow.



OUTLET CONTROL

Figure 2



INLET CONTROL

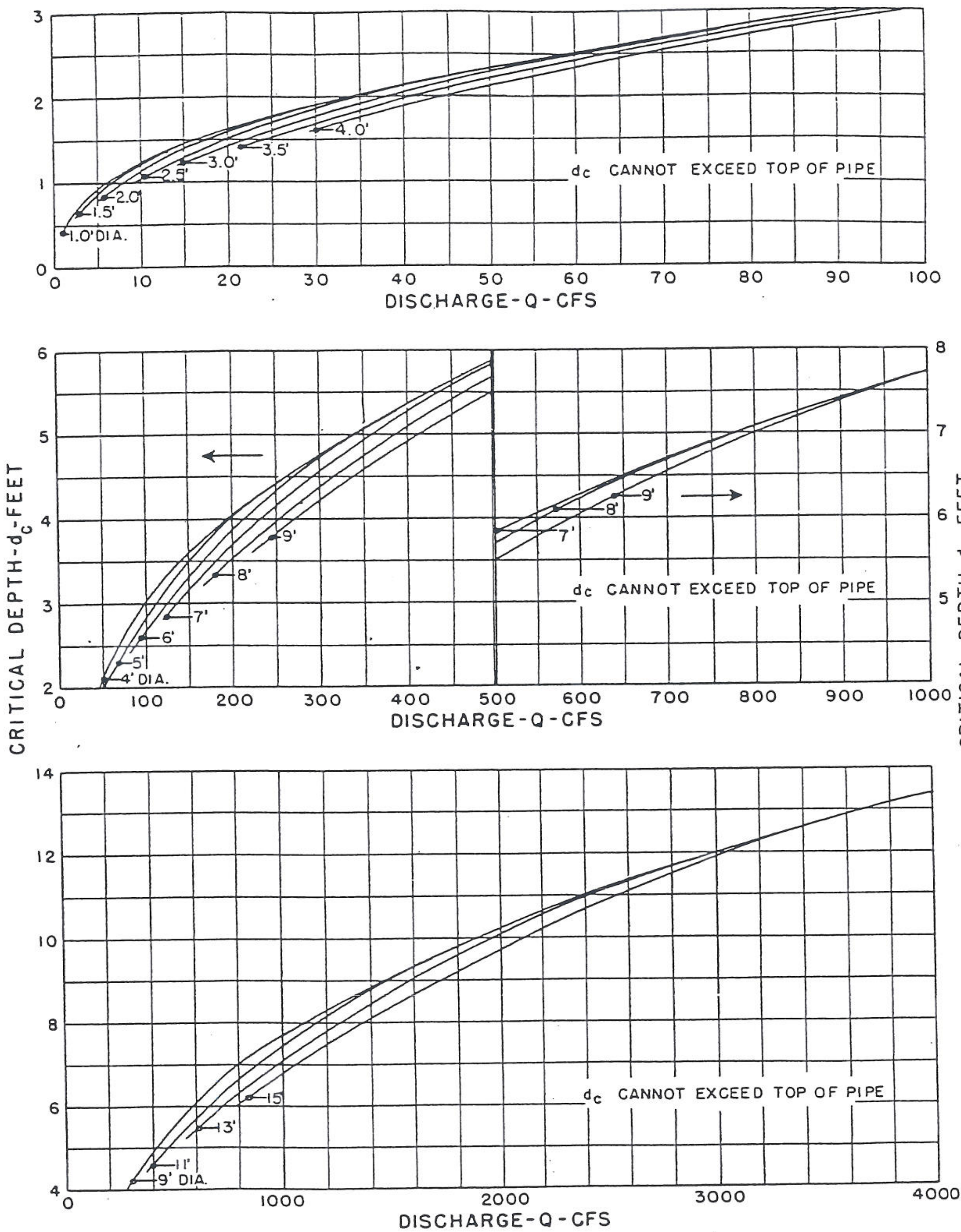
Figure 1

$Q_c = A_c \sqrt{D_c} \sqrt{g}$

Need y_c

$E_c = y_c + \frac{D_c}{2}$

CHART 16



BUREAU OF PUBLIC ROADS

JAN. 1964

CRITICAL DEPTH
CIRCULAR PIPE

TABLE 1 - ENTRANCE LOSS COEFFICIENTS

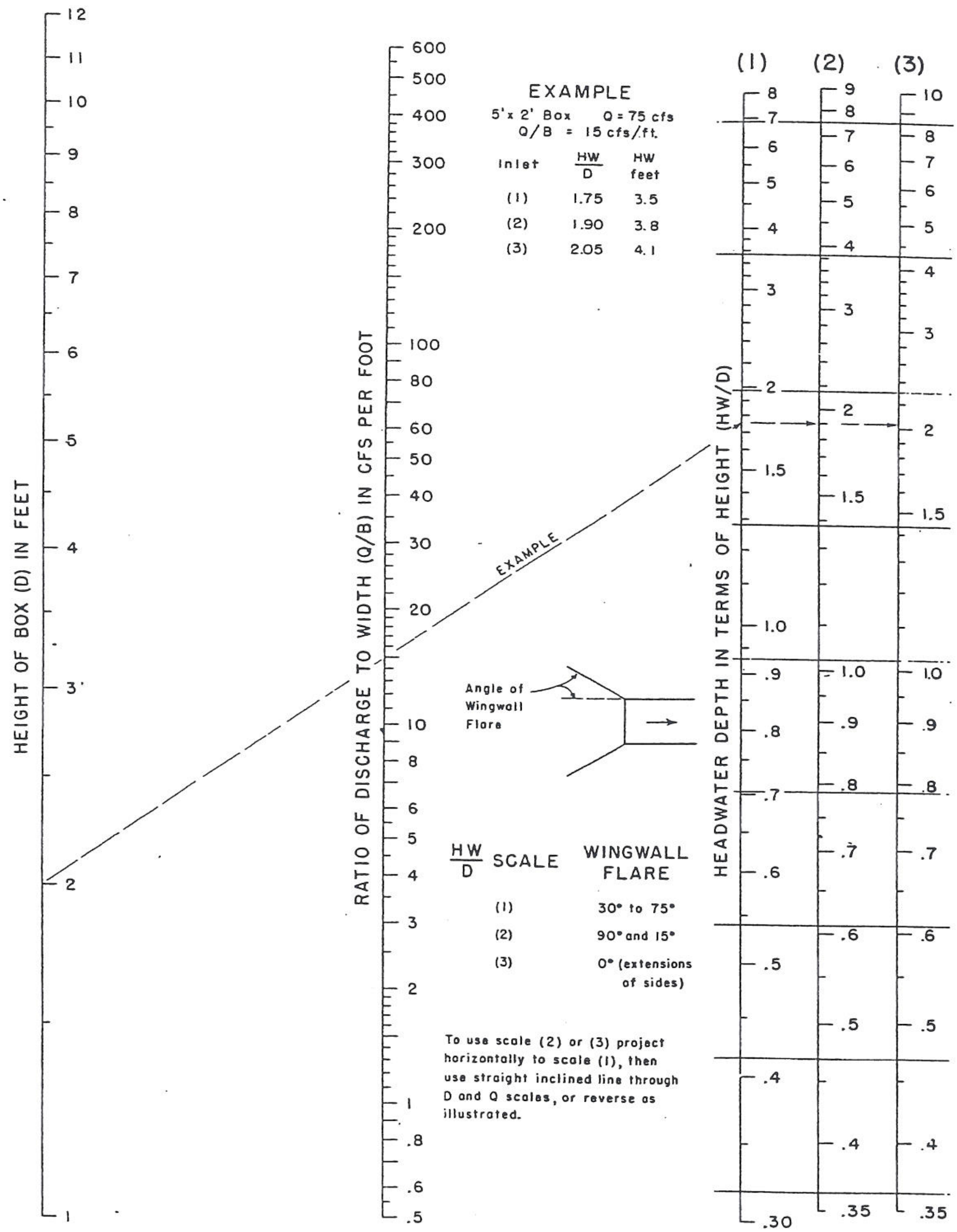
Outlet Control, Full or Partly Full

Entrance head loss $H_e = k_e \frac{V^2}{2g}$

Type of Structure and Design of Entrance	Coefficient k_e
Pipe, Concrete	
Projecting from fill, socket end (groove-end) . . .	0.2
Projecting from fill, sq. cut end	0.5
Headwall or headwall and wingwalls	
Socket end of pipe (groove-end)	0.2
Square-edge	0.5
Rounded (radius = 1/12D)	0.2
Mitered to conform to fill slope	0.7
*End-Section conforming to fill slope	0.5
Beveled edges, 33.7° or 45° bevels	0.2
Side-or slope-tapered inlet	0.2
Pipe, or Pipe-Arch, Corrugated Metal	
Projecting from fill (no headwall)	0.9
Headwall or headwall and wingwalls square-edge . .	0.5
Mitered to conform to fill slope, paved or unpaved slope	0.7
*End-Section conforming to fill slope	0.5
Beveled edges, 33.7° or 45° bevels	0.2
Side-or slope-tapered inlet	0.2
Box, Reinforced Concrete	
Headwall parallel to embankment (no wingwalls)	
Square-edged on 3 edges	0.5
Rounded on 3 edges to radius of 1/12 barrel dimension, or beveled edges on 3 sides . . .	0.2
Wingwalls at 30° to 75° to barrel	
Square-edged at crown	0.4
Crown edge rounded to radius of 1/12 barrel dimension, or beveled top edge	0.2
Wingwall at 10° to 25° to barrel	
Square-edged at crown	0.5
Wingwalls parallel (extension of sides)	
Square-edged at crown	0.7
Side-or slope-tapered inlet	0.2

*Note: "End Section conforming to fill slope," made of either metal or concrete, are the sections commonly available from manufacturers. From limited hydraulic tests they are equivalent in operation to a headwall in both inlet and outlet control. Some end sections, incorporating a closed taper in their design have a superior hydraulic performance. These latter sections can be designed using the information given for the beveled inlet, p. 5-13.

CHART 1



HEADWATER DEPTH
FOR BOX CULVERTS
WITH INLET CONTROL

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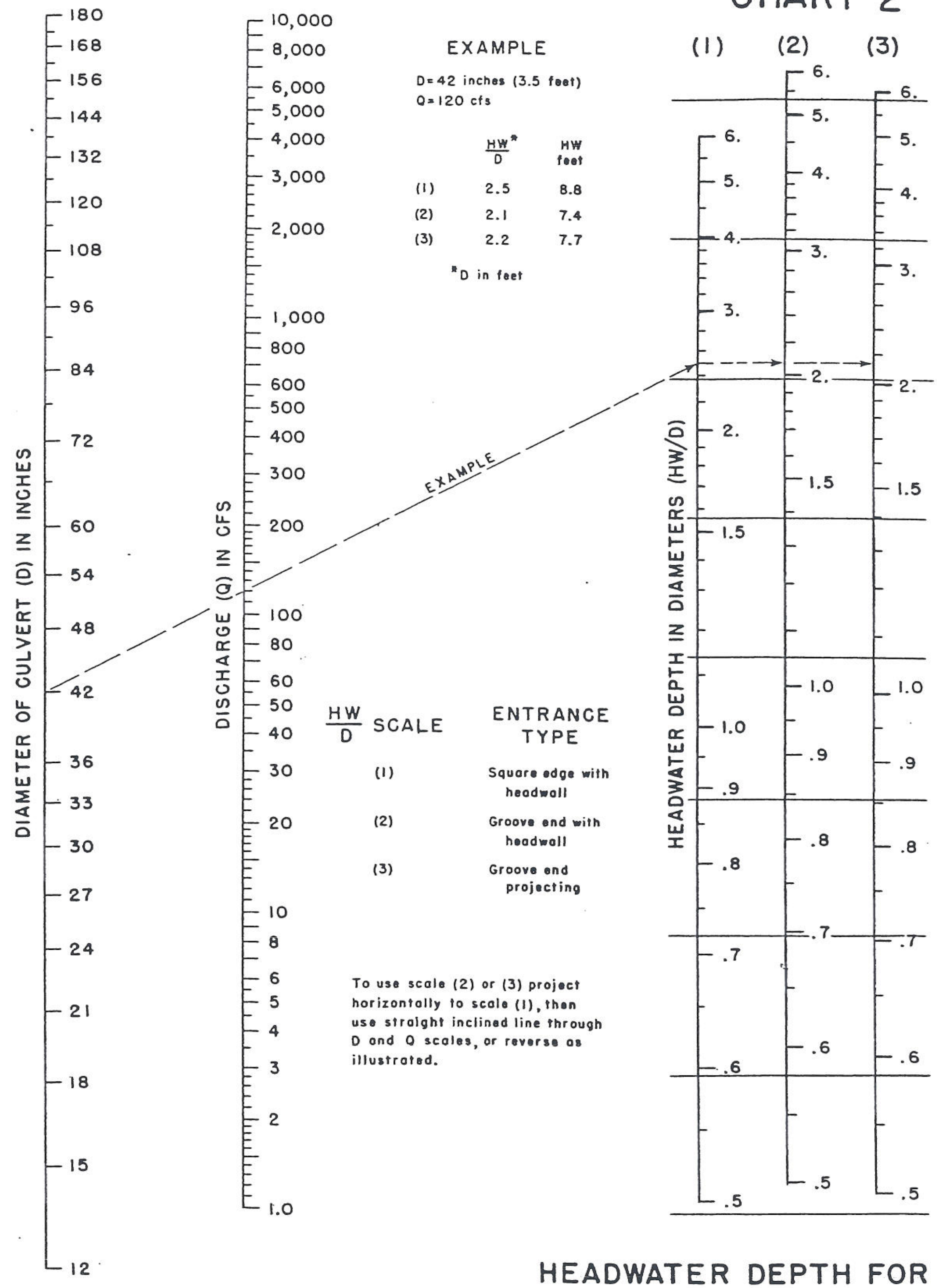
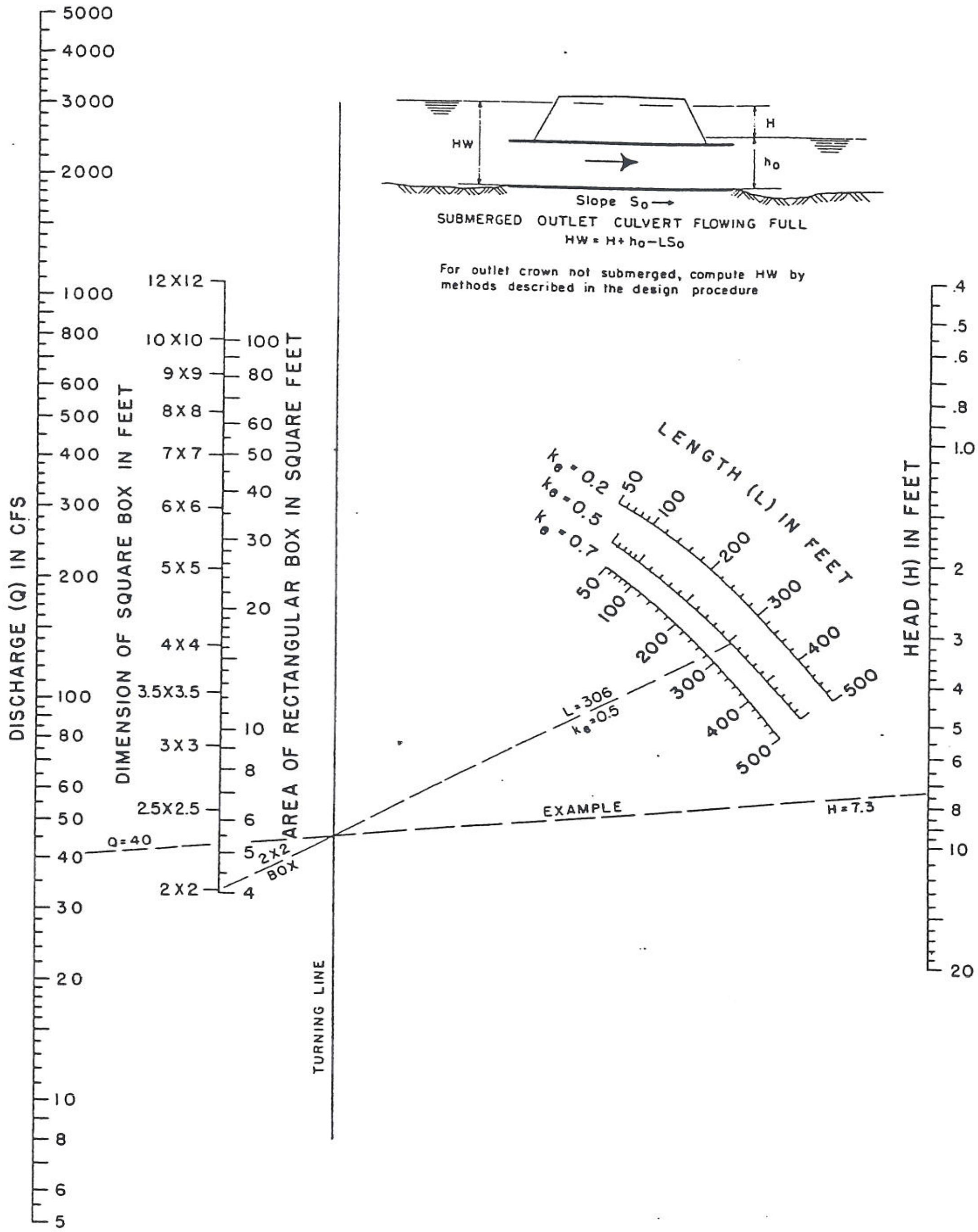
CHART 2



HEADWATER DEPTH FOR CONCRETE PIPE CULVERTS WITH INLET CONTROL

HEADWATER SCALES 2&3
REVISED MAY 1964

BUREAU OF PUBLIC ROADS JAN. 1963



HEAD FOR
CONCRETE BOX CULVERTS
FLOWING FULL
n = 0.012

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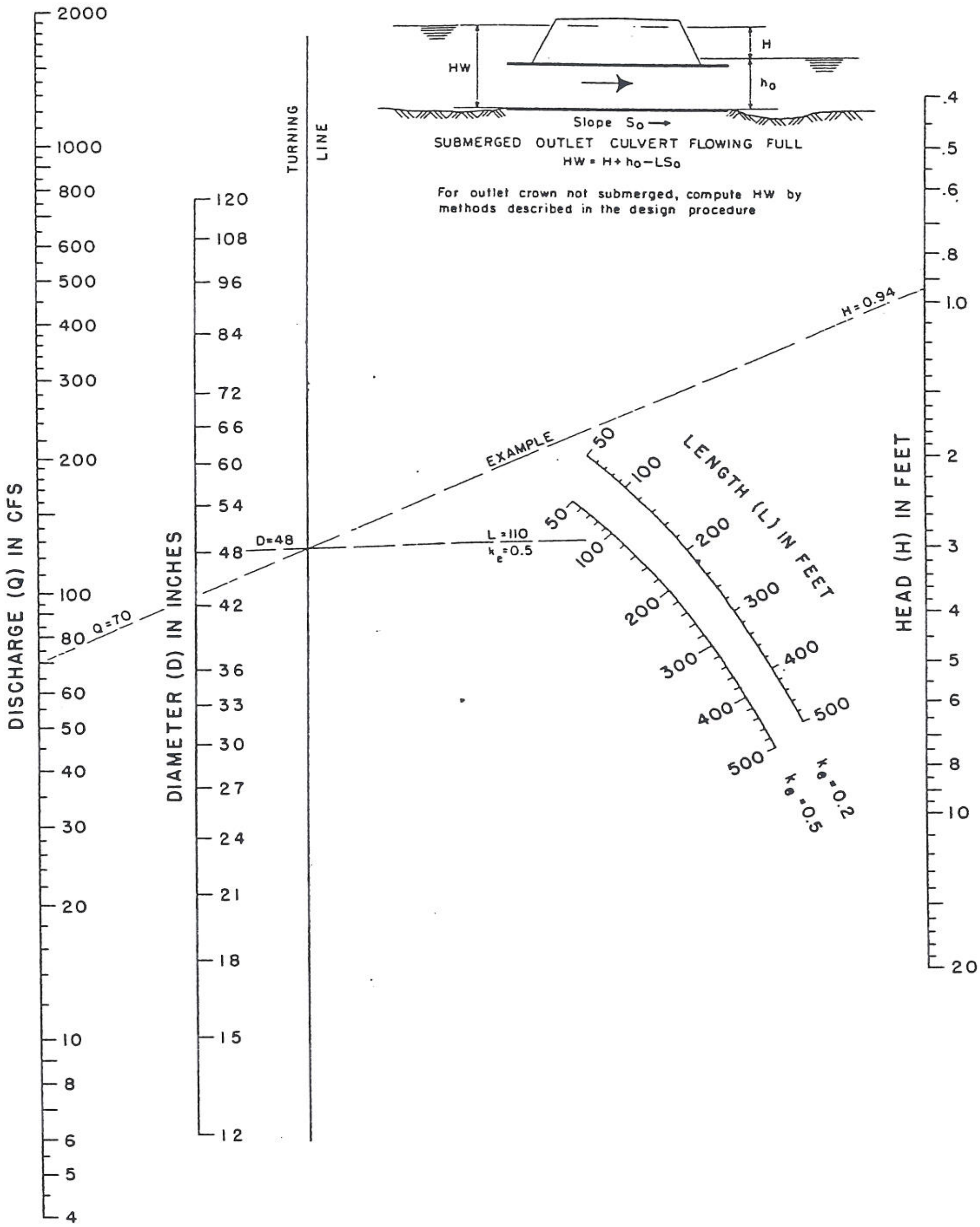

# HEAD FOR CONCRETE PIPE CULVERTS FLOWING FULL
n = 0.012

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To use scale (2) or (3), project horizontally to scale (1), then use straight inclined line through D and Q scales, or reverse as illustrated.

