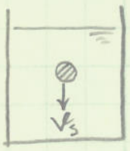
Settling

- Discrete particle settling are those that maintain there physical shape while settling and also settle independently from neighbors.



d_p = part. diam.
A_p = part cross sectional area
V_p = part. vol.
v_s = part. settling

ρ_S = solid dens.
ρ_L = liq. dens.

Force balance @ equilibrium $0 = F_W - F_B - F_D$

F_W = weight = $\forall \rho_s g$
F_B = Buoy = $\rho_L \cdot V_p \cdot g$
$F_D = \frac{C_D A_p \rho_L v_s^2}{2}$

Solve for $v_s = \sqrt{\frac{2g(\rho_s - \rho_L)}{C_D \rho_L} \cdot \left(\frac{V_p}{A_p}\right)}$

$$\frac{V_p}{A_p} = \frac{\frac{1}{6}\pi d_p^3}{\frac{1}{4}\pi d_p^2} = \frac{2}{3} d_p$$

Newtons Law
$$\boxed{v_s = \sqrt{\frac{4g}{3C_D}\left(\frac{\rho_s - \rho_L}{\rho_L}\right) d_p}}$$

C_D = function of particle R_e

$$R_e = \frac{d_p \cdot v_s}{\nu}$$
$\frac{\mu}{\rho} \nearrow$

$$C_D = \frac{24}{R_e} + \frac{3}{\sqrt{R_e}} + 0.34$$

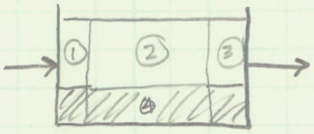
If $R_e < 1$ (Laminar Flow), $C_D = \frac{24}{R_e}$, $v_s = \frac{g}{18.75\mu}(\rho_s - \rho_L) d_p^2$ (Stokes Law)

$R_e > 2000$ (Turb Flow), $C_D \approx 0.4$; $v_s = \sqrt{3.3 g d_p (S_g - 1)}$

(Transition Zone): $C_D = \frac{24}{R_e} + \frac{3}{\sqrt{R_e}} + 0.34$; $v_s = \left[\frac{\frac{4}{3} g (S_s - 1) d_p}{\frac{24\nu}{d_p \cdot v_s} + 3\sqrt{\frac{\nu}{d_p \cdot v_s}} + 0.34}\right]^{1/2}$

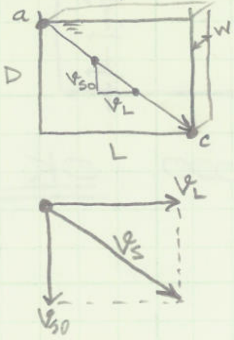
Trial & Error (GOAL SEEK)
- Assume v_s
- Calc. $R_e = \frac{d_p v_s}{\nu}$
- calc. C_D
- Calc v_s assumed − v_s calc.
- If $\Delta v_s \neq 0$, Try again.

Ideal Settling Tank of Discrete particles



① Inlet Zone: all particles uniformly distrib. along entire depth
② Settling Zone: sedimentation takes place
③ Outlet Zone: no sedimentation occurs, no particles carry over
④ Sludge Zone:

Settling Zone:



Consider crit. particle (smallest we want to remove) with settling vel. v_{s0}=

For particle to be removed it has to follow trajectory a-c.

Similar triangles: $\frac{v_L}{L} = \frac{v_{s0}}{D} \Rightarrow v_{s0} = \frac{V_L D}{L} \Rightarrow \frac{V_L D}{L} \cdot \frac{W}{W}$

Design overflow rate $\frac{m^3/h}{m^2}$, $\frac{gpm}{ft^2}$ $\quad v_{s0} = \frac{Q}{A}$

$$v_{s0} = \frac{Q}{A} \cdot \frac{D}{D} = \frac{QD}{\forall} = \frac{D}{\forall/Q} = \frac{D}{t} \Rightarrow \boxed{v_{s0} = \frac{D}{t}}$$

$L = \frac{v_L \cdot D}{v_{s0}}$, ideal settling zone

$L_T = \underset{\simeq 1.5}{K} \cdot \frac{V_L D}{v_{s0}}$, total tank length, K take into acc. inlet and outlet

