

3-23 (continued)  
Shears, Moments, and Deflections

CONCENTRATED LOADS UNSYMMETRICALLY PLACED

$V_{max}$  when  $a < b$  ..... =  $\frac{P}{l}(l - a + b)$   
 $V_{max}$  when  $a > b$  ..... =  $\frac{P}{l}(l - b + a)$   
 $R_1$  when  $a < x < (l - b)$  ..... =  $\frac{P}{l}(b - a)$   
 $V_{max}$  when  $a > b$  ..... =  $R_1 a$   
 $V_{max}$  when  $a < b$  ..... =  $R_2 b$   
 $V$  when  $x < a$  ..... =  $R_1 x$   
 $V$  when  $a < x < (l - b)$  ..... =  $R_1 x - P(x - a)$

CONCENTRATED LOADS UNSYMMETRICALLY PLACED

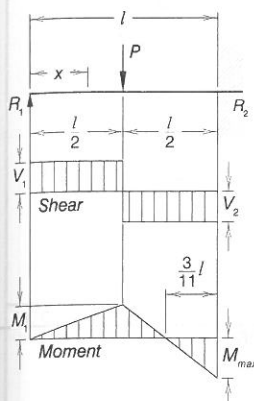
$R_1 = V_1$  ..... =  $\frac{P_1(l - a) + P_2 b}{l}$   
 $R_2 = V_2$  ..... =  $\frac{P_1 a + P_2(l - b)}{l}$   
 $R_1$  when  $a < x < (l - b)$  ..... =  $R_1 - P_1$   
 $V_{max}$  when  $R_1 < P_1$  ..... =  $R_1 a$   
 $V_{max}$  when  $R_2 < P_2$  ..... =  $R_2 b$   
 $V$  when  $x < a$  ..... =  $R_1 x$   
 $V$  when  $a < x < (l - b)$  ..... =  $R_1 x - P_1(x - a)$

CONCENTRATED LOADS UNSYMMETRICALLY PLACED AT OTHER END — UNIFORMLY DISTRIBUTED LOAD

Uniform Load ..... =  $wl$   
 $V_{max}$  ..... =  $\frac{3wl}{8}$   
 $V_{max}$  ..... =  $\frac{5wl}{8}$   
 $R_1$  ..... =  $R_1 - wx$   
 $V$  ..... =  $\frac{wl^2}{8}$   
 $x = \frac{3}{8}l$  ..... =  $\frac{9}{128}wl^2$   
 $V$  ..... =  $R_1 x - \frac{wx^2}{2}$   
 $x = \frac{l}{16}(1 + \sqrt{33}) = 0.422l$  ..... =  $\frac{wl^4}{185EI}$   
 $V$  ..... =  $\frac{wx}{48EI}(l^3 - 3lx^2 + 2x^3)$

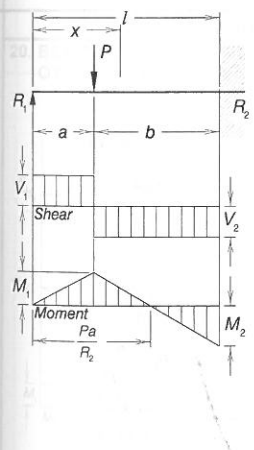
Table 3-23 (continued)  
Shears, Moments, and Deflections

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER — CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load ..... =  $\frac{3P}{2}$   
 $R_1 = V_1$  ..... =  $\frac{5P}{16}$   
 $R_2 = V_2 = V_{max}$  ..... =  $\frac{11P}{16}$   
 $M_{max}$  (at fixed end) ..... =  $\frac{3Pl}{16}$   
 $M_1$  (at point of load) ..... =  $\frac{5Pl}{32}$   
 $M_x$  (at  $x < \frac{l}{2}$ ) ..... =  $\frac{5Px}{16}$   
 $M_x$  (when  $x > \frac{l}{2}$ ) ..... =  $P(\frac{l}{2} - \frac{11x}{16})$   
 $\Delta_{max}$  (at  $x = \frac{l}{\sqrt{5}} = 0.447l$ ) ..... =  $\frac{Pl^3}{48EI\sqrt{5}} = 0.00932 \frac{Pl^3}{EI}$   
 $\Delta_x$  (at point of load) ..... =  $\frac{7Pl^3}{768EI}$   
 $\Delta_x$  (at  $x < \frac{l}{2}$ ) ..... =  $\frac{Px}{96EI}(3l^2 - 5x^2)$   
 $\Delta_x$  (at  $x > \frac{l}{2}$ ) ..... =  $\frac{P}{96EI}(x - l)^2(11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT THE OTHER — CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$  ..... =  $\frac{Pb^2}{2l^3}(a + 2l)$   
 $R_2 = V_2$  ..... =  $\frac{Pa}{2l^3}(3l^2 - a^2)$   
 $M_1$  (at point of load) ..... =  $R_1 a$   
 $M_2$  (at fixed end) ..... =  $\frac{Pab}{2l^2}(a + l)$   
 $M_x$  (at  $x < a$ ) ..... =  $R_1 x$   
 $M_x$  (when  $x > a$ ) ..... =  $R_1 x - P(x - a)$   
 $\Delta_{max}$  (when  $a < 0.414l$  at  $x = l \frac{(l^2 + a^2)}{(3l^2 - a^2)})$  ..... =  $\frac{Pa}{3EI} \frac{(l^2 - a^2)^3}{(3l^2 - a^2)^2}$   
 $\Delta_{max}$  (when  $a > 0.414l$  at  $x = l \frac{a}{2l + a}$ ) ..... =  $\frac{Pab^2}{6EI} \frac{a}{\sqrt{2l + a}}$   
 $\Delta_a$  (at point of load) ..... =  $\frac{Pa^2 b^3}{12EI l^3} (3l + a)$   
 $\Delta_x$  (when  $x < a$ ) ..... =  $\frac{Pb^2 x}{12EI l^3} (3a^2 - 2lx^2 - ax^2)$   
 $\Delta_x$  (when  $x > a$ ) ..... =  $\frac{Pa}{12EI l^3} (l - x)^2 (3l^2 x - a^2 x - 2a^2 l)$