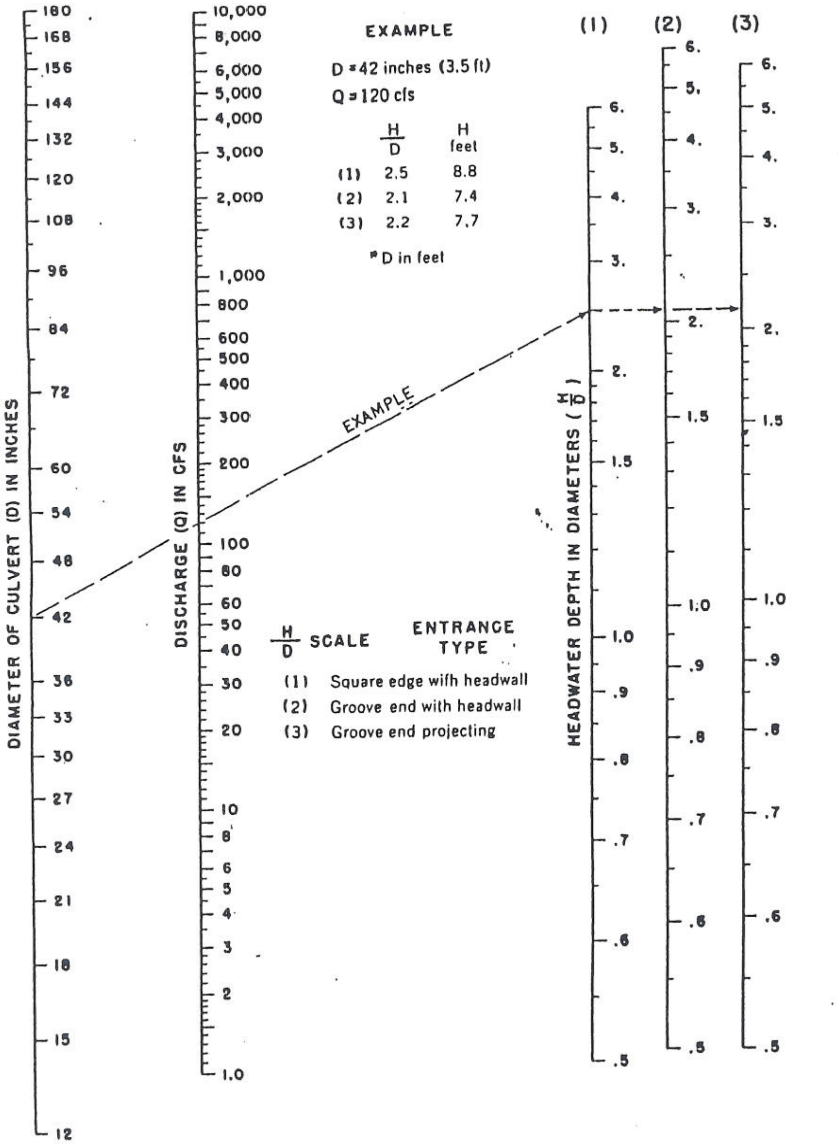


Figure B-8. **Headwater depth for concrete pipe culverts with entrance control. (U.S. Bureau of Public Roads.)**

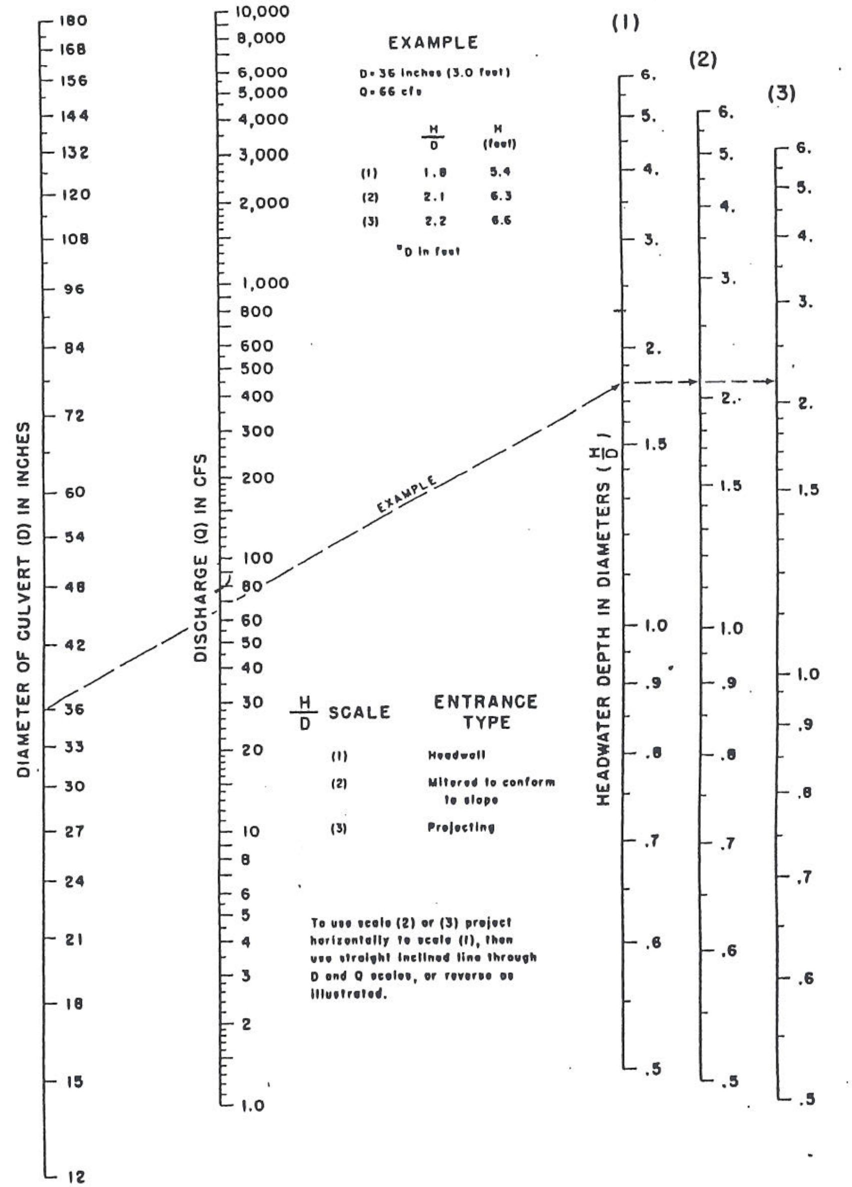
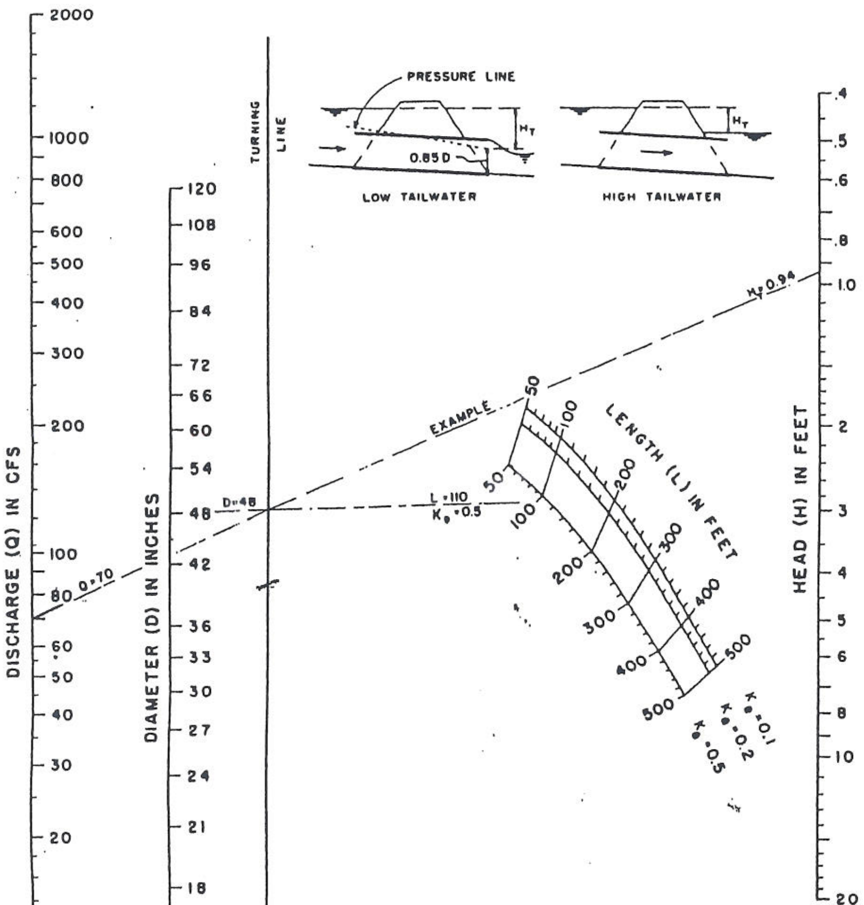


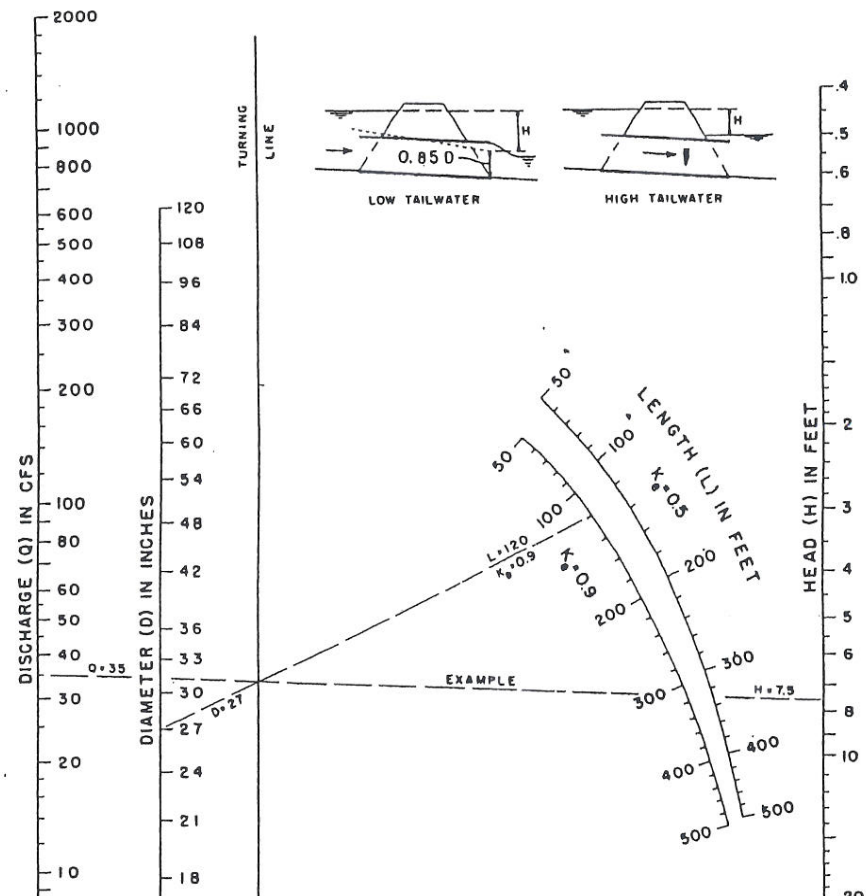
Figure B-9. **Headwater depth for corrugated-metal pipe culverts with entrance control. (U.S. Bureau of Public Roads.)**



EQUATION: $H_T = \left[\frac{2.5204(1+K_e)}{D^4} + \frac{466.18\,n^2 L}{D^{16/3}}\right]\left(\frac{Q}{10}\right)^2$

H_T = Head in feet
K_e = Entrance loss coefficient
D = Diameter of pipe in feet
n = Manning's roughness coefficient
L = Length of culvert in feet
Q = Design discharge rate in cfs

Figure B-10. Head for concrete pipe culverts flowing full, n=0.012. (U.S. Bureau of Public Roads.)



EQUATION: $H_T = \left[\frac{2.5204(1+K_e)}{D^4} + \frac{466.18\,n^2 L}{D^{16/3}}\right]\left(\frac{Q}{10}\right)^2$

H_T = Head in feet
K_e = Entrance loss coefficient
D = Diameter of pipe in feet
n = Manning's roughness coefficient
L = Length of culvert in feet
Q = Design discharge rate in cfs

Figure B-11. Head for corrugated-metal pipe culverts flowing full, n=0.024. (U.S. Bureau of Public Roads.)

Re-entrant $K_e \approx 1$
Flush $K_e \approx 0.5$

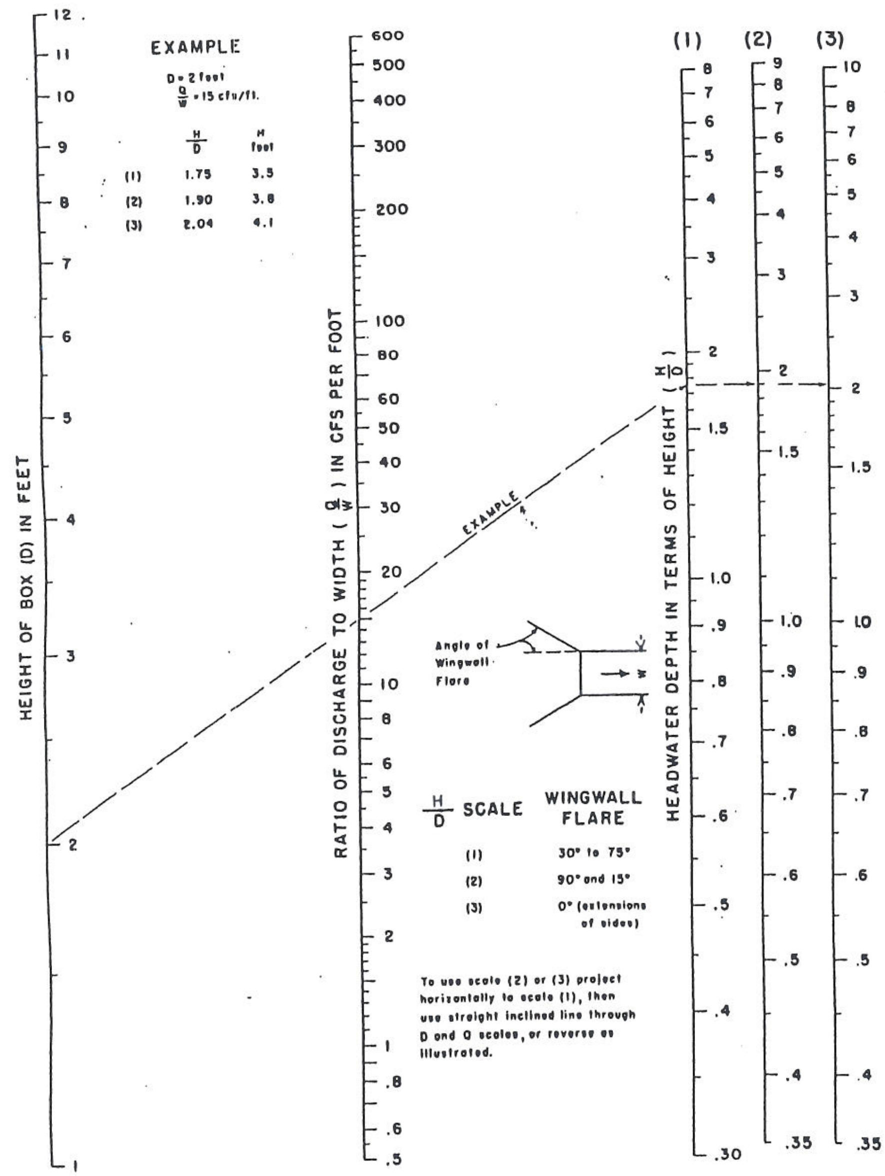


Figure B-12. Headwater depth for box culverts with entrance control. (U.S. Bureau of Public Roads.)