---

Ex. Design a grit chamber, $Q = 4.4 \frac{m^3}{s}$, min sand particle = 0.1 mm = 0.01 cm ①
$T = 10°c$ $\quad \mu = 1.3077 \times 10^{-3} \frac{kg}{m \cdot s}$; $\nu = 1.371 \times 10^{-6} \frac{m^2}{s}$; $S_s = 2.5$ (sand)

Additional criteria for horz. settling tanks:

$$\left.\begin{aligned} \text{Tank } (Re)_t &= \frac{v_L \cdot R}{\nu} < 20000 \\ F_r &= \frac{v_L^2}{gR} > 10^{-5} \end{aligned}\right\}$$

v_L = longitudinal velocity
$R = \frac{A_x}{P} = \frac{\text{cross sectional area}}{\text{wetted perimeter}}$
ν = kinematic viscosity

Solution:

① Find settling velocity, v_{s0}. Use the graph Fig 25-2 as first approx

$v_{s0} = 0.6 \frac{cm}{s}$ $\quad Re = \frac{d_p \cdot v_s}{\nu} = \frac{10^{-4}\,m \times (6 \times 10^{-3})\frac{m}{s}}{1.371 \times 10^{-6}\,m^2/s} = 0.44 < 1$ (laminar)

∴ Use stokes Law: $v_s = \frac{g}{18.75\mu}(\rho_s - \rho_L) d_p^2$

$$= \frac{9.81\,(2500 - 1000)\frac{kg}{m^3} \times (1 \times 10^{-4}\,m)^2}{18.75\,(1.3077\,kg/m \cdot s)} = 6 \times 10^{-3}\ m/s$$

$v_{s0} = 6 \times 10^{-3} \frac{m}{s} \times 3600 \frac{s}{h}$; $v_{s0} = 21.6 \frac{m}{h}$ (between 10 and 25) OK

② Select two rectangular tanks, each handling $Q = 2.2 \frac{m}{s}$

$L_T = \frac{v_L \cdot D}{v_{s0}}$; $v_L = 3 \frac{m}{min}$, $D = 3.7m$

$L_T = \frac{0.05 \frac{m}{s} \times 3.7\,m}{6 \times 10^{-3} \frac{m}{s}} \times 1.5 = 46.25\,m \simeq 46m \Rightarrow$ $L_T = 46\,m$

$A = \frac{Q}{v_{s0}} = \frac{2.2\,m^3/s}{6 \times 10^{-3}\,m/s} = 366.7\,m^2$ $(L \times W)$ $\quad W_T = \frac{366.7}{46} = 8\,m$

Check $v_L \Rightarrow v_L = \frac{Q}{A_x} = \frac{2.2 \frac{m^3}{s}}{(8 \times 3.7)m^2} = 0.074 \frac{m}{s} = 4.45 \frac{m}{min}$

Check $\frac{L}{W} < 4 \Rightarrow 5.75 < 4$ OK $\quad$ Check $\frac{L}{D} > 6 \Rightarrow 12.4 > 6$ OK

Back

Check $Re_T = \frac{v_L}{\nu} \cdot \frac{A_x}{P} = \frac{0.0074\,m/s}{1.371 \times 10^{-6} \frac{m^2}{s}} \times \frac{8 \times 3.7}{2(3.7) + 8} = 103{,}744 > 86000$ Not OK Too high

Assignment 8 Pipe Friction NAME ______________________________

Due: 1 week

8.1. Compare the head losses in a 5000-ft long pipe line with an inside diameter of 12 inches, a flow of 6.0 cfs and the following conditions:

a) a smooth wall
b) a wall with a roughness height of 10 mm.

Use the Moody Diagram to estimate the friction factor.

Ans:

a) f = ________________

hL=________________

b) f = ________________

hL=________________

Solution

8.2. Determine the Manning's *n* that produces the same head losses as in cases a) and b) in Problem 1.

a) n = ____________________

b) n = ____________________

Solution

Lecture 8

Review of Dimensional Analysis with Example of the Friction Equation

Dimensional Analysis is a commonly used method of organizing experimental variables to:

a) help plan the experiments and
b) obtain more general dimensionally homogenous equations.