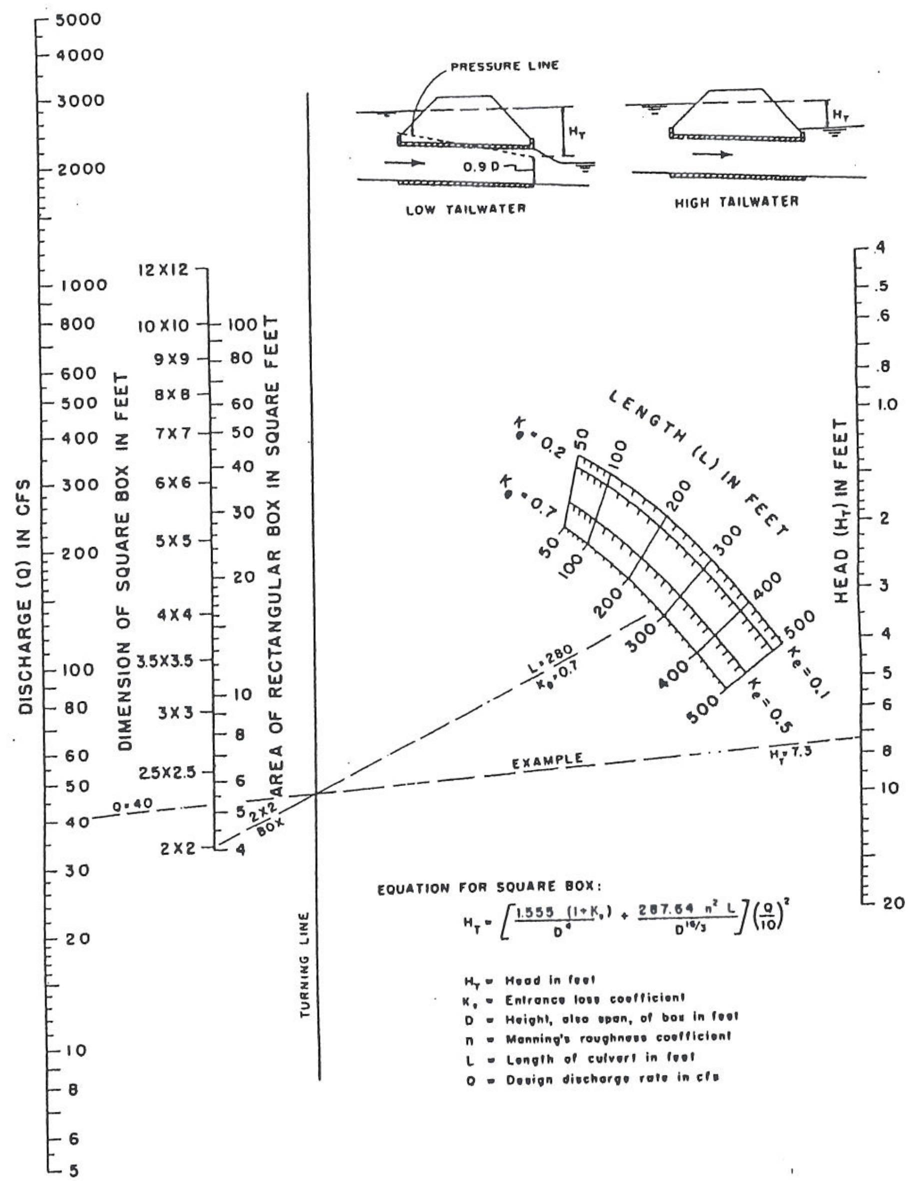


Figure B-13. Head for concrete box culverts flowing full, $n=0.013$. (U.S. Bureau of Public Roads.)

Figure 3 shows the terms of equation 2, the energy line, the hydraulic grade line and the headwater depth, HW. The energy line represents the total energy at any point along the culvert barrel. The hydraulic grade line, sometimes called the pressure line, is defined by the elevations to which water would rise in small vertical pipes attached to the culvert wall along its length. The energy line and the pressure line are parallel over the length of the barrel except in the immediate vicinity of the inlet where the flow contracts and re-expands. The difference in elevation between these two lines is the velocity head, $\frac{V^2}{2g}$.

The expression for H is derived by equating the total energy upstream from the culvert entrance to the energy just inside the culvert outlet with consideration of all the major losses in energy. By referring to figure 3 and using the culvert invert at the outlet as a datum, we get:

$$d_1 + \frac{V_1^2}{2g} + LS_o = d_2 + H_v + H_e + H_f$$

where

d_1 and d_2 = depths of flow as shown in fig. 3

$\frac{V_1^2}{2g}$ = velocity head in entrance pool

LS_o = length of culvert times barrel slope

then

$$d_1 + \frac{V_1^2}{2g} + LS_o - d_2 = H_v + H_e + H_f$$

and

$$H = d_1 + \frac{V_1^2}{2g} + LS_o - d_2 = H_v + H_e + H_f$$

From the development of this energy equation and figure 3, head H is the difference between the elevations of the hydraulic grade line at the outlet and the energy line at the inlet. Since the velocity head in the entrance pool is usually small under ponded conditions, the water surface or headwater pool elevation can be assumed to equal the elevation of the energy line. Thus headwater elevations and headwater depths, as computed by the methods given in this circular, for outlet control, can be higher than might occur in some installations. Headwater depth is the vertical distance from the culvert invert at the entrance to the water surface, assuming the water surface (hydraulic grade line) and the energy line to be coincident, $d_1 + \frac{V_1^2}{2g}$ in figure 3.

NAME: Donald Jerolleman
DUE: TUES NOV. 30/10

13/20

9-7. Sketch the possible flow profiles in the channels shown in Fig. 9-13.

given SLOPE ZONE

∅ = possible critical controls

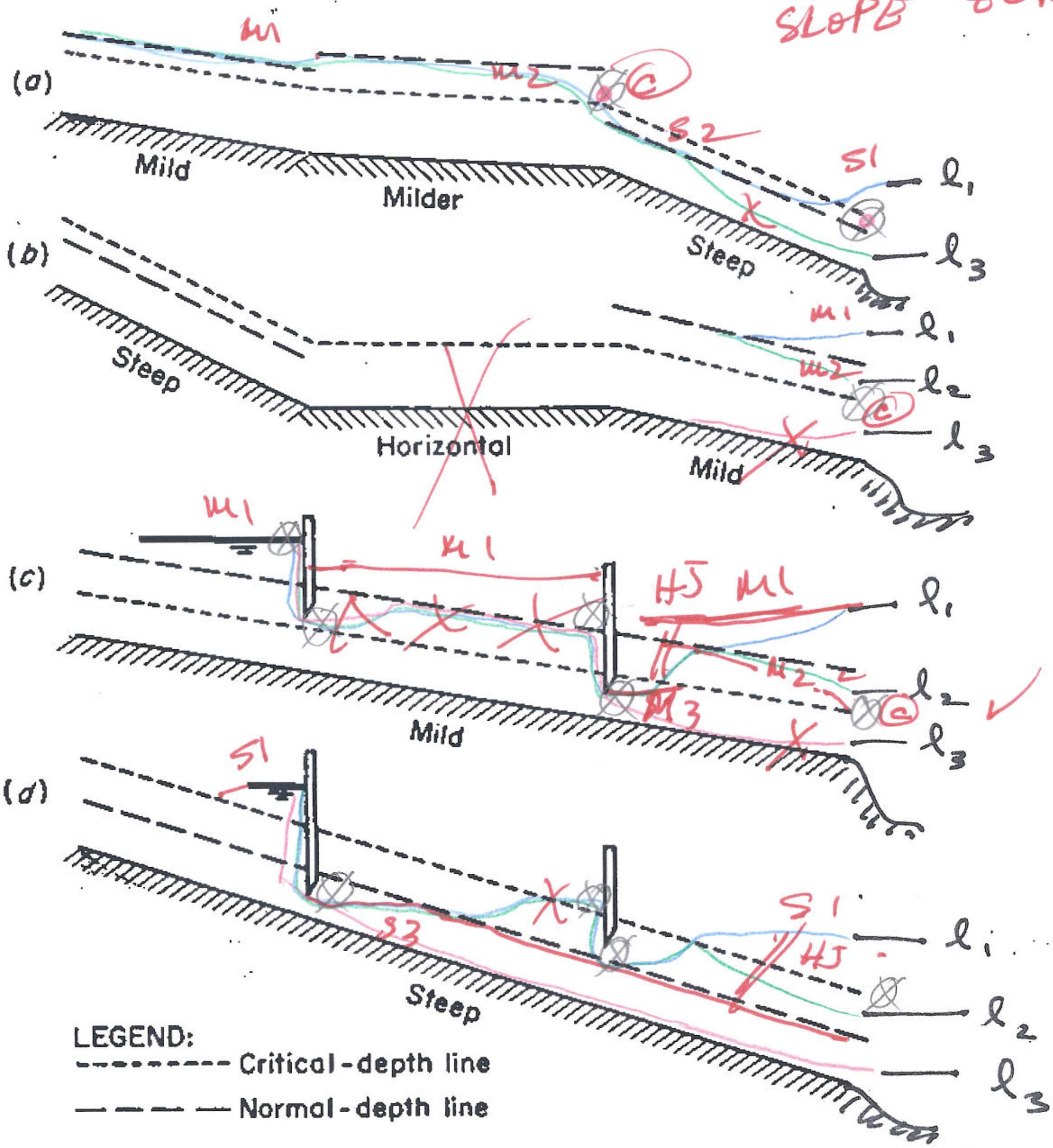


FIG. 9-13. Channels for Prob. 9-7. The vertical scale is exaggerated.

72/100

Tutorial 9 Donald Jecollemon p. 1 of 2

1) Select High & Low Flows

$Q_{High} = CiA = 0.42(1.5\ in/hr)(8000\ \text{Acres}) = 5040\ cfs$?

$Q_{Low} = 3\ cfs$

Drawing -25

2) Side Slope $z = 2$

3) Bed Slope $S_o = 0.0006$

4) Lining = Concrete → $n = 0.013$ → $V_{max} = 20\ ft/s$

5) min Vel → $V_{min} = 2\ ft/s$

6) optimum $b/y = 2[(1+z)^{1/2} - z] = 0.472$ ✓ equ 17.3

7) Calc. Depth

$C_a = \frac{nQ}{c' S_o^{1/2}} = \frac{0.013(5040)}{1.486(0.0006)^{1/2}} = 1800\ cfs$ ✓

$y_n = C_a^{3/8} \frac{[b/y + 2(1+z^2)^{1/2}]^{1/4}}{(b/y + z)^{5/8}} = 14.1\ ft$ ✓

8) Find b: $b = b/y\,(y_n) = 0.472(14.1) = 6.65\ ft$ ✓

9) Check $V_{max/min}$, N_F

Bmax = ? -3

$A = y(b + zy) = 491\ ft^2$ ✓

$V_{max} = Q_{max}/A = 5040/491 = 10.3\ ft/s < 20$ ∴ OKAY

$N_F = \frac{V}{\sqrt{gD}}$; $D = \frac{A}{B} = \frac{y(b+zy)}{b+2zy} = 7.787'$ ✓

∴ $N_F = 10.3/(32.2(7.787))^{1/2} = 0.65 < 0.8$ ∴ OKAY ✓

10) ADD Freeboard: $FB_1 = 0.439 \ln Q_{cfs} - 1.5 = 2.24\ ft$ ✓

$FB_2 = 0.476 \ln Q - 0.2 = 3.86\ ft$ ✓

11) ADD Drainage, Frost Control & Safety Fences (see diagram)

12) Design Sub Channel for low flow (best)

$z = \frac{\sqrt{3}}{3} = 0.577$

$Q = \frac{c'}{n}\sqrt{3}\,y^2\left(\frac{y}{2}\right)^{2/3} S_o^{1/2}$ → $0.982 = y^{8/3}$ → $y_{min} = 0.99\ ft$ ✓

$b^* = \frac{2}{\sqrt{3}}y = 1.15\ ft$ ✓ $A^* = \sqrt{3}\,y = 1.71\ ft$

$V_{min} = Q_{min}/A_{min} = 3/1.71 = 1.75\ ft/s < 2$ ✓ ∴ must allow for maintnance

1) $y_c = \sqrt[3]{\frac{\left(\frac{Q}{w}\right)^2}{g}}$; $E_c = \frac{3}{2} y_c$

$$E_0 = y_0 + \frac{Q^2}{2g(y_1 w)^2} = E_c + S$$

$$E_1 = y_1 + \frac{Q^2}{2g(y_1 w)^2} = E_c + S$$

$$y_2 = \frac{y_1}{2}\left(\sqrt{1+8N_{F_1}^2} - 1\right)$$

2)(a) $y_0 + \frac{Q^2}{2g(y_0 w)^2} = \frac{3}{2}\left(\frac{(Q/w)^2}{g}\right)^{1/3} + S$

$$\rightarrow 5 + \frac{Q^2}{2(32.2)(5(3))^2} = \frac{3}{2}\left(\frac{Q^2/3^2}{32.2}\right)^{1/3} + 2$$

wolframalpha.com $\rightarrow$ $\boxed{Q = 52.9 \text{ ft}^3/\text{s}}$

(b) $y_1 + \frac{Q^2}{2g(y_1 w)^2} = \frac{3}{2}\left[\frac{(Q/w)^2}{g}\right]^{1/3} + 2$

$$y_1 + \frac{(52.9)^2}{2g\, y_1^2 (3^2)} = \frac{3}{2}\left[\frac{(52.9/3)^2}{32.2}\right]^{1/3} + 2$$

$$y_1 + \frac{4.828}{y_1^2} = 5.194 \rightarrow y_1^3 - 5.194 y^2 + 4.828 = 0$$

wolframalpha.com $\rightarrow$ $\boxed{y_1 = 1.08 \text{ ft}}$

(c) $E_1 - y_1 = \frac{V_1^2}{2g} = 5.194 - 1.08 = \frac{V_1^2}{2(32.2)} \rightarrow \boxed{V_1 = 16.3 \text{ ft}/\text{s}}$

$$N_{F_1} = \frac{V_1}{\sqrt{g y_1}} = \frac{16.3}{\sqrt{32.2(1.08)}} = \boxed{2.76}$$

$$y_2 = \frac{y_1}{2}\left(\sqrt{1+8N_{F_1}^2} - 1\right) = \frac{1.08}{2}\left(\sqrt{1+8(2.76)^2} - 1\right) = \boxed{3.71 \text{ ft}}$$

Assignment 12 **Name** Donald Scrolleman Mark(18 /25)

20 - 2 late - 2

WRITE YOUR ANSWERS HERE!

12.1. y_u = 11.98 ft (5)
y_l = 2.3 ft
Solution

$$F_c = y_c + \frac{D_c}{2} = 12' \; ; \; D_c = \frac{A_c}{B} = \frac{z y_c^2}{2 z y_c} = \frac{y_c}{2} \;\therefore\; 12' = y_c + \frac{\frac{y_c}{2}}{2} = \frac{5}{4} y_c \rightarrow y_c = 9.6 \text{ ft}$$

Zero order approximation $\frac{y_u}{y^{(0)}} = 12' \rightarrow y^{(1)} = E - \frac{Q^2}{2gA^2} = E - \frac{Q^2}{2g(zy^2)^2} = 12 - \frac{400^2}{2g(3\times 12^2)^2}$ {First Correction} cont' on (22)

12.2. y_u = 19.98 ft (5)
y_l = 1.739 ft
Solution
You can use a spreadsheet for this problem.

12.3. y_c = 9.6 ft (5)
Solution

12.4. y_c = 5.64 ft (5)
Solution

-6

12.5. p_c/γ = (5)
Solution

12.1) $E = 12\,ft$, $z = 3$, $Q = 400\ ft^3/s$, $A = zy^2$, $E = y + \frac{Q^2}{2gA^2} = y + \frac{Q^2}{2g(zy^2)^2}$

y_c:
$E_c = y_c + \frac{D_c}{2}$, $B = 2zy_c$, $D_c = \frac{A_c}{B} = \frac{zy_c^2}{2zy_c} = \frac{y_c}{2}$

$\therefore E_c = y_c + \frac{y_c/2}{2} = y_c + \frac{y_c}{4}$

$12 = y_c + \frac{y_c}{4} = \frac{5y_c}{4} \rightarrow y_c = 9.6\ ft$

y_u: $y(0) \cong E = 12\,ft$ $E = y_1 + \frac{Q^2}{2gA^2} = y_1 + \frac{Q^2}{2g(zy(0)^2)^2}$

$12 = y_1 + \frac{400^2}{2(32.2)(3(12)^2)^2} \rightarrow$ $\boxed{y_1 = y_u = 11.98\ ft}$

* Use y_u to find A : $A = zy^2 = zy_u^2 = 3(11.98)^2 = 431\ ft^2$

$12 = y_u + \frac{400^2}{2(32.2)(432)^2} \rightarrow y_u = 11.98\ ft$

---

y_l: $E = y + \frac{V^2}{2g} \rightarrow V = \sqrt{(E-y)2g}$ Assume $y(0) = 10\%\ E$

$\rightarrow y(0) = 0.1(12) = 1.2\ ft$; $V = \sqrt{(12-1.2)2(32.2)} = 26.37\ ft/s$

Use V to find $A = \frac{Q}{V} = \frac{400}{26.37} = 15.17\ ft^2$

$A = zy^2 \rightarrow y = 2.25\ ft$

use y to get new V

$V = \sqrt{(12-2.25)2g} = 25.1\ ft/s$ use V to get new A

$A = \frac{Q}{V} = \frac{400}{25.1} = 15.9\ ft^2$ $A = zy^2 \rightarrow y = 2.3\ ft$

$\therefore$ $\boxed{y_l = 2.3\ ft}$

12. 2) $E = 20\ ft$, $Q = 1400\ ft^3/s$, $E = y + \frac{V^2}{2g}$, $V = \frac{Q}{A}$, $A = y(b + zy)$

$$E = y + \frac{Q^2}{2gA^2} = y + \frac{Q^2}{2gy^2(b+zy)^2}$$

$$20 = y + \frac{Q^2}{2(32.2)\,y^2\,(20+2y)^2}$$

useing wolframalpha.com

$y_n = 19.98$ $\qquad$ $y_c = 1.739$

---

12. 3) $z = 3$, $E = 12\ ft = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2} = y + \frac{Q^2}{2g\,(zy^2)^2}$

y_c:

$$E_c = y_c + \frac{D_c}{2} = y_c + \frac{\left(\frac{A_c}{B}\right)}{2} = y_c + \frac{\frac{A_c}{2zy_c}}{2} = y_c + \frac{\frac{zy_c^2}{2zy_c}}{2} = y_c + \frac{\frac{y_c}{2}}{2}$$

$E_c = y_c + \frac{y_c}{4} = 12 = \frac{5}{4} y_c \rightarrow$ $\boxed{y_c = 9.6\ ft}$ ✓

* can find Q_c by $E_c = y_c + \frac{Q_c^2}{2g(zy_c^2)^2}$

12.4) $d_o = 9ft$ $\quad E = 8ft$, $E = E_c = y_c + \frac{D_c}{2}$

$Q_c = Q_{max} = V_c A_c$, $y_c = \frac{2}{3} E_c = \frac{2}{3}(8) = 5.33\ ft$

$\frac{y_c}{d_o} = \frac{5.33}{9} = 0.59$ see Fig 2-1 $\rightarrow \frac{D_c}{d_o} \sim 0.49 \rightarrow D_c = 4.41\ ft.$

$y_c + \frac{D_c}{2} = E \rightarrow E = \underline{7.54\ ft < 8\ ft}$ ✓

$\therefore y_c = 5.33 + \frac{2}{3}(8 - 7.54) =$ $\boxed{5.64\ ft}$

To Find Q_c, A_c, V_c

$\frac{y_c}{d_o} = \frac{5.64}{9} = 0.63 \rightarrow$ Fig 2-1 $\rightarrow \frac{D_c}{d_o} \sim 0.55 \rightarrow D_c = 4.95\ ft$

$E = y_c + \frac{D_c}{2} = 5.64 + \frac{4.95}{2} = \underline{8.115 > 8\ ft}$

$V_c = \sqrt{g D_c} = \sqrt{g(4.95)} = \underline{\underline{12.6 \text{ ft/s}}}$

$Q_c = V_c A_c \rightarrow$ Fig 2-1

$\frac{y_c}{d_o} = 0.63 \rightarrow \frac{A}{A_o} = \frac{A_c}{A_o} = 0.68 \rightarrow A_c = 0.68\left(\frac{\pi (d_o)^2}{4}\right) = 43.3$ (d_o = 9 ft)

$Q_c = V_c A_c = 12.6(43.3) = \underline{\underline{545.6 \text{ ft}^3/\text{s}}}$

2/2

12.5 $r = 8$ ft $\quad y_c = 12$ ft $\quad V = V_c$

$Q = KL(2g)^{1/2}(H_e)^{3/2}$ $\qquad H_e = H + \frac{V_a^2}{2g}$

$K = 0.5$ for high dams w/ ~ø crest pressure.

$\frac{p}{\gamma} = d\left(1 - \frac{V^2}{gr}\right) = ?$ $\qquad V = \sqrt{gD}$

NAME ______________________

$E = y + \frac{Q^2}{2gA^2}$

Assignment 12
Open Channel Flow

1. Find the alternate flow depths for a specific energy of 12 feet in a triangular channel that has z=3 and a flow of 400 cfs.

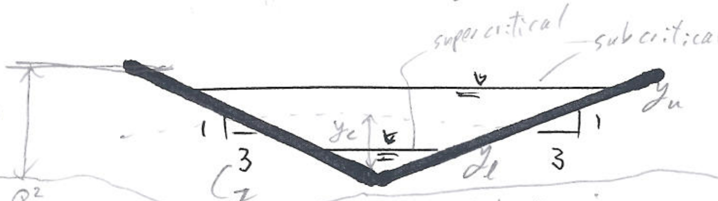
$A = z y^2$

① solve y_c: $E_c = y_c + \frac{D_c}{2} = 12'$

$D_c = \frac{A_c}{B} = \frac{z y_c^2}{2 z y_c} = \frac{y_c}{2}$

$\therefore 12' = y_c + \frac{(y_c/2)}{2} = \frac{5}{4} y_c$

$y_c = (12(\frac{4}{5})) = 9.6$



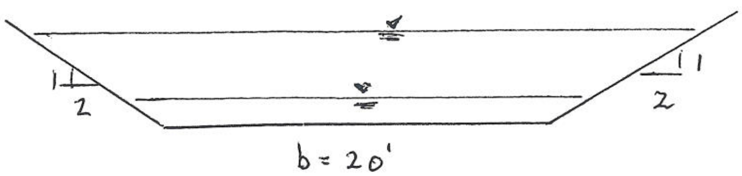
zero order approx.

$y_u = 12' \rightarrow y^{(1)} = E - \frac{Q^2}{2gA^2}$

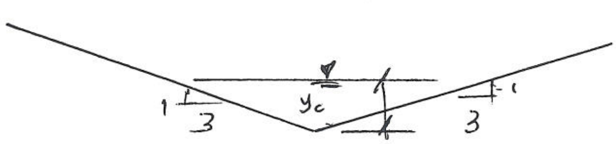
$y^{(0)}$

$= E - \frac{Q^2}{2g(zy^2)^2} = 12 - \frac{400^2}{2g(3\times12^2)^2}$ = First correction; Put back in & get the second correction (Picard method)

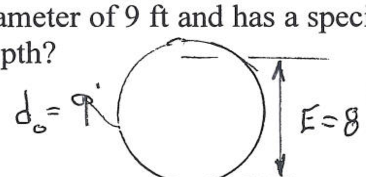
2. Find the alternate flow depths for a specific energy of 20 feet in a trapezoidal channel that is 20 feet wide at the bottom with 2H:1V side slopes and has a flow of 1400 cfs.



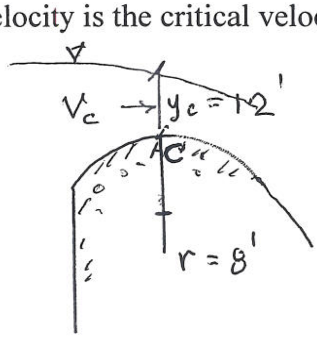
3. Find the critical depth in a triangular channel that has z = 3 and has a specific energy of 12 feet. What is the flow corresponding to this critical depth?



4. Find the critical depth in a circular pipe that has diameter of 9 ft and has a specific energy of 8 feet. What is the flow corresponding to this critical depth?



5. A spillway with a crest has radius of curvature of 8 ft and a critical depth of 12 ft. Estimate the pressure head at the surface of the crest.
Note: Assume that the velocity is the critical velocity.

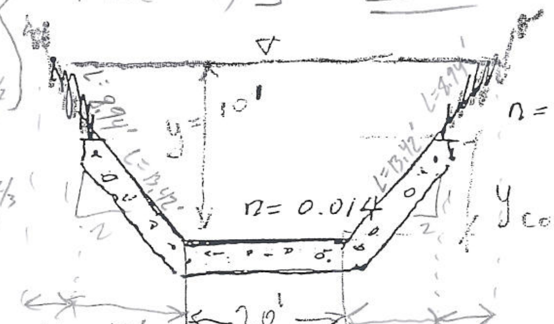


Donald Sciolleman 9/10

Assignment

1.Find the equivalent n for the trapezoidal channel shown below.
Given: Q = 500 cfs; z = 2; b = 20 ft; y = 10 ft.

Assume: $V_1 = V_2 = V_3 \ldots$, $S_1 = S_2 = S_3 = \ldots$



$\left[\sum P_i n_i^{3/2}\right]^{2/3} = \left[2(8.94)(0.035)^{3/2} + 2(13.42)(0.014)^{3/2} + 20(0.014)^{3/2}\right]^{2/3} = 0.3359$

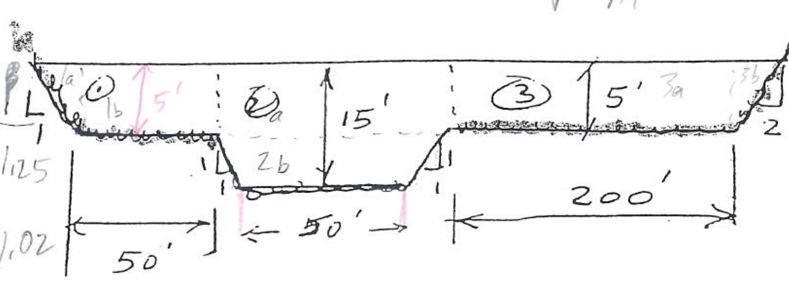
$\left[\sum P_i\right]^{2/3} = \left(2(8.94) + 2(13.42) + 20\right)^{2/3} = 16.12$

$n_T = \left(\frac{P_i n_i^{3/2}}{\sum P_i}\right)^{2/3}$

$n_T = \frac{0.3351}{16.12} = 0.0208$

2. Find the equivalent Q, n, α and β for the compound section shown below.
Given: So = 0.0003.

$\alpha_T = \frac{\sum \alpha_i V_i^3 A_i}{V^3 A_T}$ $\beta_T = \frac{\sum \beta_i V_i^2 A_i}{V^2 A_T}$



Section	A	P	R ($\frac{A}{P}$)	n	α	β
1	1a+1b			0.04	1.5	1.25
2	2a+2b			0.025	1.05	1.02
3				0.04	1.5	1.25
Σ	A_T	P_T	$\frac{\Sigma A}{\Sigma P}$			

	$AR^{2/3}$	$\frac{c'}{n_i} A_i R^{2/3}$
1		
2		
3		

$n_1 = 0.04$

$n_2 = 0.025$

$n_3 = 0.045$

$n_T = \frac{A_T R_T^{2/3}}{\sum \left(\frac{A_i R_i^{2/3}}{n_i}\right)}$

$\frac{c'}{n_i} A_i R_i^{2/3} S_o^{1/2}$	$V_i = \frac{Q_i}{A_i}$
Q_1	
Q_2	
Q_3	
$\Sigma Q_i = Q_T$	

con't on attached sheet

Section	A (ft²)	P (ft)	R (A/P)	n	α	β	$A_i R^{2/3}$	$\frac{c'}{n_i} A_i R^{2/3}$ (c' = 1.486)	$\frac{c'}{n_i} A_i R_i^{2/3} S_0^{1/2}$ (Q; $0.0003^{1/2} = 0.01732$)	$V_i = \frac{Q_i}{A_i}$
1	262.5	57.07	4.6	0.04	1.5	1.25	726.04	26972	467.17	1.780
2	950	78.28	12.14	0.025	1.05	1.02	2627.6	156186	~~2705.14~~ X	2.848
3	1025	211.18	4.854	0.045	1.5	1.25	2938.5	97036	1680.66	1.640
Σ	A_T 2237.5	P_T 346.5	$\frac{\Sigma A}{\Sigma P} = R_T$ 6.457	$*n_T$ 0.04115	α_T 1.404	β_T 1.173			$\Sigma Q_i = Q_T$ 4852.97	$\bar{V} = Q_T/A_T$ 2.169

$$n_T = \frac{A_T R_T^{2/3}}{\Sigma\left(\frac{A_i R_i^{2/3}}{n_i}\right)} = \frac{2237.5(6.457)^{2/3}}{188555} = 0.04115$$

$$\Sigma \frac{A_i R_i^{2/3}}{n_i} = 18151 + 105104.7 + 65300$$
$$= 188555$$

$$\alpha_T = \frac{\Sigma\, \alpha_i V_i^3 A_i}{\bar{V}^3 A_T} = \frac{[1.5(1.78)^3(262.5)] + [1.05(2.848)^3(950)] + [1.5(1.640)^3(1025)]}{(2.169)^3(2237.5)}$$

$$= 1.404$$

$$\beta_T = \frac{\Sigma \beta_i V_i^2 A_i}{\bar{V}^2 A_T} = \frac{[1.25(1.78)^2(262.5)] + [1.02(2.848)^2(950)] + [1.25(1.64)^2(1025)]}{2.169^2(2237.5)}$$

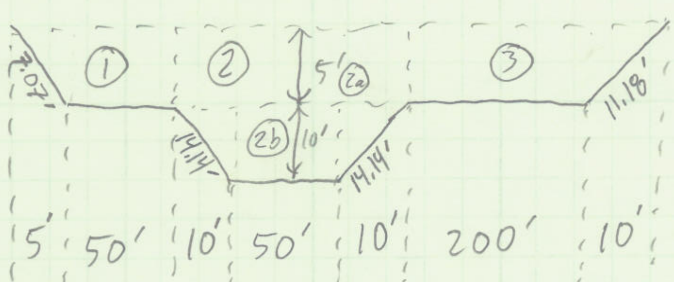
$$= 1.173$$

$Q_T = 4852.97$ ft³/s X low

$n_T = 0.04115$ 0.027

$\alpha_T = 1.404$ low

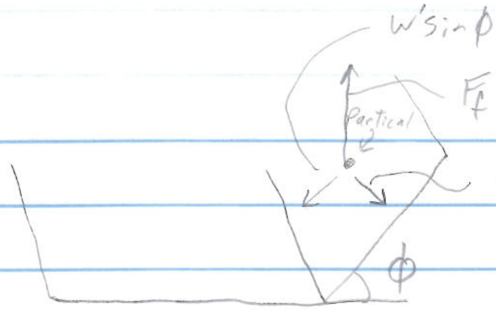
$\beta_T = 1.173$ low



Hydro

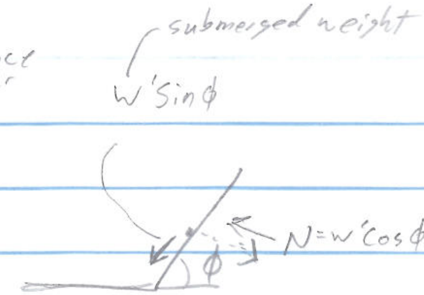
Nov 8

①



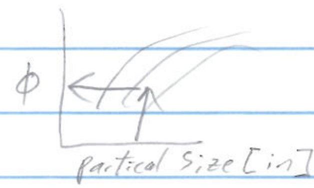
resistance shear

$\tau_{so} = \tau_{sr}$



$\tau_{sr} = K\,\tau_b$ $\qquad$ $K = \sqrt{1-\left(\frac{\sin\phi}{\sin\theta_f}\right)^2}$

see handout $\quad$ Fig. 5.33



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p.126 Notes steps to solve for shears

1) Find Flow
2) Find Manning's n $\left(\text{Table 7-3} ; \overset{\text{OR}}{} n = 0.034\left(D_{50}\,[ft]\right)^{1/6} \text{ for } \begin{smallmatrix}\text{Coarse sand,}\\ \text{gravel,}\\ \text{riprap}\end{smallmatrix}\right)$
3) Get θ_f Fig. 5.33
4) $\tau_b = \tau_o$ $\quad$ OR $\quad$ $\tau_b = 0.4\left(D_{25 >}[in]\right) = C_b\,\gamma\,y\,S_o$
5) $\tau_{sr} = K\,\tau_b = \tau_{so} = C_s\,\gamma\,y\,S_o$
6) $y = \left(\dfrac{\tau_b}{C_b\,\gamma\,S_o}\right)$ $\quad$ depth to prevent serious errosion on bottom

Not for clay →

7) $\sin\phi = \sin\theta_f\sqrt{1-\left(\frac{C_s}{C_b}\right)^2}$ $\quad$ * gets ϕ
   * Can get C_s & C_b from COE charts
   * for now assume $C_s = 0.76$, $C_b = 0.97$

$z = \dfrac{1}{\tan\phi}$

8) $Q = \dfrac{c'}{n} A R^{2/3} S_o^{1/2} = (\text{For here}) = \dfrac{c'}{n}\dfrac{\left(y\,(b+zy)\right)^{5/3}}{\left(b+2y\sqrt{1+z^2}\right)^{2/3}} S_o^{1/2}$
   * Solve for b
9) Add Free board

10) Check F_N

11) check $\frac{b}{y} > 4$ if yes OKAY

if not try adjusting C_s & C_b ← higher is more Conservative

Table 5.8 Permissible Canal Velocities (Fortier and Scobey, 1926)
also see ven te Chow.

Original material excavated for canals	Mean velocity, after aging of canals with flow depths ≤ 3 ft					
	Clear water, no detritus		Water transporting colloidal silts		Water transporting noncolloidal silts, sands, gravels or rock fragments	
	(ft/sec)	(m/sec)	(ft/sec)	(m/sec)	(ft/sec)	(m/sec)
1. Fine sand (noncolloidal)	1.5	0.46	2.5	0.76	1.5	0.46
2. Sandy loam (noncolloidal)	1.75	0.53	2.5	0.76	2.0	0.61
3. Silt loam (noncolloidal)	2.0	0.61	3.0	0.91	2.0	0.61
4. Alluvial silt (noncolloidal)	2.0	0.61	3.5	1.07	2.0	0.61
5. Ordinary firm loam	2.5	0.76	3.5	1.07	2.25	0.69
6. Volcanic ash	2.5	0.76	3.5	1.07	2.0	0.61
7. Fine gravel	2.5	0.76	5.0	1.52	3.75	1.14
8. Stiff clay	3.75	1.14	5.0	1.52	3.0	0.91
9. Graded, loam to cobbles (noncolloidal)	3.75	1.14	5.0	1.52	5.0	1.52
10. Alluvial silt (colloidal)	3.75	1.14	5.0	1.52	3.0	0.91
11. Graded, silt to cobbles (colloidal)	4.0	1.22	5.5	1.68	5.0	1.52
12. Coarse gravel (noncolloidal)	4.0	1.22	6.0	1.83	6.5	1.98
13. Cobbles and shingles	5.0	1.52	5.5	1.68	6.5	1.98
14. Shales and hard pans	6.0	1.83	6.0	1.83	5.0	1.52

2. Determine the soil properties of the bed and banks of the design reach and of the channel upstream.

3. Determine sediment yield for the reach and compute sediment concentration for design flow.

4. Check to see if the allowable velocity procedure is applicable using the Channel Evaluation Procedural Guide, Figure 5.27.

5. Determine the basic channel velocities from Figure 5.28a and multiply them by the appropriate correction factors as found in Figure 5.28b. Compare the design velocities with the allowable velocities determined from Figures 5.28a and 5.28b.

6. If the allowable velocities are greater than the design velocities, the design is satisfactory. Otherwise, if the allowable velocities are less than design velocities, it may be necessary to consider a mobile boundary condition and evaluate the channel using appropriate sediment transport theory and programs.

5.3.4 TRACTIVE FORCE DESIGN

Lane (1953a,b) developed an analytical design approach for shear distribution in trapezoidal channels. The tractive force, or shear force, is the force which the water exerts on the wetted perimeter of a channel due to the motion of the water.