The method is based on the Buckingham Pi Theorem that can be summarized as follows:

If there are n dimensional variables with m different units, then it is possible to reduce the set of variables to (n-m) dimensionless or Π_i *variables.*

This is accomplished by:

1) Listing a set of possible dimensional variables and their units, that we think could affect the experimental result, in the form

$$f(x_1, x_2, x_3, \ldots x_m \ldots x_n) = 0$$

2) Selecting m repeating variables $x_1, x_2, x_3, \ldots x_m$ which amongst them contain all of the m units, L, m, time, θ

(Pi variables)

3) Setting up (n-m) dimensionless Π_i variables in the form,

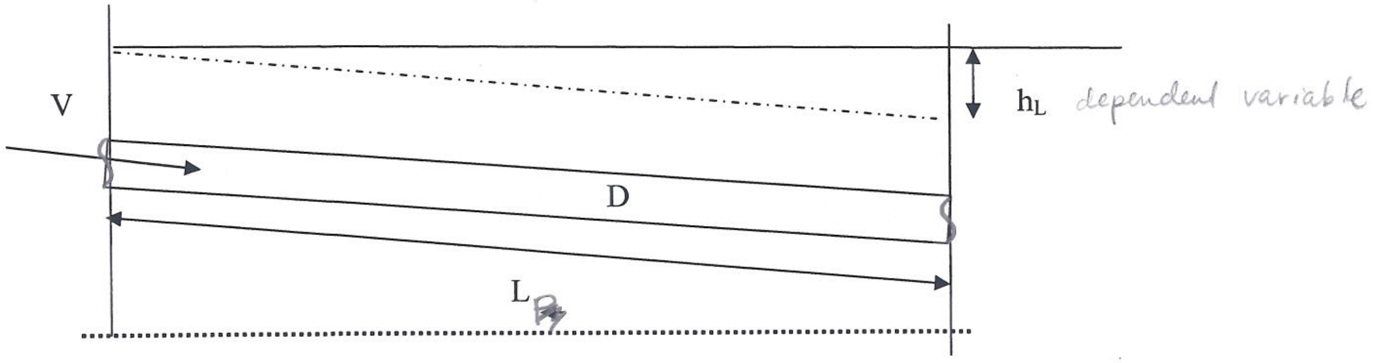
variable we are interested in

$$\Pi_i = x_1^a\, x_2^b\, x_3^c \ldots x_m^d\, x_i \qquad i = 1 \text{ to } (n-m)$$

4) Solving for all (n-m) values of *a, b, c ..d* using the fact that the sum of the exponents on each of the units, e.g. L, M, T, ... must be zero in each of the Π_i variables, i.e. $\{L^0, M^0, T^0, \ldots\}$.

eqn's to solve for a, b, c

Example: Consider the energy loss in the pipe shown below.



L, M, or T

1. List possible variables and their units

	Variable	Symbol	Units	
x_1	**Diameter**	**D**	**L**	1
x_2	**Fluid Density**	ρ	M/L^3	2
x_3	**Mean Velocity**	**V**	**L/T**	3
	Pipe Length	L_p	**L**	4
	Roughness height	ε	**L**	5
	Pipe Shape	**P/D**	-	6
	Gravity	**g**	L/T^2	7
	Viscosity	ν	L^2/T	8
	Surface tension	σ	$F/L = M/T^2$	9
	Elasticity or Bulk Modulus	**K**	$F/L^2 = M/(LT^2)$	10
	Head loss {the dependant variable}	$h_L = -\Delta(p/\gamma + h_z)$	**L**	11

$$f(\overbrace{D,\ \rho,\ V,}\ L,\ \varepsilon,\ P/D,\ g,\ \nu,\ \sigma,\ K,\ h_L) = 0$$

$n = 11$ variables

$m = 3$ units

No. of Π_i Variables $= n - m = 11 - 3 = 8$

2. **Select m = 3 repeating variables, e.g. D, ρ, V which together contain M, L an T units.**

check this

3. **Set up n-m Π_i variables as**

$$\Pi_1 = D^{a1}\ \rho^{,b1}\ V^{c1}\ L_p = \frac{L_p}{D}$$
$$\Pi_2 = D^{a2}\ \rho^{,b2}\ V^{c2}\ \varepsilon$$
$$\Pi_3 = D^{a3}\ \rho^{,b3}\ V^{c3}\ P/D$$
$$\Pi_4 = D^{a4}\ \rho^{,b4}\ V^{c4}\ g$$
$$\Pi_5 = D^{a5}\ \rho^{,b5}\ V^{c5}\ \nu$$
$$\Pi_6 = D^{a6}\ \rho^{,b6}\ V^{c6}\ \sigma$$
$$\Pi_7 = D^{a7}\ \rho^{,b7}\ V^{c7}\ K$$
$$\Pi_8 = D^{a8}\ \rho^{,b8}\ V^{c8}\ h_L$$