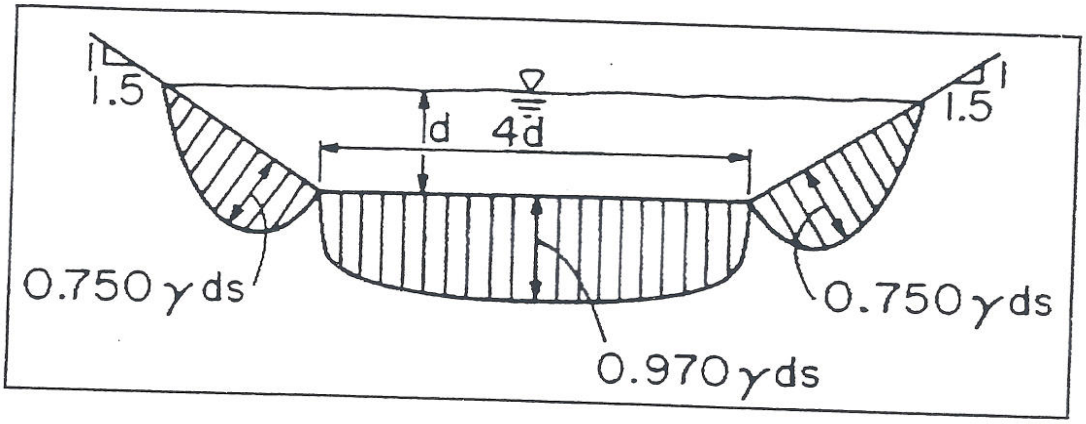


Figure 5.29 Maximum Unit Tractive Force Versus b/d (from Simons and Sentürk, 1992), b is the Bottom Width and d is the Depth

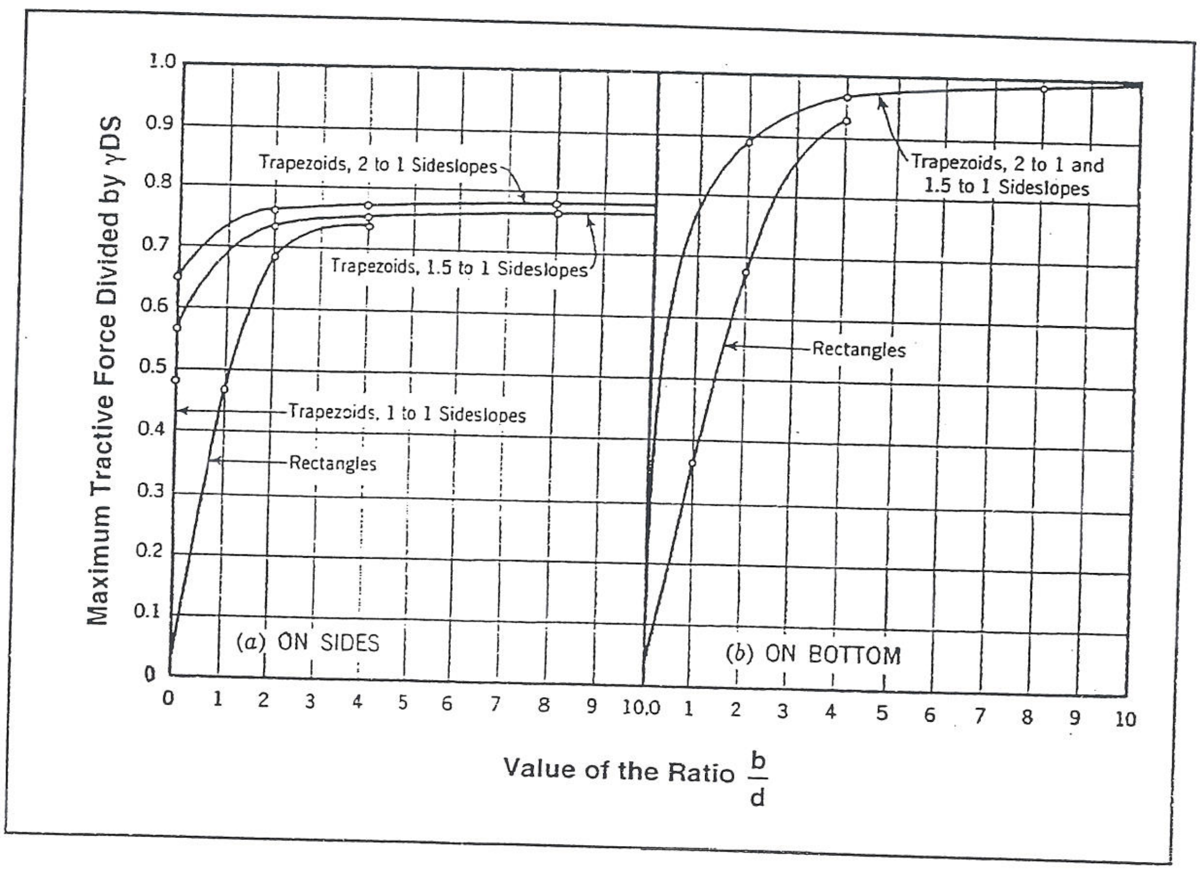


Figure 5.30 Maximum Tractive Forces in a Channel (from Lane, 1953b)

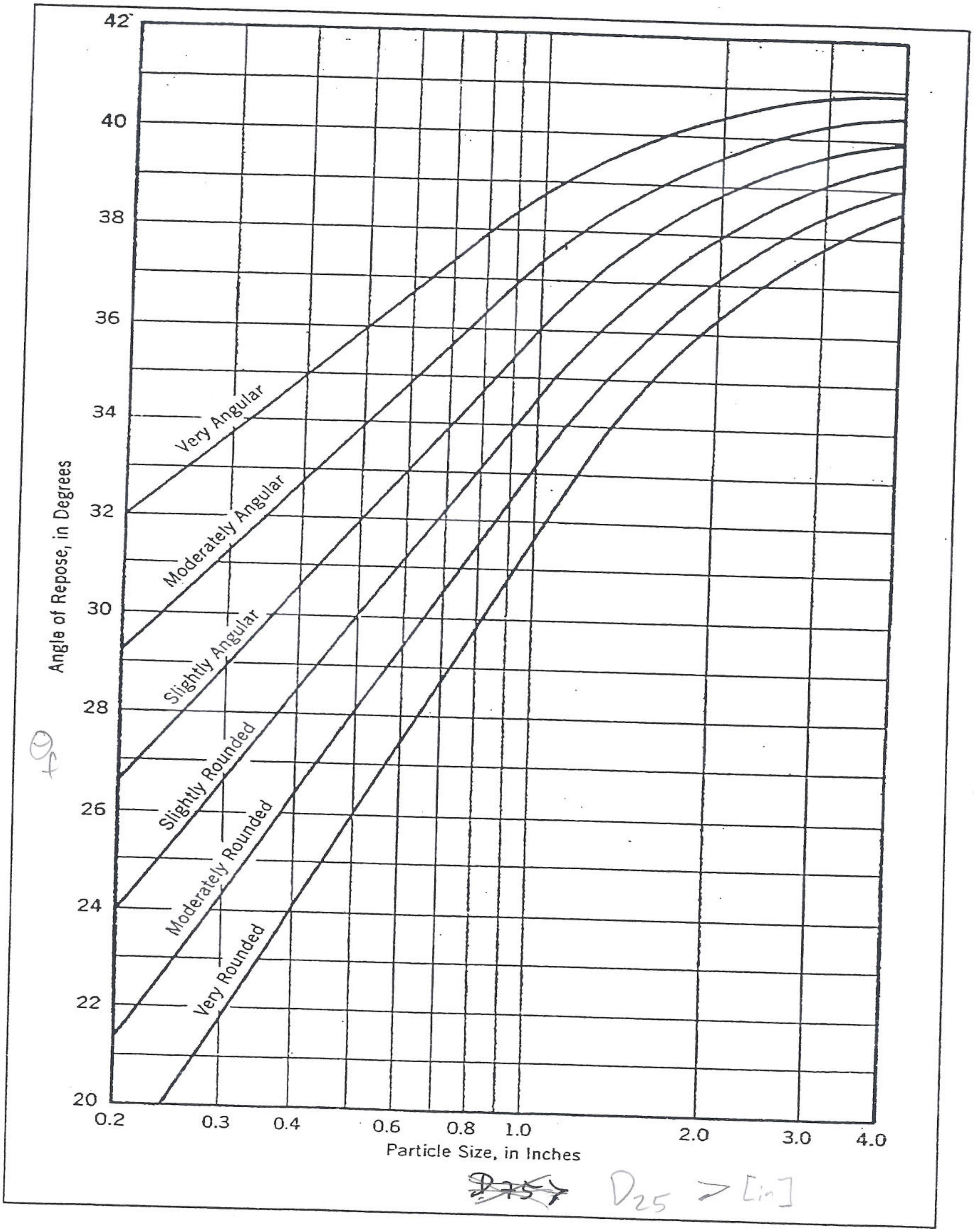


Figure 5.33 Angle of Repose of Noncohesive Material (from Lane, 1953b)

Lecture 27
Rapidly Varied Steady Flow -
Spillway and Stilling Basin Design

Assignment Due Date : In class assignment.
Reference Corps of Engineers Manuals and Handouts.

Design Case Study
See separate handout.

Function of a Spillway
The function of a spillway is to safely pass the excess flood waters around, through of over a dam.

Types of Spillways
The following are examples of commonly used spillways:

1) Crest, e.g. Ogee, WES, weir,
2) Side Channel,
3) Drop Inlet, e.g. Morning Glory
4) Sluice,
5) Over-and-Under,
6) Fuse-plug,
7) Syphon,
8) Stepped,
9) In-built.

The spillway may be gated of free flow. In any case the gates are assumed to be fully open at the PMF.

Design Considerations
1. The most important design criterion for a spillway is the design flood. The selection of this flood must consider the consequences of exceeding the spillway capacity. Generally it is assumed that if the dam is overtopped it will fail. If this would cause any risk to human life then the probable maximum flood (PMF) must be used. This flood is determined by hydrologic studies of the existing floods, regional flood analysis, regional rainfall analysis, probable maximum rainfall analysis (maximum moisture content in air column and maximum efficiency of conversion to precipitation) and rainfall runoff models and flood routing models.
In rivers with very long and reliable flow records, the 1:10,000 year flood is sometimes used as the design flood. In this case, the extrapolation of the flood frequency curve is based on the probability function that best fits the available annual series of peak flows. The most common probability functions are the log-normal Pearson III and the Gumbel distribution.
2. Another criterion in designing a spillway is the maximum allowable reservoir level during the passage of the probable maximum flood. This is established by the overall project cost-benefit analysis. The cost side includes: the cost of building a higher dam, the cost of land and flood rights, environmental and transportation costs and present value of future costs such as operation s and maintenance. The benefits include: increased storage, increased hydroelectric

power, increased attenuation of flood peaks. Based on acceptable interests rates and inflation rates the annual benefits (income) must exceed the amortized capital costs plus the operating and maintenance costs. In fact the owners would like to maximize the return on their investment; in this case the height of dam that maximizes the return on the capital investment would be the design height that is selected. In other cases the dam height that give the maximum benefit to cost ratio is selected.

3. It is also necessary to know the tailwater level (river stage downstream from the dam) for the entire range of floods from the low flows to PMF. This is usually presented as a Stage versus Q curve which is also called a rating curve. This curve may change with time after the construction of the reservoir. For example, the river morphology will change due to removal of sediment load in the reservoir; this may cause degradation of the channel and lowering of the tailwater level. This information is needed to design the stilling basin and other outlet works for the dam. It is also need to estimate uplift on the structure and back pressure on turbines. Fish migration structures are designed for a specified tailwater range.

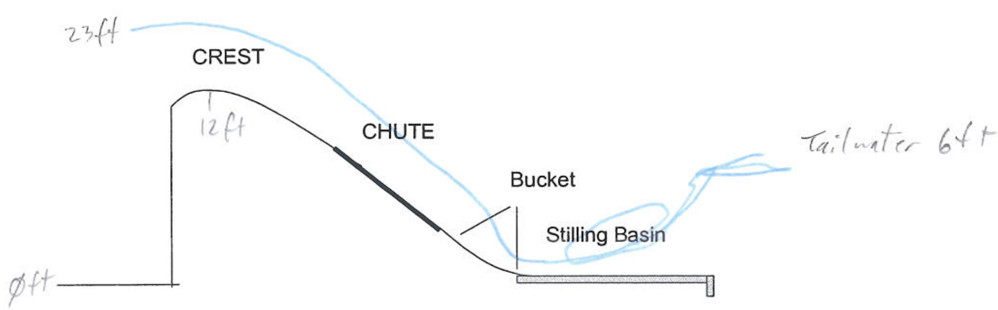
A tailwater rating curve can be established using existing flow and stage records; however, if these are not available it will be necessary to use models like HEC-RAS to estimate the rating curve - in this case calibration with actual stage-flow data is essential. The tailwater rating curve may exhibit hysteresis, i.e. on the rising limb of the flood the stage may be lower than normal and on the falling limb it may be higher than normal where normal refers to the stage that would exist for the same steady flow.

4. The normal pond elevation is often used to establish the sill of the spillway. It may also correspond to the ice loading elevation.

Crest Spillway Design

A complete spillway typically consists of the following elements (see Figure below):

a) the crest section,
b) the chute,
c) the bucket,
d) the stilling basin.



Crest Design

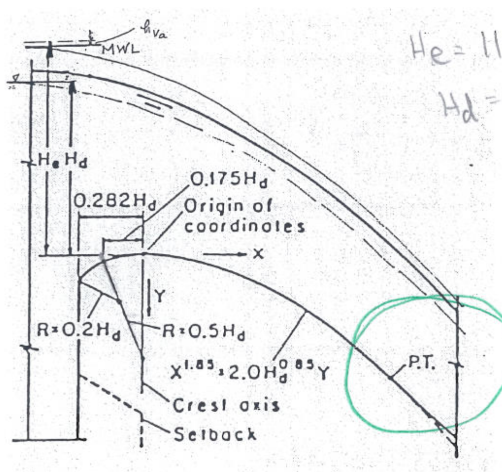
We need to select a spillway form that has low cost and high capacity. An early attempt to obtain an efficient shape was to take the lower nappe of the flow over an aerated sharp-crested weir as the shape of the concrete crest (see figure below). Of course the weirs were scale models of the actual spillway. The idea was to have nearly zero normal force between the water

and the concrete and therefore have almost no frictional resistance for the selected head on the weir. This gave a parabolic spillway shape.

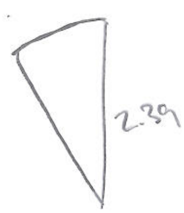
To generalize these results and make the crest easier to construct, the Waterways Experimental Station (WES) proposed the following dimensionless equation for the downstream portion of the crest (see Figure below):

$$\mathbf{Y/H_d = K' (X/H_d)^n} \qquad (27\text{-}1)$$

where H_d is the design head (not necessarily the maximum head); X, Y are Cartesian coordinates of the crest as shown below; K_d and n are constants that depend on the upstream batter and the relative height of the spillway (see attached table). For, typical vertical face spillway **K' =1/2 and n = 1.85.**



He = 11.14'
Hd = 4.78



WES suggests a compound curve for the upstream portion of the spillway. The radii and offsets are proportional to H_d.

X	Y
1	.132
2	.477
3	1.010
4	1.719
5	2.597

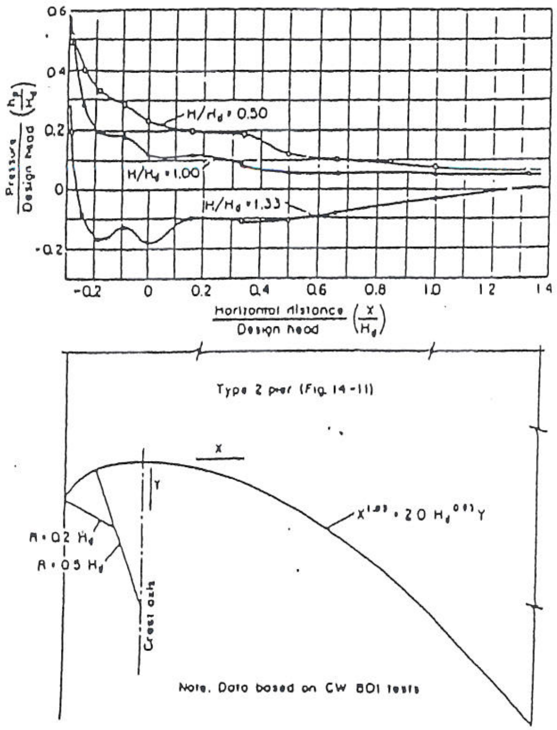


Fig. 14-13. Crest pressures on WES high overflow spillways (*continued*). (*b*) Along center line of pier bay. (*U.S. Army Engineers Waterways Experiment Station* [20], *Hydraulic Design Chart* 111-16/1, *WES* 3-55.)

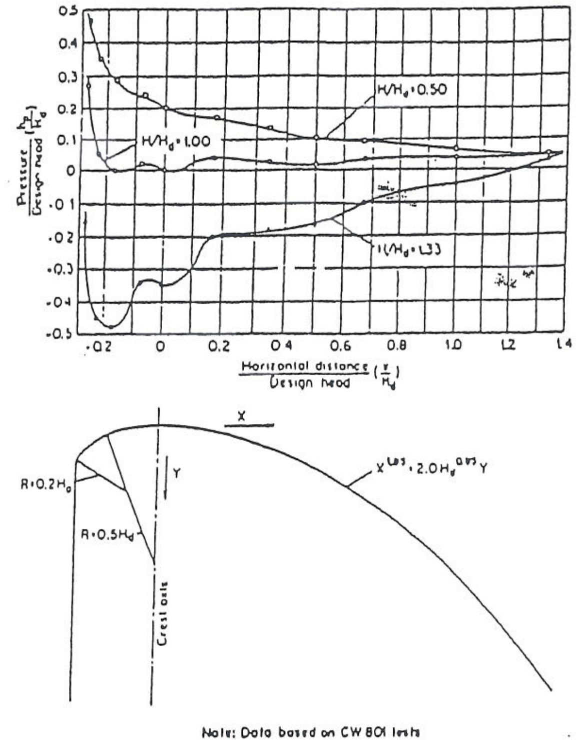


Fig. 14-13. Crest pressures on WES high overflow spillways. (*a*) No piers. (*U.S. Army Engineers Waterways Experiment Station* [20], *Hydraulic Design Chart* 111-16, *WES* 9-54)

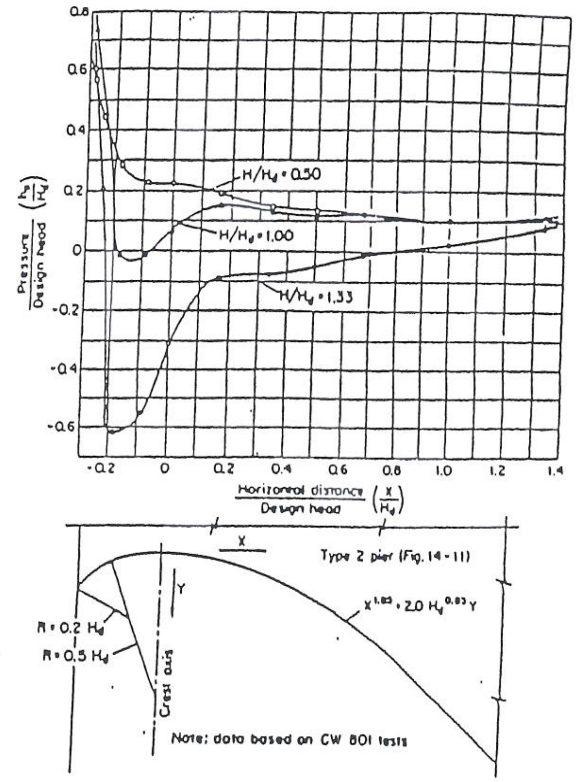
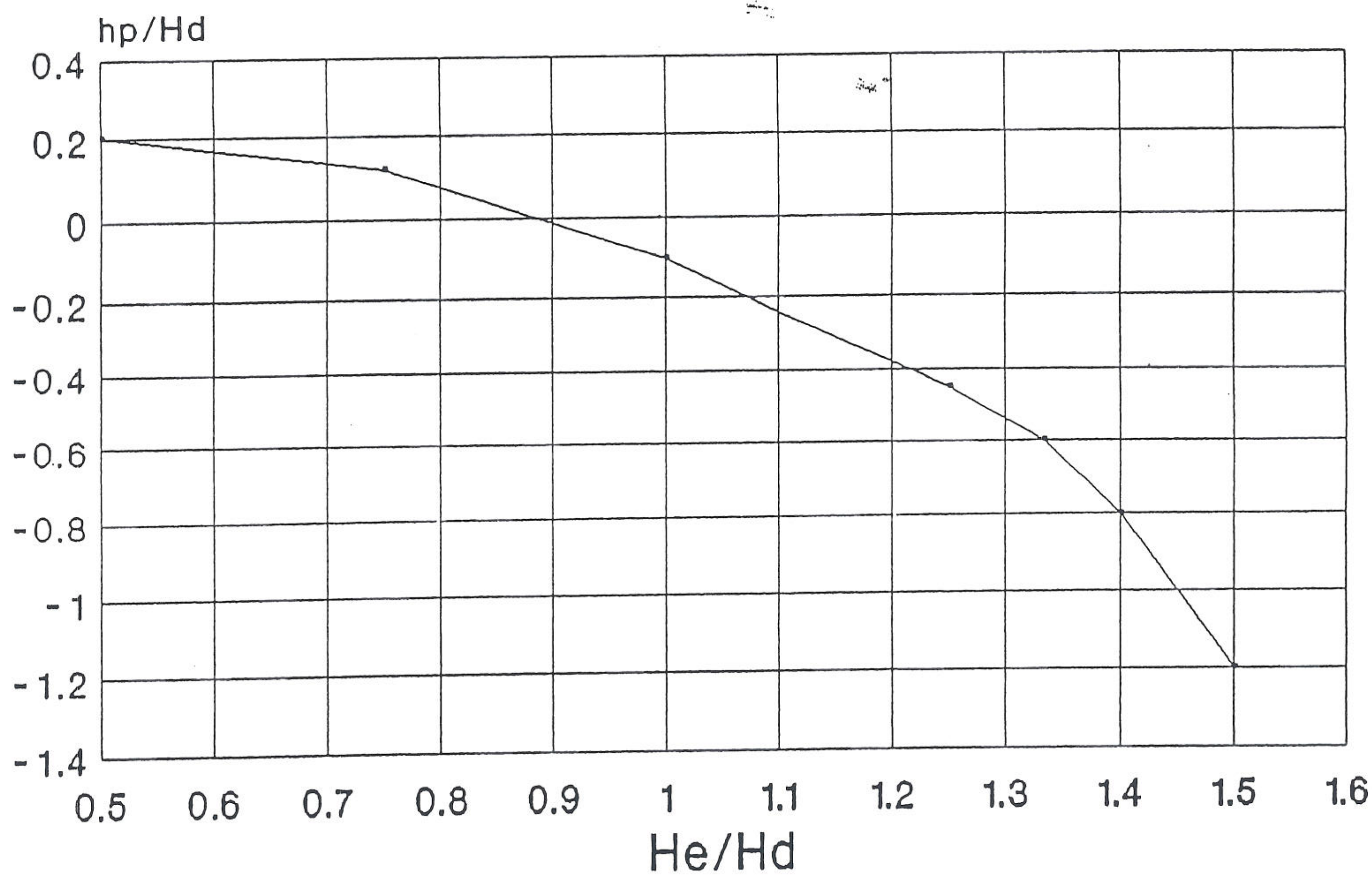


Fig. 14-13. Crest pressures on WES high overflow spillways (*continued*). (*c*) Along piers. (*U.S. Army Engineers Waterways Experiment Station* [20], *Hydraulic Design Chart* 111-16/2, *WES* 3-55.)

FIG 26-1

Minimum Crest Pressure Head

After Ven te Chow



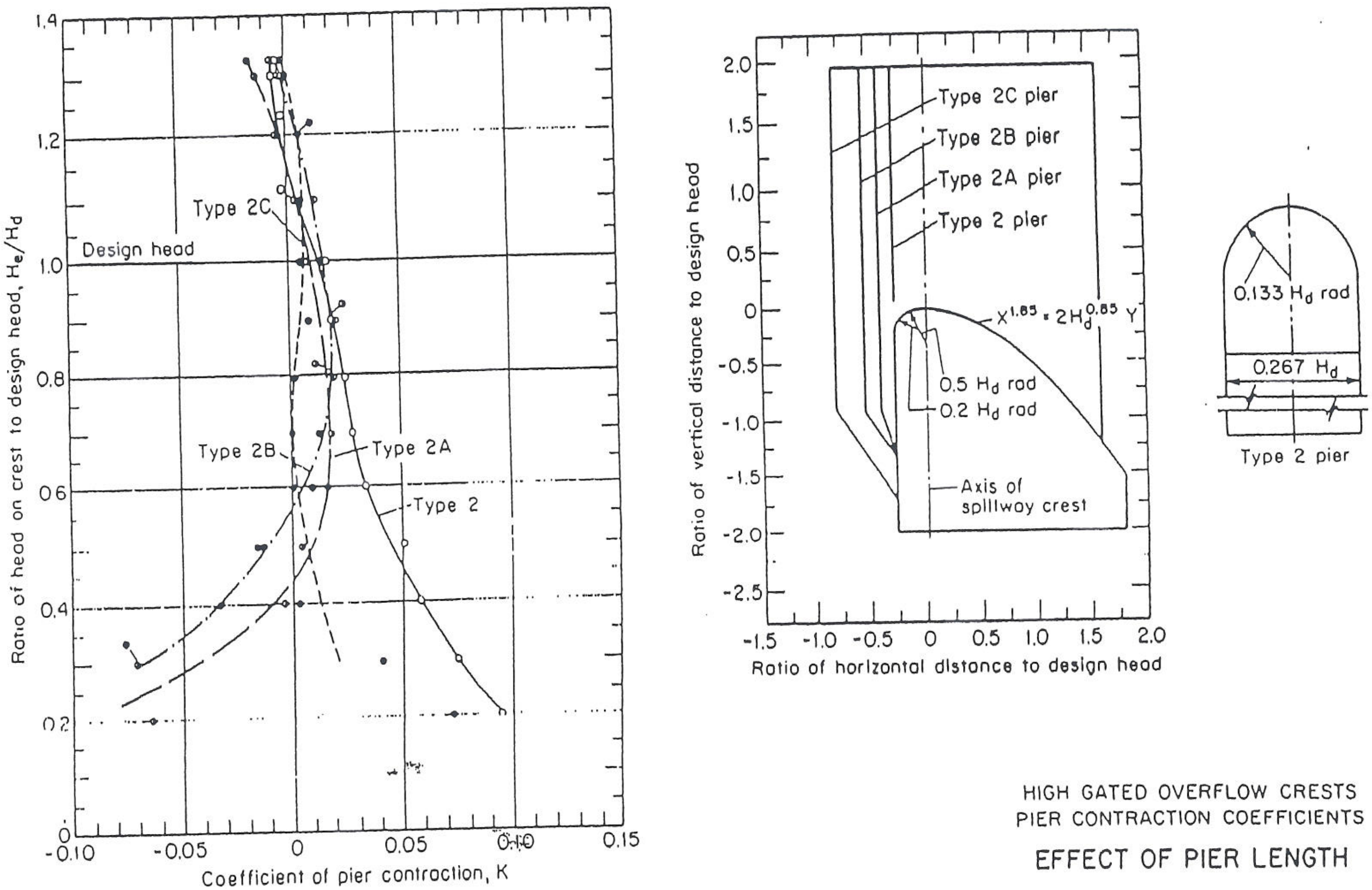


FIG. 11-10. Coefficient of contraction for the round-nose pier in high dams. (*U.S. Army Engineers Waterways Experiment Station* [20], *Hydraulic Design Chart* 111-6, *WES* 4-1-53.)

Determination of the Maximum Energy Head $H_{e\,max}$

For assigment
Forbay = 23'
Assume $V_a \sim 3$ ft/s
crest = 12'
$H_{e_{max}} = 11.14$ ft

The maximum head on the crest of the spillway during the passage of the PMF is

For bay level

$$H_{e\,max} = (\text{Maximum Pond Level - Crest Elevation}) + V_a^2/2g$$

where V_a =approach velocity $= Q_{max}/A_{forbay}$;the Crest Elevation; usually the normal pond level.

Discharge Equation

The discharge over a WES spillway is given by

$$Q = C_d\, L_e\, H_e^{\;1.5} \qquad 27\text{-}2$$

where H_e = the energy head above the spillway crest; L_e = effective length of the spillway crest; C_d = discharge coefficient which is a function of the ratio of $\{H_e/H_d\}$, e.g.

$$C_d = C_{do}\, \{H_e/H_d\}^{0.12} \qquad 27\text{-}3$$

where C_{do} = the discharge coefficient for $H_e = H_d$. In U.S. units $C_{do} = 3.97$.

Selection of Design Head H_d

The design head H_d is the scaling parameter for all of the elements of the spillway crest. It is selected to reduce the concrete in the crest section, to maximize the Q but to do this without causing cavitation due to low negative pressures on the crest. Since the size of the crest increases with H_d, the larger the H_d the more concrete that will be needed.

From the discharge coefficient it can be seen that there is an advantage of increase Q due to selecting

$h_p \sim -18$ to -20 ft (allowable neg. pressure) (use -20 for assigment)

$$H_d < H_{e\,max}. \qquad H_d = \frac{H_{emax}}{\left\{1 - \frac{h_p}{C_p H_{e_{max}}}\right\}} \qquad 27\text{-}4$$

For TYPE II Piers $C_p \sim 1.35$

For assigment $H_d = 4.78 \sim 4.8'$

However, since the radii of curvature are proportional to H_d, as H_e increases relative to H_d the negative pressure on the crest also increases, as indicated by

$$p/\gamma \sim d(1 - V^2/(g\,k\,H_d) \sim d(1 - C\,H_e/H_d) \text{ since } V_c^2/2g \sim H_e/3$$

Figure 27-1 (attached) was developed using experimental data on the lowest pressure head on a WES spillway with different ratios of $\{H_e/H_d\}$. Figures 26-14 a,b,c (from ven te Chow) show some of the dimensionless WES experimental plots of pressure head along the bed of the crests for different ratios of $\{H_e/H_d\}$ with and without piers. As a guide the lowest pressure should be

>> the vapour pressure of approximately **- 33 ft** for sea level installations. Due to irregularities in the concrete bed and walls a safe negative pressure is approximately, **- 18 to -20 ft.**

The design H_d that will give the highest discharge coefficient and still be safe from cavitation is the one that gives $p_{min}/\gamma \sim$ - 18 ft at the maximum head on the crest, $H_{e\ max}$.

$$H_d.\ H = H_{e\ max}.\ /\{1 - h_p./(1.35\ H_{e\ max}.)\} \qquad 27\text{-}5$$

Example:
Given: $H_{e\ max} = 60$ ft; use $p_{min}/\gamma \sim$ - 20 ft.
Find H_d.

Selection of Piers
The pier width and nose are determined based on $\boldsymbol{H_d}$. For example a Type II WES Pier has a thickness of 0.266 $\boldsymbol{H_d}$ and Radius of 0.133 $\boldsymbol{H_d}$.

Crest Length
The effective crest length L is

$$Le = \boldsymbol{L_a} - \boldsymbol{N_p K_p H_e} \qquad 27\text{-}6$$

where $\boldsymbol{L_a}$ = actual (clear) crest length; $\boldsymbol{N_p}$ = number of pier contractions; $\boldsymbol{K_p}$ = pier contraction coefficient. The effective length is found from

$$Le = Q/\{ C_d H_e^{1.5}\} \qquad 27\text{-}7$$

contraction loss @ piers for project = (-0.01)

and then $\boldsymbol{L_a} = Le + \boldsymbol{N_p K_p H_e}$ 27-8

Start of Chute

The Chute starts when the slope of the crest function = the assigned chute slope (1/m). The minimum value of m depends to some extent the stability analysis of the gravity section of the entire spillway.

$$dY/dX = 1/m$$

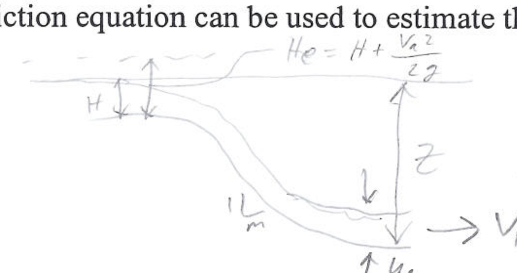
Bucket Radius

The bucket radius depends on the velocity and flow. Chow gives an empirical equation for

$$R_b = 10^{\left\{\frac{V_1 + 6.1 H_e + 16}{3.6 H_e + 64}\right\}}$$ U.S. units

Velocity at Entrance to Stilling Basin

The energy principle along with an appropriate friction equation can be used to estimate the velocity at the bottom of the spillway:



$$V_1 = [2g\,(Z - y_1 - h_f)]^{1/2} \qquad 27\text{-}9$$

where Z = [TEL in pond (Forbay WSEL) - Stilling Basin Floor Elevation]; y_1 = depth at start of stilling basin; h_f = energy loss from pond to stilling basin entrance. Note: $y_1 = Q/(V_1 W)$ where W is the width of the stilling basin.

The USBR developed an alternative (can approx. w/ H ?) to the above equation:

use but check → $$V_1 = [2g\,(Z - H_e/2]^{1/2} \qquad 27\text{-}10$$

which is explicit and eliminates the friction term.

Lecture 28
Design of Stilling Basins

Function of a Stilling Basin

The function of a stilling basin is to dissipate the excess kinetic energy at the toe of the spillway to avoid damage to the downstream channel or property.

Types of Stilling Basins

1. Hydraulic Jump,
2. Impact,
3. Flip-bucket
4. Plunge Pool,
5. Submerged Bucket and Roller,
6. Stepped spillway,
7. Baffled Chute,
8. Raft.

Design Criteria

1. The stilling basin must protect the dam and spillway from failure for all floods up to the PMF.
2. The stilling basin should protect the downstream channel and property for serious damage for the regional flood, e.g. 1:100 year flood.

Theory

Hydraulic Jump Stilling Basins: An hydraulic jump is the transition for supercritical to subcritical flow. Due to the inherent instability of the decelerating flow, a large portion of the kinetic energy is converted to turbulent energy and subsequently lost as heat energy. A portion of the kinetic energy is also converted tp potential energy. There are several types of hydraulic jumps. For stilling basin design, two of these are very important, i.e. the free jump and the forced jump. The former occurs when there are no appurtenances to aid in the formation of jump and the latter refers to jumps that have appurtenances such as baffle blocks, chute blocks and sills, to assist in the formation of the jump.

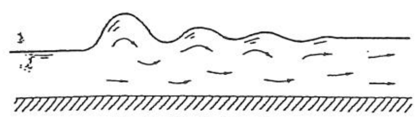
The characteristics of a free hydraulic jump depends on the Froude Number at the start of the jump. The table below summarizes 5 phases of the free jump. The US Bureau of Reclamation (USAR), St. Anthony Fall Laboratory (SAF) and WES have designed stilling basins especially for each phase (see USAR, "Design of Small Dams"; USCOE, "Hydraulic Design Criteria" and EM1100-1602 & 1603; ven te Chow, "Open Channel Hydraulics").

The momentum principle gives the downstream (sequent depth) of a free jump,

$$y_2 = y_1 \{[1 + 8N_{F1}^2]^{1/2} - 1\}/2 \qquad 28.1$$

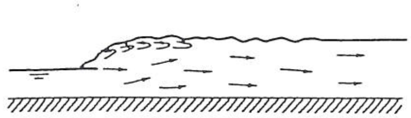
and the energy equation gives the energy loss as

$$\Delta E = E_1 - E_2 = (y_2 - y_1)^3/\{4\, y_1\, y_2\} = [V_1^2/2g].(y_2 - y_1)^3/\{2N_{F1}^2\, y_1^2\, y_2\} \qquad 28.2$$

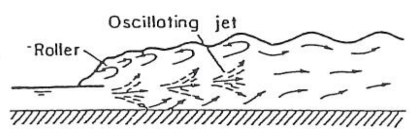


$F_1 = 1-1.7$ Undular jump

v.t. Chow



$F_1 = 1.7-2.5$ Weak jump



$F_1 = 2.5-4.5$ Oscillating jump

$F_1 = 4.5-9.0$ Steady jump

$F_1 > 9.0$ Strong jump

Stilling Basin Design

The velocity at the toe of the spillway can be estimated from the energy and continuity principles applied between the forebay and the toe, i.e.,

$$V_1 = [\,2g\,(Z - y_1)]^{1/2} \qquad \& \qquad y_1 = Q/(W\,V_1) \qquad 28.3$$

or the USBR empirical equation can be used to obtain V_1 directly,

$$V_1 = [\,2g\,(Z - H/2)]^{1/2} \qquad \& \qquad y_1 = Q/(W\,V_1) \qquad 28.4$$

Froude No. at the toe is $N_{F1} = V_1/(g\,y_1)^{1/2}$ is supercritical.

The purpose of the stilling basin is the dissipation of the excess energy at the toe of the spillway.
One means of doing this is to force a hydraulic jump to occur before the flow re-enters the river channel.

★ If can use, save construction cost

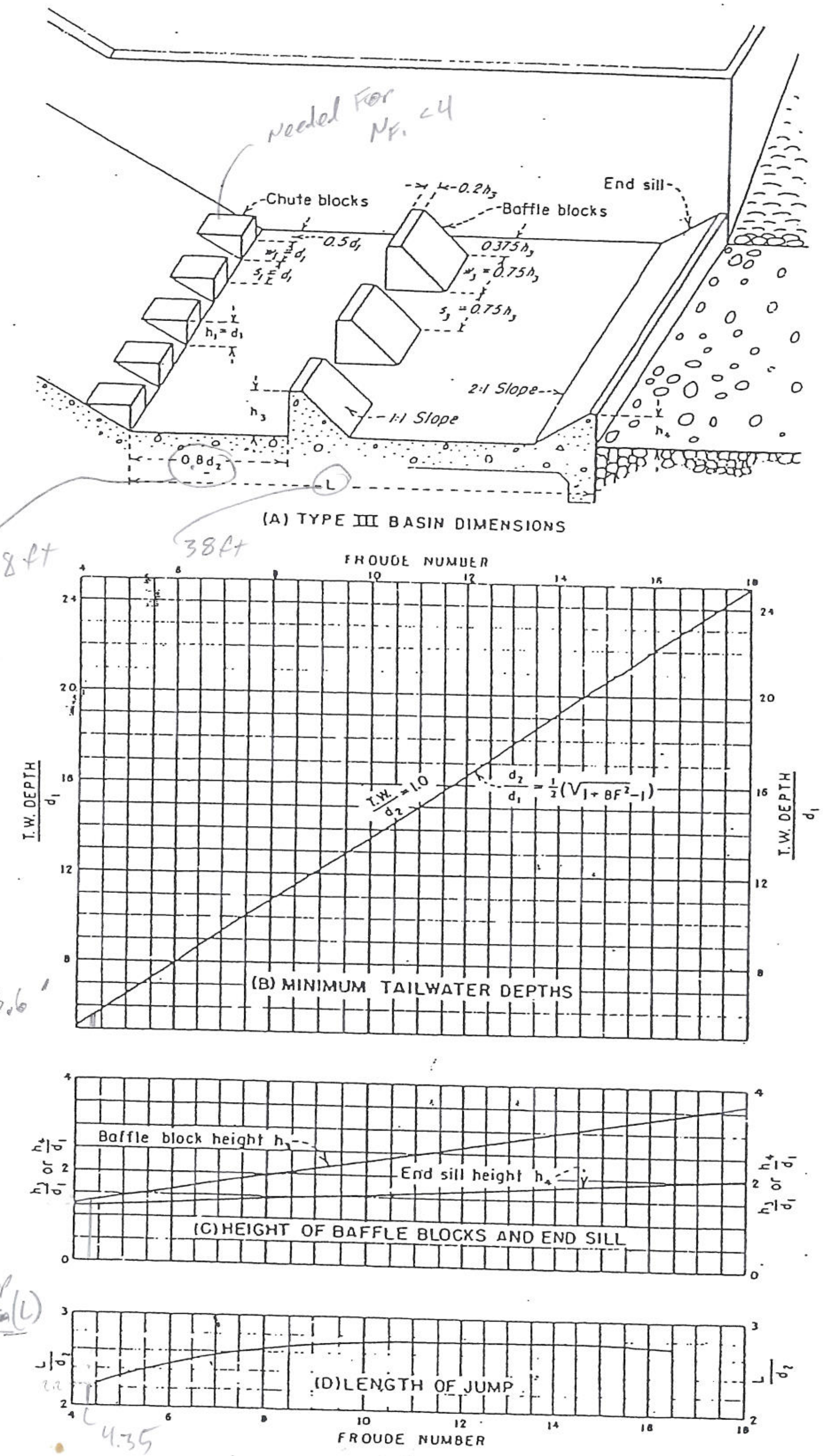


Figure 266. Stilling basin characteristics for use with Froude numbers above 4.5 where incoming velocity (V_1) does not exceed 50-60 feet per second. 288—D—2426.