L: $a_1 - 3b_i + c_1 + 1 = 0$

M: $b = 0$

T: $-c_1 = 0$

4. Solve for all a, b, c exponents, e.g. $\Pi_5 = D^{a5}\ \rho'^{b5}\ V^{c5}\ \nu$ gives:

L^0, $a_5 - 3\,b_5 \quad + c_5 + 2 \;=\; 0$

M^0, $\qquad b_5 \qquad\qquad = \; 0$

T^0, $\qquad\qquad - c_5 - 1 \;=\; 0$

therefore we have $a_5 = -1 \;;\; b_5 = \; 0;\; c_5 = -1$

so that

$$\Pi_6 = D^{-1}\ V^{-1}\ \nu$$

Without effecting the generality of the result, we can rise any dimensionless variable to any exponent and it still retains its dimensionless property; so we can take the reciprocal which give the familiar Reynolds Number,

$$N_R = D\ V / \nu$$

Similarly the other dimensionless variables can be expressed as follows:

$N_F = V/\{gD\}^{1/2}$ **= Froude No.** dont use in full pipe

→ $N_E = h_L/\{V^2/2g\}$ **= Euler No.**

$N_W = \rho D V^2/\sigma$ **= Weber No.** Surface tension

$N_M = V/\{K/\rho\}^{1/2}$ **= Mach No.** speed of sound in pipe

e/D = Relative Roughness

P/D = Shape factor

L/D = Length factor.

5. Finally a new function can be set up as,

$$F_a\,(N_E\,,\ N_R\,,\ N_F,\ N_M,\ N_W\,, \varepsilon/D,\ P/D,\ L/D\,) = 0$$

Taking out N_E as the dependent variable we get

$$N_E = F_b\,(N_R\,,\ N_F,\ N_M,\ N_W\,, \varepsilon/D,\ P/D,\ L/D\,)$$

This is the new equation for the experimental testing.

6. Experiments and theoretical studies can be used to find the actual form of the above dimensionless function.

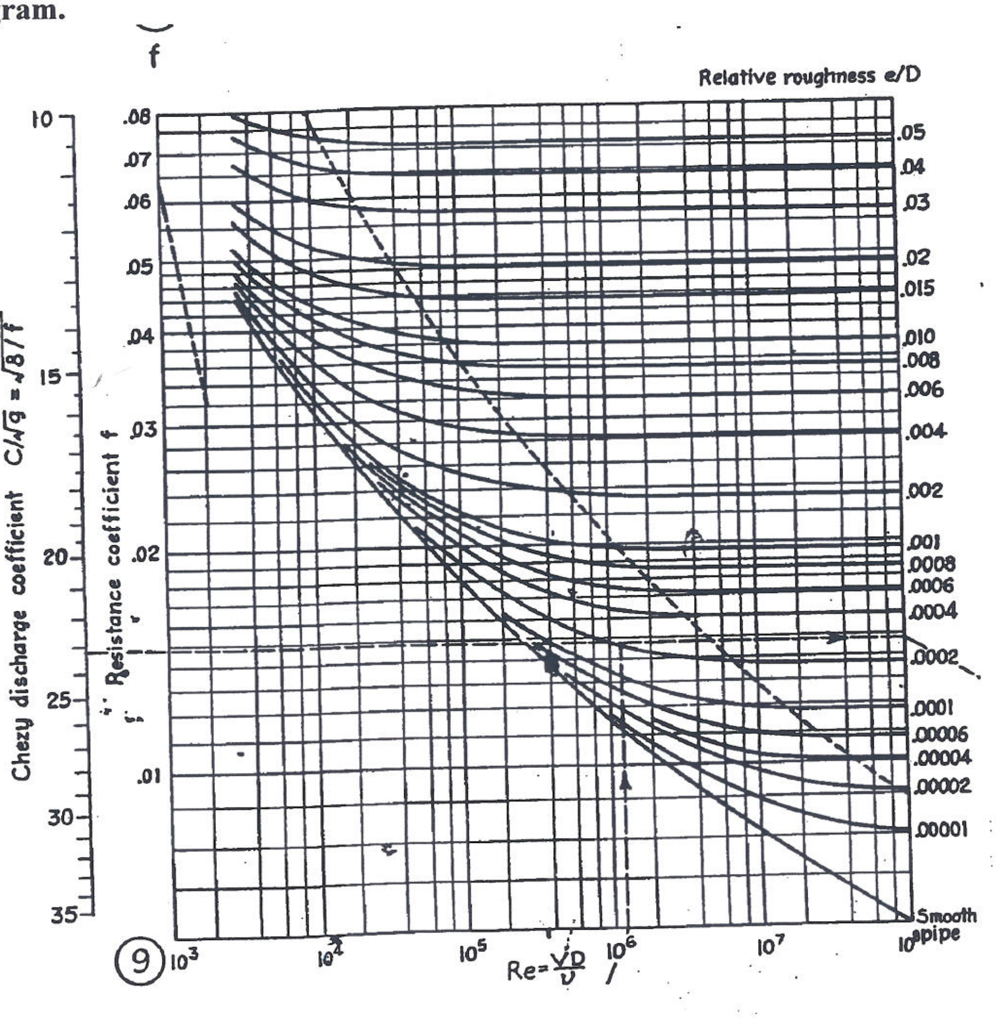
e.g. i) Experiments show that h_L is proportional to L;
ii) Gravity is included in N_E ;
iii) If there is no air/water surface N_W , has no effect;
iv) Also if only circular pipes are used P/D is constant;

This leaves

$$N_E = h_L/\{V^2/2g\} = \{L/D\}\, f_c\, (N_R\, ,\ \varepsilon/D)$$

$$h_L = f_c\, (N_R\, ,\ \varepsilon/D)\, L\, V^2/\{2gD\}$$

which is the Darcy friction equation. The function f_c (N_R , ε/D) is given in the Moody's diagram.



There are several other friction equations that are used in pipe flow, e.g.

Hazen-Williams

The head loss formulas we have presented up to now are general because they are applicable for any fluid and any system of units. Other more restrictive empirical equations are also useful for their limited range of application. The most notable one, used for decades by waterworks engineers in the United States, is the Hazen-Williams formula. In English units, the formula is given in Eq. (5-12):

$$V = 1.318 C_h R^{0.63} S^{0.54} \tag{5-12}$$

where V = mean velocity in ft/s
C_h = Hazen-Williams friction coefficient (depends on pipe roughness)
R = hydraulic radius in ft
$S = h_f/L$ (slope of energy grade line)

To solve for head loss using the Hazen-Williams equation, a little algebraic manipulation of Eq. (5-12) yields

$$h_f = 3.02 L D^{-1.167} \left(\frac{V}{C_h}\right)^{1.85} \tag{5-13}$$

The resistance coefficient C_h depends on the surface characteristics of the pipe

Table 5-2 Hazen-Williams C_h Values for Different Kinds of Pipe (5)

Character of Pipe	C_h
New or in excellent condition cast-iron and steel pipe with cement or bituminous linings centrifugally applied, concrete pipe centrifugally spun, cement-asbestos pipe, copper tubing, brass pipe, plastic pipe, and glass pipe	140
Older pipe listed above in good condition, and cement mortar-lined pipes in place with good workmanship, larger than 24 in. in diameter	130
Cement mortar-lined pipe in place, small diameter with good workmanship or large diameter with ordinary workmanship; wood stave; tar dipped cast-iron pipe new or old in inactive water	120
Old unlined or tar-dipped cast-iron pipe in good condition	100
Old cast-iron pipe severely tuberculated, or any pipe with heavy deposits	10–80

Manning's Equation

Civil Engineers commonly use the Manning's Equation to computer friction losses in open channels and storm sewers. This equation relates the velocity V to the friction slope (S_f) and the section geometry by: Slope

$$V = c' R^{2/3} \overset{\downarrow}{S_f}^{1/2} / n$$

where c' is a conversion factor =1 in SI units and 1.486 in US units; n is the Manning's roughness coefficient and R = hydraulic radius of the section.

Now the head loss due to friction is

$$h_f = S_f L = L \left(\frac{nV}{c' R^{2/3}} \right)^2$$

Recall that Q = V A

Therefore a common form of the Manning's Equation is

$$Q = c' A R^{2/3} S_f^{1/2} / n$$

Typical n values are:

Smooth Concrete 0.013
Rough (old) Concrete 0.015
CMP 0.024
Mississippi River ~ 0.025
Grass lined channels ~ 0.03
Natural Rivers 0.02 to 0.04

Strickler Equation gives an approximate relation between the grain roughness (D50 = median grain size) and n:

$$n = 0.034\ (D50\ ft)^{1/6}$$

Lecture 9

Review of Minor Losses in Pipes

Minor losses are due to non-uniform flow in pipes. These result from the production of turbulent eddies at pipe expansions, contractions, obstructions, valves, orifices, and bends.