0.2 h_3, 0.75 h_3, 0.75 h_3, h_3

$h_3 = 1.3 y_1$

$\frac{h_4}{d_1} = 1.2$ → $h_4 = 3.6'$

$\frac{h_3}{d_1} = 1.3$ → $h_3 = 4'$

$\frac{\text{Length of jump } (L)}{y_2}$

$\frac{L.J.}{17.24} = 2.2$ $L_J = 38$ ft.

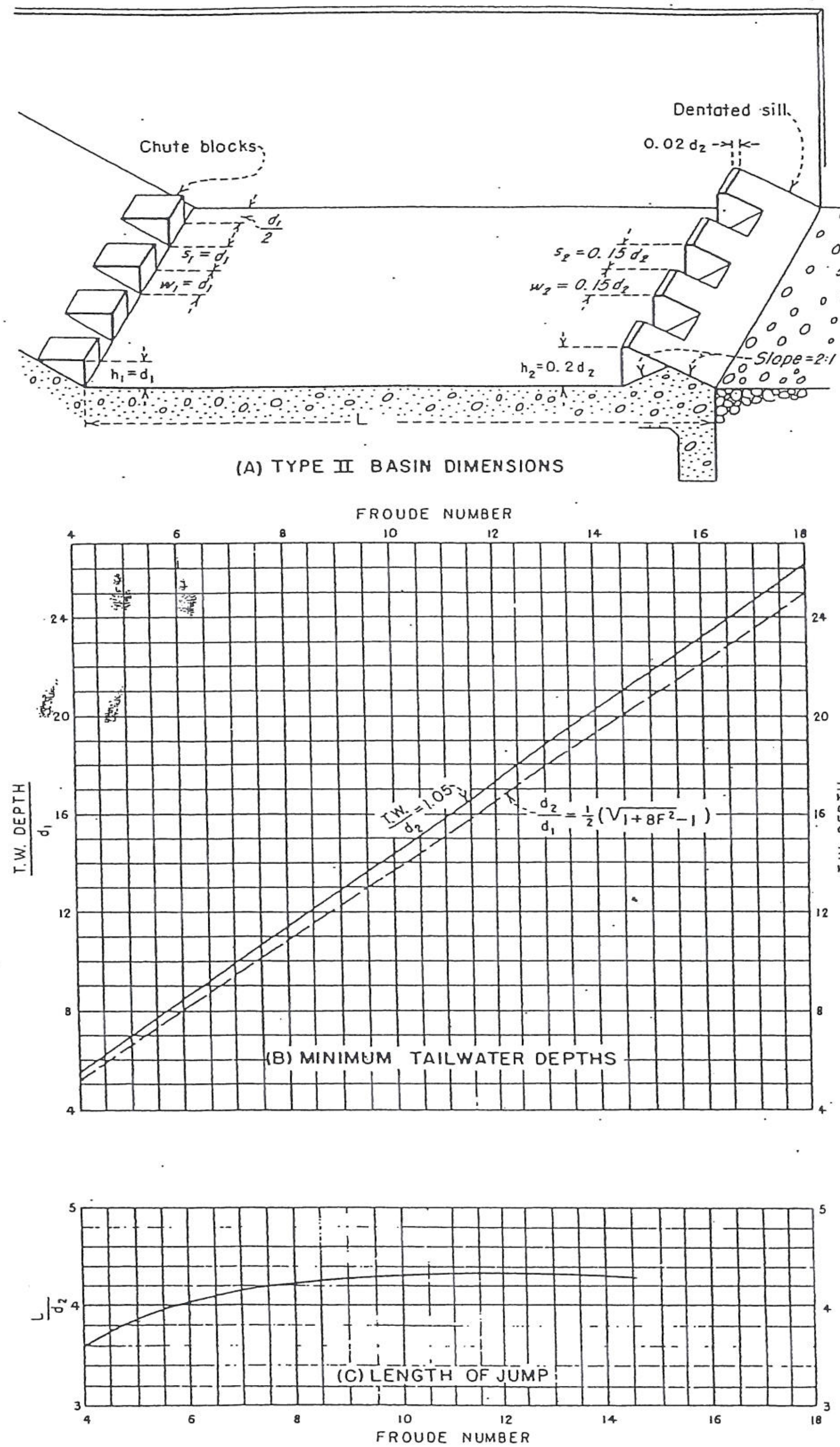


Figure 267. Stilling basin characteristics for use with Froude numbers above 4.5. 288—D—2427.

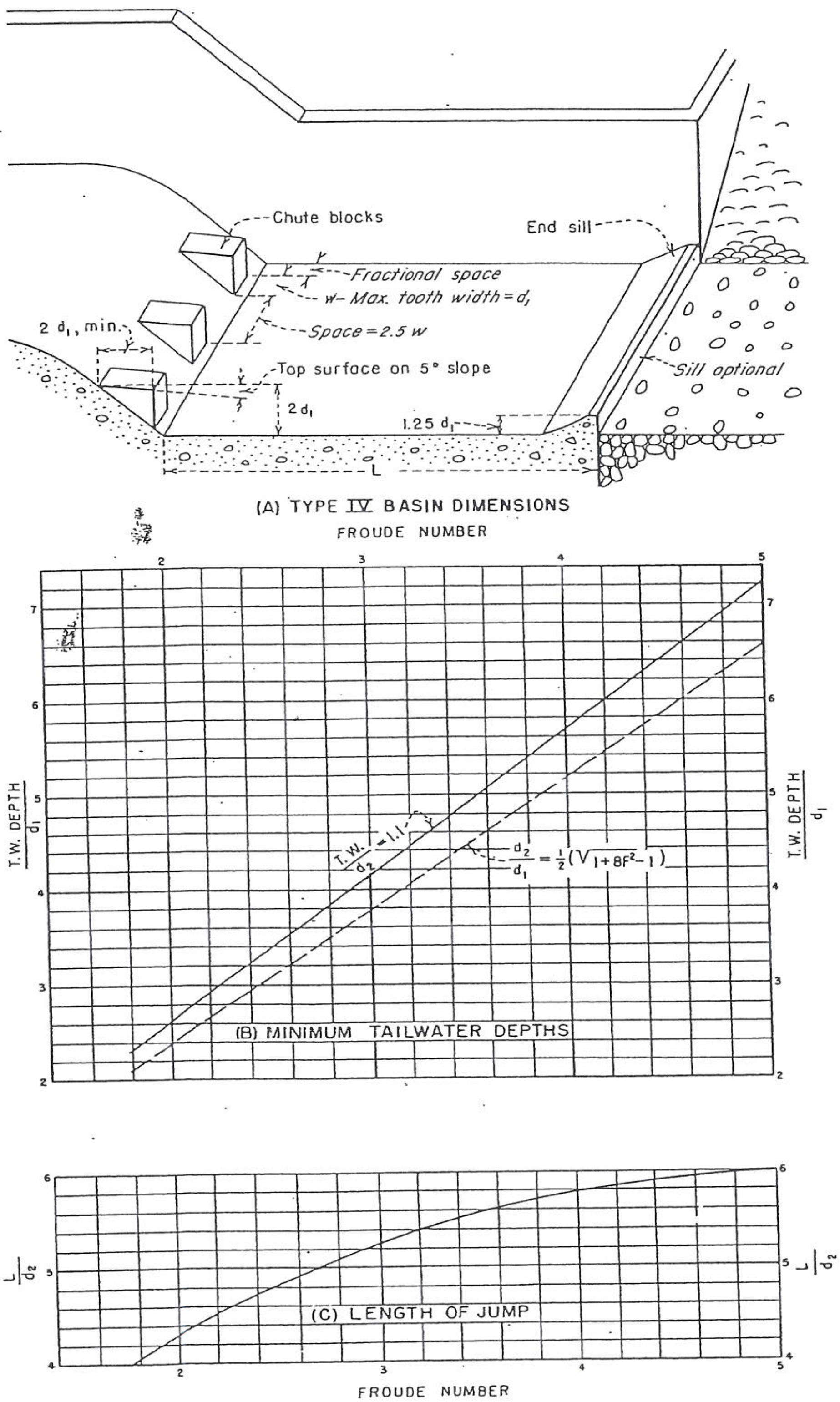


Figure 265. Stilling basin characteristics for Froude numbers between 2.5 and 4.5. 288–D–2425.

drainage system cannot be considered entirely effective because of the possibility of clogging or silting of the drain outlets, the floor slab is usually made sufficiently heavy to resist the flotation effect on the floor.

For design, the stilling basin floor is considered to be a free body in static equilibrium with foundation reactions balancing active loads. Uplift forces caused by hydrostatic head on the bottom of the slab are counterbalanced by the weight of the concrete and the effective weight of the water in the basin. Differential horizontal hydrostatic forces are opposed by the sliding resistance of the horizontal leg of the slab on the foundation. Equilibrium against rotation is achieved by equating any unbalanced forces with a foundation reaction force positioned so that the mo-

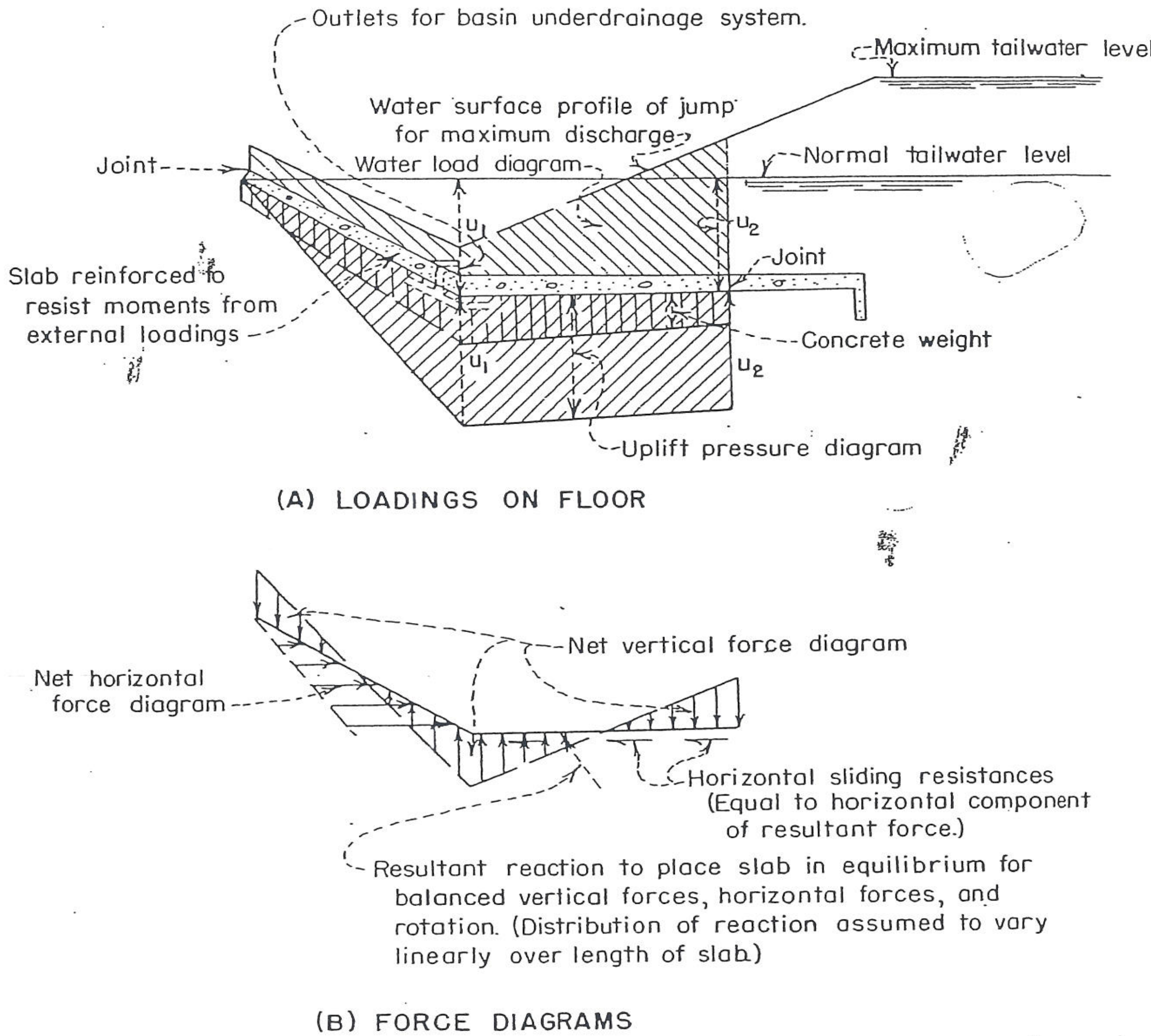


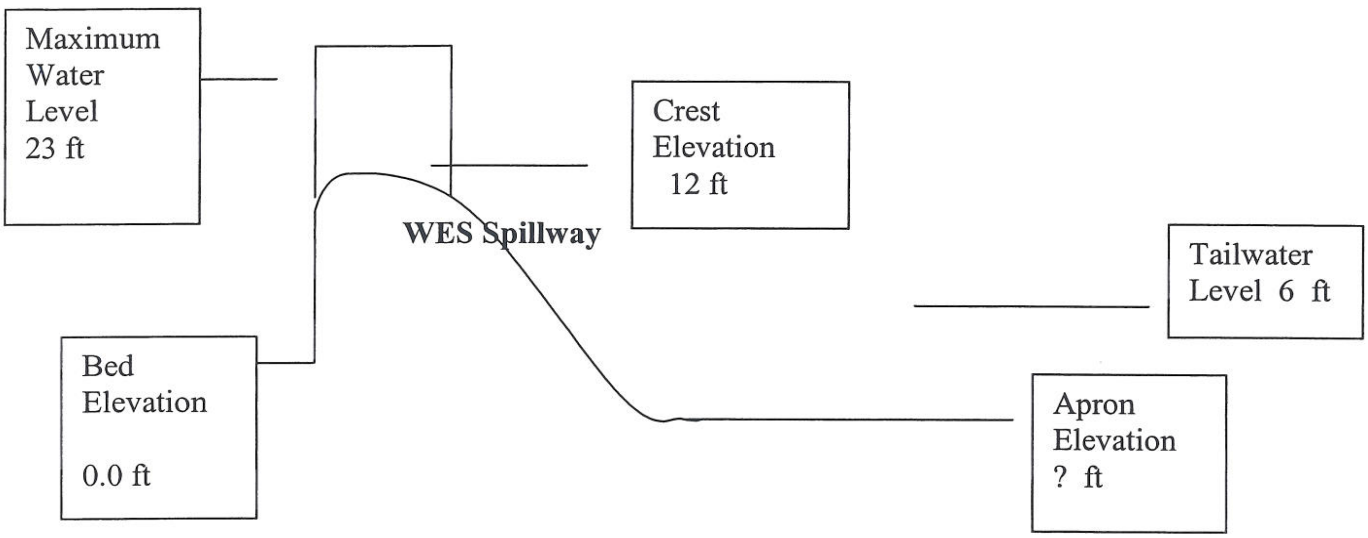
Figure 299. Illustration of uplift forces acting on a stilling basin floor. 288–D–2522.

In-Class Tutorial

Complete the design of the Spillway and Stilling Basin in the problem statement below.

A typical cross-section of the spillway is shown below. The crest follows the WES standard design. There are 100 piers each 6 ft wide. Assume that the minimum pressure head on the spillway is -20 ft. design Type III basin, size of blocks ... p. 399

a) The maximum discharge from the spillway is 385,000 cfs.
b) Determine if a hydraulic jump conditions on the apron.