The head loss is generally represented in the form,

$$h_L = K_L V^2/2g \qquad 9.1$$

where h_L = loss in energy head through the local transition,
K_L = loss coefficient;
V = characteristic velocity at the transition, e.g. in general the highest velocity is used.

Common cases:

Abrupt Flow expansion

Let V_1 = upstream velocity; V_2 = downstream velocity. Application of the continuity, momentum and energy principles leads to

$$h_L = K_L V_1^2/2g$$

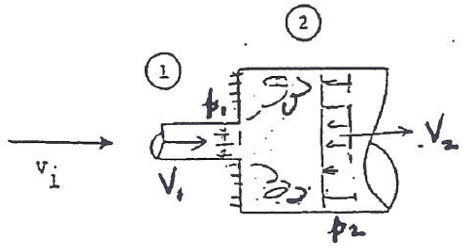
9.2

and

$$h_L = (V_1 - V_2)^2/2g$$

$$h_L = (1 - A_1/A_2)^2 V_1^2/2g$$

$$K_L = (1 - A_1/A_2)^2$$



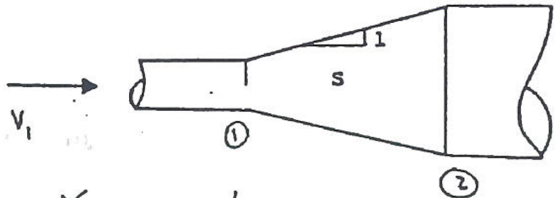
As $A_1/A_2 \to 0$ then $K_L \to 1$.

Gradual Expansions (diffuser cone)

highest value of V

$$h_L = K_L \, V_1^2/2g$$

$$K_L = \{1/8 + 7(20 - s)/136\} \, (1 - A_1/A_2)^2$$



$K_{L_{min}} \simeq \frac{1}{8}$

Contractions **-Flush Abrupt**

In this case it is best to use $h_L = V_2$ instead of V_1 for the references V.

$$h_L = K_L \, V_2^2/2g$$

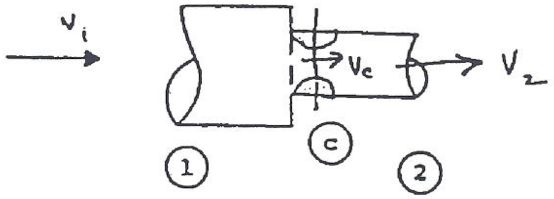
$$K_L = (A_2/A_c - 1)^2 = (1/C_c - 1)^2$$

In the limit of a large pipe reducing to a very small pipe we have

$C_c \sim 0.61$ and $K_L \sim 0.4$

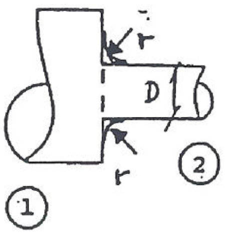
Rouse ("Engineering Hydraulics") gives the following

A_2/A_1	K_L
0	0.5
0.2	0.45
0.4	0.36
0.6	0.21
0.8	0.07
1.0	0

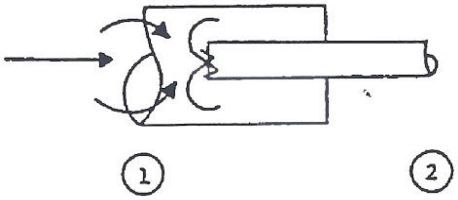


For a smooth transition (see sketch $r/D \geq 0.14$)

$K_L \doteq 0.05$ or less



Re-entrant Contractions (Borda)



$C_c = 0.5$ and $K_L = 1.0$

Bends Bend Losses: Morris (Text) suggests a loss coefficient of

$$K_L = \frac{\theta^o}{90}[280 f^3(21.8 + 43.7\frac{D}{r}) + \frac{1}{5\sqrt{r/D}}]$$

where r = bend radius; D = diameter of the pipe; f = Darcy friction factor; θ = angle of bend.

Example:

Determine the flow through the culvert shown below.
Assume that there is no overtopping of the road. Also assume that the downstream cross-sectional area is large compared to the pipe area.