BAFFLES

Hydro Last Class

Continuity $Q = VA$

Energy Relationsip $H_{T_1} = H_{T_2} + h_{L_{1\to 2}}$

Momentum (used in hydraulic jumps) $\Sigma F_x = \rho Q V_x$

$\nearrow y_2 = \frac{1}{2} y_1 \left(\sqrt{1 + 8 N_{F_1}^2} - 1 \right)$

can only use if $S_o = \emptyset$, no steps, no blocks, $h_f = \emptyset$, rectangular chann

Uniform Flow

Mannings equ. → to find y_n, or b (width of channel)

Design (a) ** Lined channel, show detail of drainage & subgrade

check for sub critical flow

(b)* unlined channel 3 methods

- Regime
- max shear method (max unit permissible force)
- max vel. method

↳ often gives crazy demensions

if only given flow & grain size use Regine

is given S_o, & detail of grain size, use max shear method

* may be asked to design in one method & check w/ max Vel. method

(c) Culverts design (see 5 problem from class) Lecture 26

- Know if working w/ inlet control or outlet control

(d) Spillways & stilling basins

Water surface Profiles Gradually Varied Flow (GVF)

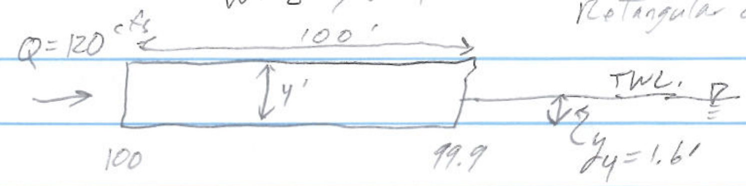
***** - Sketch & classify profiles

*** - Numerical integration method (usualy need to find y_n & y_c

- Standard step method

answers given in spread sheet

Problem #5



Sol'n: calc slope $S_o = \frac{y_1 - y_2}{L} = \frac{100 - 99.9}{100} = 0.001$

If $S_o < S_c$ it is outlet control

$y_c = \sqrt[3]{\frac{\left(\frac{Q}{W}\right)^2}{g}}$ (For rectangular) $= 1.91' > y_4$ so critical flow possible

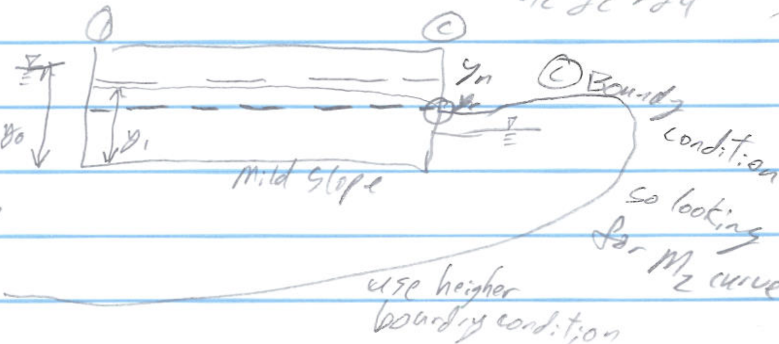
$S_c = \left(\frac{n V_c}{c' R_c^{2/3}}\right)^2$, V_c (rectangle) $= \sqrt{g y_c} = 7.85$ fps, $n = 0.015$

$R_c = \frac{A_c}{P_c} = \frac{y_c W}{2 y_c + W}$ (No top on it "not full") ($2W$ if flowing Full) $= 1.29$

P_c — wetted perimiter

$S_c = 0.00445 > S_o$ ∴ outlet control (actually critical/outlet control b/c $y_c > y_4$)

$y_n = 3.25$ ft



$y_n > y_c$ ∴ mild slope

$y_c = 1.9$ $y_4 = 1.6$ ∴

* Can not comput upstream in supercrit, must go downstream

Proceedure for est. upstream depth (guess)

Try: $y_1 \sim \frac{1}{2}(y_c + y_n) = 2.55'$

$H_{T_c} = 99.9 + y_c(1.5) = 102.77$ "E_c" (el. = 99.9; depth = y_c; 1.5 converts depth to energy)

down stream energy

guess $y_1 \sim 2.55'$

upstream energy H_{T_1} (calc by method (a)) $= h_{e1} + y_1 + \frac{V_1^2}{2g} \rightarrow 103.09$

based on y_1 guess & A use continuity $\rightarrow V_1 = 5.88$ ft/s

standard step

H_{T_1} (method (b)) $= H_{T_c} + h_{L_{c\to 1}} = 102.77 + L(S_{av})$

$S_{av} = \frac{1}{2}(S_c + S_1) = 0.0032$ (friction slope @ 1 based on y_1: $S_1 = \left(\frac{nV_1}{c' R_1^{2/3}}\right)^2$)

$h_{L_{c\to 1}} = 0.0032(100') = 0.32$

$\therefore H_{T_1}^{(b)} = 102.77 + 0.32 = 103.09'$

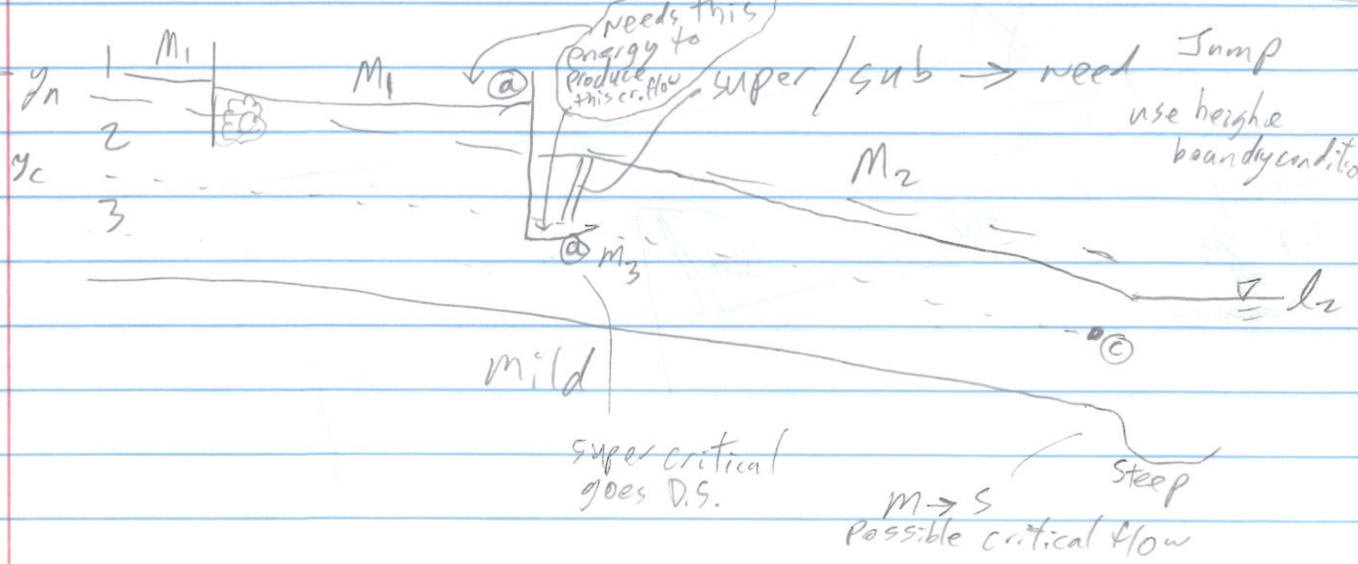
Second guess is to change H_{T_1} (a) up or down

$h_{en} = K_{en}\frac{V_1^2}{2g}$ b/c flush entrence K

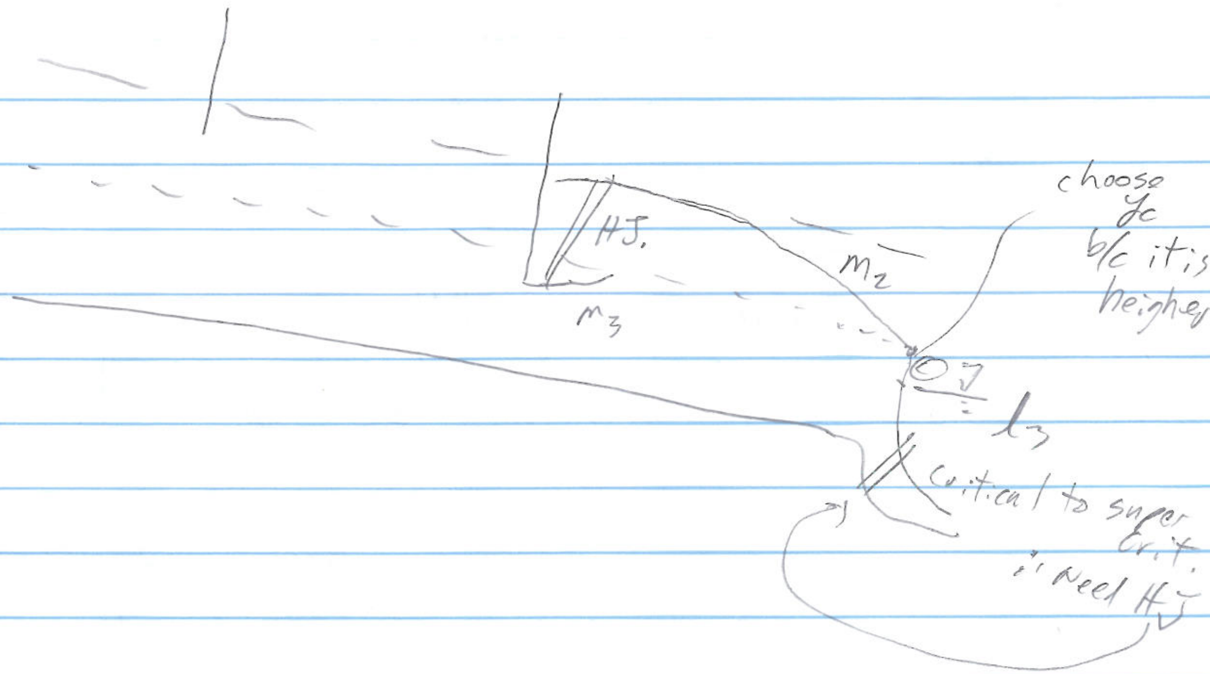
$H_{T_0} = H_{T_1}$ + entrence loss

energy b/f entrence

#3 profile



© = possible boundry conditions



choose y_c b/c it is heigher
H.J.
M_2
M_3
C
l_3
critical to super crit. ∴ Need H.J.

ENCE 4319
LABORATORY # 2 – Hydraulic Jump
10-26-10

Objectives:

1. To verify the classical hydraulic jump theory.
2. To conduct and experimental error analysis.

Theory:

The equations to be used in the analysis are the following:

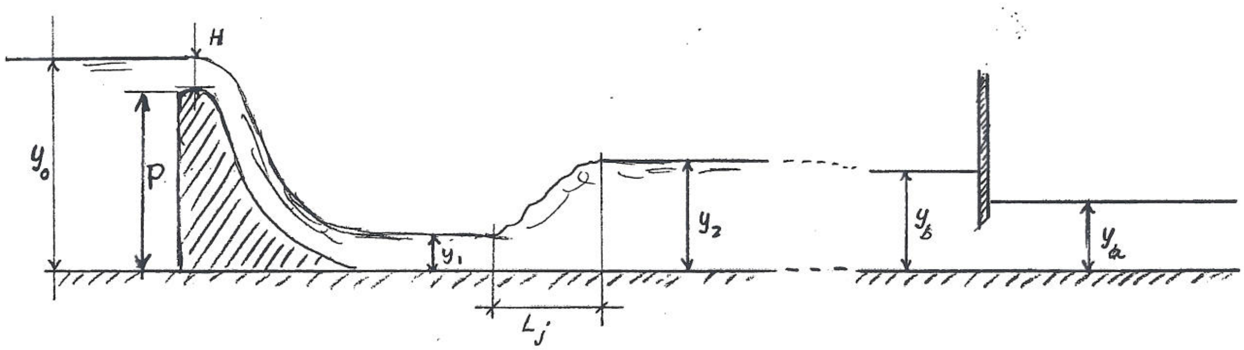
1. Flow through the spillway: $Q = \frac{1}{2} CW\sqrt{2g}H^{3/2}$

2. Flow under the gate: $Q = y_a y_b W\sqrt{\frac{2g}{y_a + y_b}}$ (assumed to be the "true" flow)

3. Hydraulic jump: $y_2 = \frac{y_1}{2}\left(\sqrt{1+8N_{F_1}^2} - 1\right)$

   where $N_{F_1} = \frac{V_1}{\sqrt{gy_1}}$

4. Length of the jump: $\frac{L_j}{y_1} = 10N_{F_1} - 6$



Procedure:

1. Each group of three students will take the following measurements (in mm): P, y_o, y_1, y_2, L_j, y_a, y_b.
2. Each group will provide the information to the ASCE President, who will prepare a summary table in Excel, with all the values provided by each group. The ASCE President will email the summary table to all students in the class.
3. Each student will calculate a table similar to the table presented on p. 2.

ENCE 4319 - Hydraulics Lab
Tutorial 8
5-Mar-08

w = 3 in
= 0.0762 m
C = 1

	Dimensions of the hydraulic jump								Flow rate, L/s				
Group	P (mm)	Y_0 (mm)	H (mm)	Y_1 (mm)	Y_2 (mm)	L_j (mm)	y_b (mm)	y_a (mm)	Spillway	Gate	Qs/Qg	Qg/Qs	NF1
1	179.0	225.0	46.0	11.8	78.0	480.0	76.0	21.0	1.66	1.73	0.96	1.04	5.65
2	180.0	227.0	47.0	11.8	80.0	500.0	82.0	21.0	1.72	1.81	0.95	1.05	5.92
3	180.0	228.0	48.0	11.9	77.5	600.0	77.5	21.0	1.77	1.75	1.01	0.99	5.65
4	179.0	227.0	48.0	12.1	80.0	340.0	82.0	20.8	1.77	1.80	0.99	1.01	5.65
5	179.0	227.0	48.0	12.3	80.0	405.0	82.0	20.8	1.77	1.80	0.99	1.01	5.51
6	179.0	227.5	48.5	12.3	80.0	356.0	80.5	21.8	1.80	1.85	0.97	1.03	5.69
7	180.0	228.5	48.5	13.7	77.5	435.0	80.9	22.0	1.80	1.87	0.96	1.04	4.89
8	179.0	228.0	49.0	14.0	81.0	348.0	78.6	21.5	1.83	1.80	1.02	0.98	4.56
9	178.0	227.0	49.0	13.8	78.0	430.0	78.7	21.2	1.83	1.78	1.03	0.97	4.60
10	178.0	228.0	50.0	13.7	79.0	430.0	84.0	21.2	1.89	1.85	1.02	0.98	4.84
Mean	179.10	227.30	48.20	12.74	79.10	432.40	80.22	21.23	1.79	1.80	0.99	1.01	5.30
Std. Deviation	0.738	0.978	1.111	0.932	1.265	79.716	2.460	0.411	0.062	0.045	0.028	0.028	0.512
Std. Error	0.233	0.309	0.351	0.295	0.400	25.209	0.778	0.130	0.019	0.014	0.009	0.009	0.162
Std. Error/Mean	0.00130281	0.00135997	0.00728607	0.02314322	0.00505689	0.05829915	0.00969837	0.00612341	0.01089874	0.00795412	0.00880198	0.00881628	0.031

In the Excel table above, which contains the data collected in 2008, if P was recorded on cells B10 through B19, then for column B, corresponding to P:
Mean = @AVERAGE(B10:B19)
Std. Deviation = @STDEV(B10:B19)
Std. Error = (Std. Deviation)/(N)$^{0.5}$, where N = # observations (in this case, N = 10)

4. Each student will prepare a lab report (due 11-2-10) that should include the following points:
   - Introduction
   - Theory
   - Experimental setup and test procedure (include a photograph of the hydraulic jump)
   - Data analysis, including experimental error analysis, as presented in class.
   - Results, verification of models
   - Conclusions
   - References.

Hydro Lab Oct. 26 ①

Group	P	y_0	H ($y_0 - P$)	y_1	L_j	y_2	y_b	y_a
1	178 mm	222 mm		10.2 mm	335 mm	74 mm	78 mm	16.9 mm
2	181	221		7.2	38 cm	57	60	17.9
3	178	222		10	270	74	73	16.8
4	179	222		9.4	340	75	82	16.9
5	179	221		10	349	70	77.2	16.8
6	178	220		9	341	75	72	17.6
7	185	222		9	370	70	80	22
8	179	222		9.1	330	65	73	18
9	179	222		9.1	330	65	73	18

Error in Q $= \frac{1}{2} C \sqrt{2g}\, W H^{3/2}$; set $K = \frac{1}{2}\sqrt{2g}\, W$

$$Q = KCH^{3/2}$$

Take ln : $\ln Q = \ln K + \ln C + \frac{3}{2} \ln H$

First derivative: $\frac{dQ}{Q} = \frac{dC}{C} + \frac{3}{2}\frac{dH}{H}$

Def. of Relative error in Q : $\frac{\delta Q}{Q} = \sqrt{\left(\frac{\delta C}{C}\right)^2 + \left(\frac{3}{2}\frac{\delta H}{H}\right)^2}$ (error in Q; makes relative error in Q)

$\frac{\delta C}{C}$ (standard error in C; mean) = relative standard error of the ratio $\frac{Q\ \text{spillway}}{Q\ \text{gate}}$ = 0.0088

$\frac{\delta H}{H}$ = relative stnd. error of H = 0.00729

Plug in

∴ $\frac{\delta Q}{Q} = 0.014$, expected rel. std. error due to errors in C & H

Effective error: $\text{Error} = \sqrt{\delta Q_{model}^2 + S_{m_Q}^2}$

↑ std. error of the mean of true flow (flow under the gate)

$\frac{\delta Q}{Q}$; "true flow" = 1.8×10^{-3} m³/s

$\delta Q = 0.014(\bar{Q}_{gate}) = 2.53 \times 10^{-5}$ m³/s

$S_{m_{Q_{gate}}} = 1.4 \times 10^{-5}$ m³/s

$\therefore$ error in $Q = \sqrt{(2.53 \times 10^{-5})^2 + (1.4 \times 10^{-5})^2} = 2.89 \times 10^{-5}$ m³/s

$\Delta Q = |Q_{spillway} - Q_{gate}| = |1.8 - 1.79| \times 10^{-3} = 1.0 \times 10^{-5}$ m³/s observed

Since $\Delta Q <$ expected error in Q there is good agreement b/t model & experimental data

See blackboard paper on error analysis

General equ.: $\delta q = \sqrt{\left(\left(\frac{\partial q}{\partial x}\right)\delta x\right)^2 + \left(\frac{\partial q}{\partial y}\delta y\right)^2 + \ldots \left(\frac{\partial q}{\partial z}\delta z\right)^2}$

For rectangular channel — Hydraulic Jump model: $y_2 = \frac{y_1}{2}\left(\sqrt{1 + \frac{8q^2}{g}\left(\frac{1}{y_1^3}\right)} - 1\right)$, $q = \frac{Q}{W}$

? Error in y_2: $\delta y^2 = \sqrt{\left(\frac{\partial y_2}{\partial y_1}\delta y_1\right)^2 + \left(\frac{\partial y_2}{\partial q}\delta q\right)^2}$

From $\frac{\partial y_2}{\partial y_1} = \frac{1}{2}\left(\sqrt{1 + \frac{8q^2}{g}\left(\frac{1}{y_1^3}\right)} - \frac{12q^2}{g y_1^3 \sqrt{1 + \frac{8q^2}{g}\left(\frac{1}{y_1^3}\right)}} - 1\right)$

↑ mean value

$q = \frac{1.8 \times 10^{-3} \text{ m}^3/\text{s}}{0.0762 \text{ m}} = 2.6 \times 10^{-2}$ m³/s·m gate flow rate

plug in std. error in y_1: $\frac{\partial y_2}{\partial y_1} = \boxed{-4.1631}$

Hydro Lab Oct. 26 (2)

std. error in q $\left\{ \frac{\partial y_2}{\partial q} = \frac{4q}{g y_1^2 \sqrt{1 + \frac{8q^2}{g y_1^3}}} = 3.9913 \right.$ (Plug into y_2 error equ

Table 4 $\delta Q = 1.4 \times 10^{-5}$ m³/s (gate) $\frac{\delta Q}{W} = \delta q = 1.837(10^{-4})\frac{m^3/s}{m}$ (Plug in)

$\delta y_1 = 0.295(10^{-3})$ (Plug in)

$\therefore \delta y_2 = 1.43 \times 10^{-3}$ model

Effective error in y_2 $\doteq \sqrt{(\delta y_2 \text{ model})^2 + (S_{m_{y_2}})^2}$

↳ std. error in y_2 from Table

$= \sqrt{(1.43 \times 10^{-3})^2 + (0.4 \times 10^{-3})^2} = 1.49 \times 10^{-3}$ m (allowable)

Model: $y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8N_{F_1}^2} - 1\right)$, $\overline{N}_{F_1} = 5.3$ (Table)

$y_2 = \frac{0.01274}{2}\left(\sqrt{1 + 8(5.3)^2} - 1\right)$ $\left\{ \begin{array}{l} y_2 = 0.0957 \text{ (model)} \\ \bar{y}_2 = 0.0791 \text{ (Table)} \end{array} \right.$

$\Delta y_2 = y_2 - \bar{y}_2 = 0.0166$ m (observed)

since $\Delta y_2 >$ Error in y_2, there are discrepancies b/t observations & model

Error in L_j $\Delta L_j = \sqrt{\left(\frac{\partial L_j}{\partial y_1} \delta y_1\right)^2 + \left(\frac{\partial L_j}{\partial Q} \delta Q\right)^2}$